Introduction to Aircraft Control System

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Week – 12

Lecture – 57

Transfer Function for Longitudinal Approximations

Now, the concept that we have learned from the last few lectures, we will apply them to the aircraft system, how we can find the transfer function for the different approximations, longitudinal motions or in the lateral direction motion, we will study them in this lecture. The main motivation of having the transfer function for the aircraft systems are to apply the linear control or classical control theory to the system because this course mainly we are studying the classical control concepts. So, let us first start with the short period dynamics. The short period approximations with control held fixed means without the control the natural dynamics for the short period motion we had

$$\begin{bmatrix}\Delta\dot{\alpha}\\ \Delta\dot{q}\end{bmatrix} = \begin{bmatrix}\frac{Z_\alpha}{u_0} & 1\\ M_\alpha + M_{\dot{\alpha}}\frac{Z_\alpha}{u_0} & M_q + M_{\dot{\alpha}}\end{bmatrix}\begin{bmatrix}\Delta\alpha\\ \Delta q\end{bmatrix} \ldots\ldots Eq(1)$$

Now if the equation in the system is affected by control input from the elevator then we can modify this equation in state space form as

$$\begin{bmatrix}\Delta\dot{\alpha}\\ \Delta\dot{q}\end{bmatrix} = \begin{bmatrix}\frac{Z_\alpha}{u_0} & 1\\ M_\alpha + M_{\dot{\alpha}}\frac{Z_\alpha}{u_0} & M_q + M_{\dot{\alpha}}\end{bmatrix}\begin{bmatrix}\Delta\alpha\\ \Delta q\end{bmatrix} + \begin{bmatrix}Z_{\delta e}\\ M_{\delta e} + \frac{M_{\dot{\alpha}}Z_{\delta e}}{u_0}\end{bmatrix}\Delta\delta_e \ldots\ldots Eq(2)$$

Control due to propulsion system is neglected for the simplicity. If you notice, this is in $\dot{X} = AX + BU$ form For the system defined by Eq. (2), we have the following ODEs.

$$\Delta\dot{\alpha} = \frac{Z_\alpha}{u_0}\Delta\alpha + \Delta q + Z_{\delta e}\Delta\delta_e \ldots\ldots Eq(3)$$

$$\Delta\dot{q} = \left(M_\alpha + M_{\dot{\alpha}}\frac{Z_\alpha}{u_0}\right)\Delta\alpha + \left(M_q + M_{\dot{\alpha}}\right)\Delta q + \left(M_{\delta e} + \frac{M_{\dot{\alpha}} Z_{\delta e}}{u_0}\right)\Delta\delta_e \ldots\ldots Eq(4)$$

Taking Laplace transform of Eqs. (3) and (4), we get

$$\left(s - \frac{Z_\alpha}{u_0}\right)\Delta\alpha(s) - \Delta q(s) = \frac{Z_{\delta e}}{u_0}\Delta\delta_e(s) \ldots\ldots Eq(5)$$

$$-\left[M_\alpha + M_{\dot{\alpha}}\frac{Z_\alpha}{u_0}\right]\Delta\alpha(s) + \left[s - \left(M_q + M_{\dot{\alpha}}\right)\right]\Delta q(s) =$$

$$\left[M_{\delta e} + \frac{M_{\dot{\alpha}} Z_{\delta e}}{u_0}\right]\Delta\delta_e(s) \ldots\ldots Eq(6)$$

Dividing Eqs. (5) and (6) by $\Delta\delta_e(s)$

$$\left(s - \frac{Z_\alpha}{u_0}\right)\frac{\Delta\alpha(s)}{\Delta\delta_e(s)} - \frac{\Delta q(s)}{\Delta\delta_e(s)} = \frac{Z_{\delta e}}{u_0} \ldots\ldots \mathrm{Eq}(7)$$

$$-\left[M_\alpha + M_{\dot{\alpha}}\frac{Z_\alpha}{u_0}\right]\frac{\Delta\alpha(s)}{\Delta\delta_e(s)} + \left[s - \left(M_q + M_{\dot{\alpha}}\right)\right]\frac{\Delta q(s)}{\Delta\delta_e(s)} =$$

$$\left[M_{\delta e} + \frac{M_{\dot{\alpha}} Z_{\delta e}}{u_0}\right] \ldots\ldots \mathrm{Eq}(8)$$

We obtain set of algebraic equations in terms of transfer functions $\frac{\Delta\alpha(s)}{\Delta\delta_e(s)}$ and $\frac{\Delta q(s)}{\Delta\delta_e(s)}$. In matrix form we can write

$$\begin{bmatrix} s - \frac{Z_\alpha}{u_0} & -1 \\ -\left[M_\alpha + M_{\dot{\alpha}}\frac{Z_\alpha}{u_0}\right] & s - \left(M_q + M_{\dot{\alpha}}\right) \end{bmatrix} \begin{bmatrix} \frac{\Delta\alpha(s)}{\Delta\delta_e(s)} \\ \frac{\Delta q(s)}{\Delta\delta_e(s)} \end{bmatrix} = \begin{bmatrix} \frac{Z_{\delta e}}{u_0} \\ M_{\delta e} + \frac{M_{\dot{\alpha}} Z_{\delta e}}{u_0} \end{bmatrix}$$

