Introduction to Aircraft Control System

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Week – 12

Lecture – 58

Transfer Function for Longitudinal Approximations (Cont.)

In the last lecture, we have discussed about the transfer functions for the short period dynamics in the longitudinal motion. In today's lecture, we will be discussing the phugoid dynamics and how we can find the transfer function for the Phugoid dynamics. First, let us rewrite the natural dynamics or the Phugoid approximation.

$$\begin{bmatrix}\Delta\dot{u}\\ \Delta\dot{\theta}\end{bmatrix} = \begin{bmatrix} X_u & -g \\ \dfrac{-Z_u}{u_0} & 0 \end{bmatrix}\begin{bmatrix}\Delta u\\ \Delta\theta\end{bmatrix} \dots\dots Eq(1)$$

From the linearized equation of motion of the longitudinal dynamics $\Delta\dot{w}$ can be expressed as

$$\Delta\dot{w} = Z_u\Delta u + Z_w\Delta w + u_0\Delta q - g\sin\theta_0\Delta\theta + Z_{\delta e}\Delta\delta_e + +Z_{\delta t}\Delta\delta_t \dots\dots Eq(2)$$

Again, for the phugoid approximation

$$\Delta\alpha = \frac{\Delta w}{u_0} \Rightarrow \Delta\alpha \approx 0$$

$$\Delta w = 0 \Rightarrow \Delta\dot{w} = 0$$

If we consider, trim condition for pitch angle $\theta_0 = 0$, then Eq. (2) can be reduced to

$$\Delta q = \Delta\dot{\theta} = \frac{-Z_u}{u_0}\Delta u - \frac{Z_{\delta e}}{u_0}\Delta\delta_e - \frac{Z_{\delta t}}{u_0}\Delta\delta_t$$

Eq. (1) with control input can be written as

$$\begin{bmatrix}\Delta\dot{u}\\ \Delta\dot{\theta}\end{bmatrix} = \begin{bmatrix} X_u & -g \\ \dfrac{-Z_u}{u_0} & 0 \end{bmatrix}\begin{bmatrix}\Delta u\\ \Delta\theta\end{bmatrix} + \begin{bmatrix} X_{\delta e} & X_{\delta t} \\ -\dfrac{Z_{\delta e}}{u_0} & -\dfrac{Z_{\delta t}}{u_0} \end{bmatrix}\begin{bmatrix}\Delta\delta_e\\ \Delta\delta_t\end{bmatrix} \dots\dots Eq(3)$$

From Eq. (3), $\Delta\dot{u}$ can be written as

$$\Delta\dot{u} = X_u\Delta u - g\Delta\theta + X_{\delta e}\Delta\delta_e + X_{\delta t}\Delta\delta_t \dots\dots Eq(4)$$

Applying Laplace transform with zero initial conditions, Eq. (4) yields

$$(s - X_u)\Delta u(s) + g\Delta\theta(s) = X_{\delta e}\Delta\delta_e(s) + X_{\delta t}\Delta\delta_t(s)$$

Set $\Delta\delta_t(s) = 0$ and divide by $\Delta\delta_e(s)$ on both sides of above equation

$$(s - X_u)\frac{\Delta u(s)}{\Delta\delta_e(s)} + g\frac{\Delta\theta(s)}{\Delta\delta_e(s)} = X_{\delta e} \dots\dots Eq(5)$$

From Eq. (3), $\Delta\dot{\theta}$ can be written as

$$\Delta\dot{\theta} = \frac{-Z_u}{u_0}\Delta u - \frac{Z_{\delta e}}{u_0}\Delta\delta_e - \frac{Z_{\delta t}}{u_0}\Delta\delta_t$$

Following the similar procedure, we get

$$\frac{Z_u}{u_0}\frac{\Delta u(s)}{\Delta\delta_e(s)} + s\frac{\Delta\theta(s)}{\Delta\delta_e(s)} = -\frac{Z_{\delta e}}{u_0} \dots\dots Eq(6)$$

Using Eqs. (5) and (6)

$$\begin{bmatrix} s - X_u & g \\ \frac{Z_u}{u_0} & s \end{bmatrix}\begin{bmatrix} \frac{\Delta u(s)}{\Delta\delta_e(s)} \\ \frac{\Delta\theta(s)}{\Delta\delta_e(s)} \end{bmatrix} = \begin{bmatrix} X_{\delta e} \\ -\frac{Z_{\delta e}}{u_0} \end{bmatrix}$$

This equation is in $AX = B$ form. Now we can apply Cramers rule or brute force method

$$|A| = s^2 - X_u s - g\frac{Z_u}{u_0}$$

Using Cramers rule,

$$|A_1[B]| = \begin{vmatrix} X_{\delta e} & g \\ -\frac{Z_{\delta e}}{u_0} & s \end{vmatrix} = X_{\delta e}s + g\frac{Z_{\delta e}}{u_0}$$

$$x_1 = \frac{\Delta u(s)}{\Delta\delta_e(s)} = \frac{X_{\delta e}s + g\frac{Z_{\delta e}}{u_0}}{s^2 - X_u s - g\frac{Z_u}{u_0}} = \frac{|A_1[B]|}{|A|}$$

$$|A_2[B]| = \begin{bmatrix} s - X_u & X_{\delta e} \\ \dfrac{Z_u}{u_0} & -\dfrac{Z_{\delta e}}{u_0} \end{bmatrix}$$

$$= -\frac{Z_{\delta e}}{u_0} s + X_u \frac{Z_{\delta e}}{u_0} - X_{\delta e} \frac{Z_u}{u_0}$$

$$x_2 = \frac{\Delta\theta(s)}{\Delta\delta_e(s)} = \frac{|A_2[B]|}{|A|} = \frac{-\dfrac{Z_{\delta e}}{u_0} s + X_u \dfrac{Z_{\delta e}}{u_0} - X_{\delta e} \dfrac{Z_u}{u_0}}{s^2 - X_u s - g \dfrac{Z_u}{u_0}}$$

So, once we have the transfer function we can apply the conventional method of classical control method for designing the control algorithm. The same way also we can do that for the rest of the state space form representation what you have derived before for the lateral directional motion.

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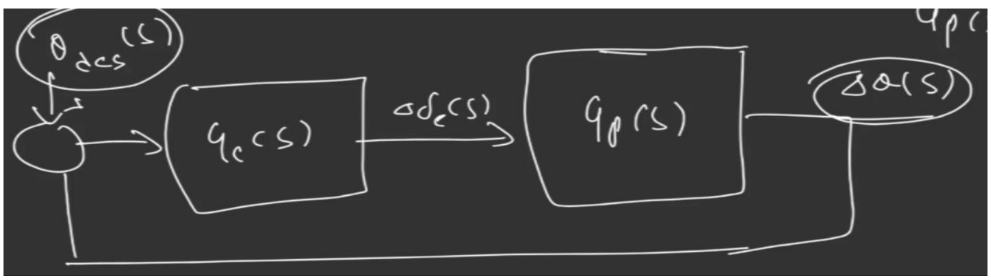

So, we can apply this concept to come up with the transfer function and if you are having transfer function we can design the control algorithm. So, here we are having $\Delta\theta(s)$, this is the controller which is giving the control input to track θ_{des}.

So, this is how we can design the closed loop control system what we have discussed in the first half of in this course there are a lot of techniques we have considered how can come up with PID controls using different concept. So, in the next two lectures we will be discussing how we can come up with the controls as well as transfer functions together. Thank you.