Hence, we can use Cramer's rule to find $\frac{\Delta\alpha(s)}{\Delta\delta_e(s)}$ or x_1 and $\frac{\Delta q(s)}{\Delta\delta_e(s)}$ or x_2.

$$x_1 = \frac{|A_1[B]|}{|A|}, \quad x_2 = \frac{|A_2[B]|}{|A|}$$

$$|A| = \begin{vmatrix} s - \frac{Z_\alpha}{u_0} & -1 \\ -\left[M_\alpha + M_{\dot{\alpha}}\frac{Z_\alpha}{u_0}\right] & s - \left(M_q + M_{\dot{\alpha}}\right) \end{vmatrix}$$

$$= s^2 - \left[M_q + M_{\dot{\alpha}} + \frac{Z_\alpha}{u_0}\right] s + \frac{Z_\alpha}{u_0} M_q - M_\alpha$$

$$|A_1[B]| = \begin{vmatrix} \frac{Z_{\delta e}}{u_0} & -1 \\ M_{\delta e} + \frac{M_{\dot{\alpha}} Z_{\delta e}}{u_0} & s - (M_q + M_{\dot{\alpha}}) \end{vmatrix}$$

$$= \frac{Z_{\delta e}}{u_0} s + M_{\delta e} - M_q \frac{Z_{\delta e}}{u_0}$$

Similarly

$$|A_2[B]| = \begin{vmatrix} s - \frac{Z_\alpha}{u_0} & \frac{Z_{\delta e}}{u_0} \\ -\left[M_\alpha + M_{\dot{\alpha}} \frac{Z_\alpha}{u_0}\right] & M_{\delta e} + \frac{M_{\dot{\alpha}} Z_{\delta e}}{u_0} \end{vmatrix}$$

$$= \left[M_{\delta e} + \frac{M_{\dot{\alpha}} Z_{\delta e}}{u_0}\right] s + \frac{Z_{\delta e}}{u_0} M_\alpha - \frac{Z_\alpha}{u_0} M_{\delta e}$$

Using the above formulas, we get

$$x_1 = \frac{\Delta\alpha(s)}{\Delta\delta_e(s)} = \frac{Z_{\delta e} s + M_{\delta e} - M_q \frac{Z_{\delta e}}{u_0}}{s^2 - \left[M_q + M_{\dot{\alpha}} + \frac{Z_\alpha}{u_0}\right] s + \frac{Z_\alpha}{u_0} M_q - M_\alpha}$$

$$x_2 = \frac{\Delta q(s)}{\Delta\delta_e(s)} = \frac{\left[M_{\delta e} + \frac{M_{\dot{\alpha}} Z_{\delta e}}{u_0}\right] s + \frac{Z_{\delta e}}{u_0} M_\alpha - \frac{Z_\alpha}{u_0} M_{\delta e}}{s^2 - \left[M_q + M_{\dot{\alpha}} + \frac{Z_\alpha}{u_0}\right] s + \frac{Z_\alpha}{u_0} M_q - M_\alpha}$$

So, if you notice here the stability derivatives we can find constant for a particular flight regime. So, if you are having this transfer function for both the system we can design the control.

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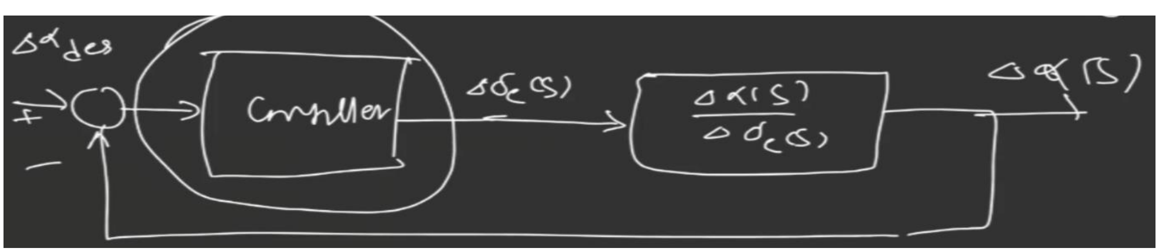

So, for example if you want to control $\Delta\alpha(s)$, the whole the transfer function will come into this picture and here we can design a control $\Delta\delta_e(s)$ and we can get the feedback which is the output actually $\Delta\alpha(s)$. So, this is how we can design the control system and this concept already we have done in the first half of this lecture. So, let us stop it here we will continue for the another system in the next lecture. Thank you.