Introduction to Aircraft Control System

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Week – 12

Lecture – 56

Transfer Function Based Linear Control System

In this lecture also we will be doing the same thing how we can find the transfer function from the given plant which are basically multi output and multi input system. So let's start with an example. A fighter aircraft flying at 200 m/s at height of 10 kilometer has short period equation of motion

$$\dot{\alpha} = -6\alpha + q$$

$$\dot{q} = -5\alpha - 0.6q - 12\delta_e \dots \dots Eq(1)$$

So this is the kind of equation we are assuming as which has the short period characteristics and here $\dot{\theta} = q$ which is the pitch rate and θ is the pitch angle and alpha is the angle of attack and δ_e is the elevator deflection which can be designed from the control algorithm. So based on the deflection of the elevator we can provide the desired moment to the system. Now the question is find transfer function $\frac{q(s)}{\delta_e(s)}$ .So let's go to solution part. So taking Laplace transform of Eq. (1) with zero initial condition we can write

$$s\alpha(s) = -6\alpha(s) + q(s) \dots \dots Eq(2)$$

$$sq(s) = -5\alpha(s) - 0.6q(s) - 12\delta_e(s) \dots \dots Eq(3)$$

From Eq. (2), $\alpha(s) = \frac{q(s)}{s+6}$, Eq. (3) can be written as

$$\frac{q(s)}{\delta_e(s)} = \frac{-12(s+6)}{s^2 + 6.6s + 8.6}$$

So now this is one way we have solved. Now we can go with another approach for what we have done in the last lecture. Alternatively, we can write Eq. (1) in state space form as

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -6 & 1 \\ -5 & -0.6 \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} 0 \\ -12 \end{bmatrix} \delta_e$$

Since q is the only output

$$y(t) = q(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix}$$

Hence,

$$\frac{q(s)}{\delta_e(s)} = C(SI - A)^{-1}B$$

$$= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s+6 & -1 \\ 5 & s+0.6 \end{bmatrix} \begin{bmatrix} 0 \\ -12 \end{bmatrix}$$

$$= \frac{-12(s+6)}{s^2 + 6.6s + 8.6}$$

So if you notice the both the expressions are same. Now let us do another problem, a little complex problem and how we can come up with the transfer function if there are multiple input in the system because as of now we have considered only one input at a time. So in the state of the system model if there are multiple input and multiple output how we can come up with the transfer function that is another challenging problem. So let us work on this in this example. Consider the following longitudinal linearized equation of motion of an aircraft at a given flight condition.

$$\dot{q} = -0.6q - 0.2\dot{\alpha} - \alpha - 1.2\delta_e$$

$$\dot{u} = 2.25\delta_{th} + 0.035\alpha - 9.81\theta - 0.18u \ldots \ldots Eq(1)$$

$$\dot{\alpha} = q - 0.2u - 0.6\alpha - 0.035\delta_e$$

$$\dot{\theta} = q$$

Where δ_{th} and δ_e are the control inputs, i.e. throttle and elevator inputs, respectively. Find the transfer functions $\frac{q(s)}{\delta_e(s)}, \frac{\alpha(s)}{\delta_{th}(s)}, \frac{\theta(s)}{\delta_e(s)}$.

Substituting $\dot{\alpha}$ in $\dot{q}$ and writing in state space form, we get

$$\begin{bmatrix} \dot{q} \\ \dot{u} \\ \dot{\alpha} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -0.85 & -0.04 & -0.88 & 0 \\ 0 & -0.18 & 0.035 & -9.81 \\ 1 & -0.2 & -0.6 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ u \\ \alpha \\ \theta \end{bmatrix} + \begin{bmatrix} -1.193 & 0 \\ 0 & 2.25 \\ -0.035 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_{th} \end{bmatrix}$$

Here

$$A = \begin{bmatrix} -0.85 & -0.04 & -0.88 & 0 \\ 0 & -0.18 & 0.035 & -9.81 \\ 1 & -0.2 & -0.6 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Since δ_e is the only control variable of interest, matrix B is taken as follows

$$B = \begin{bmatrix} -1.193 \\ 0 \\ -0.035 \\ 0 \end{bmatrix}$$

C matrix can be formed as

$$C = [1 \quad 0 \quad 0 \quad 0]$$

$$D = 0$$

Analytically, we can use $\frac{q(s)}{\delta_e(s)} = C(SI - A)^{-1}B,$ however in MATLAB we can use

$$[num, den] = ss2tf(A, B, C, D, 1)$$

Where num and den are numerator and denominator respectively and $ss2tf$ implies state space to transfer function.

$$num = 0 \quad -1.193 \quad -0.8997 \quad -0.1317 \quad 0$$

$$den = 1 \quad 1.63 \quad 1.658 \quad 0.6472 \quad 1.962$$

$$\frac{q(s)}{\delta_e(s)} = \frac{-1.193s^3 - 0.8997s^2 - 0.1317s}{s^4 + 1.63s^3 + 1.658s^2 + 0.6472s + 1.962}$$

So if you notice we just modify the B matrix and C matrix because A matrix will be same for this because this is a system matrix and we just change B and C and we can find the transfer function even if the system is multi-input and multi-output. Now we have second part where you can find $\frac{\theta(s)}{\delta_e(s)}$. So here again we can take the first column of the B matrix because the first column indicates for δ_e but here C will be changed because we are taking say fourth variable instead of first, second and third.

$$B = \begin{bmatrix} -1.193 \\ 0 \\ -0.035 \\ 0 \end{bmatrix}, \quad C = [0 \quad 0 \quad 0 \quad 1]$$

Again by using same MATLAB command, we get

$$\frac{\theta(s)}{\delta_e(s)} = \frac{-1.193s^2 - 0.9041s - 0.1325}{s^4 + 1.63s^3 + 1.658s^2 + 0.6472s + 1.962}$$

Finally, for $\frac{\alpha(s)}{\delta_{th}(s)}$, B matrix will be

$$B = \begin{bmatrix} 0 \\ 2.25 \\ 0 \\ 0 \end{bmatrix}, \quad C = [0 \quad 0 \quad 1 \quad 0]$$

Using again same MATLAB command, we get

$$\frac{\alpha(s)}{\delta_{th}(s)} = \frac{-0.45s^2 - 0.2925s}{s^4 + 1.63s^3 + 1.658s^2 + 0.6472s + 1.962}$$

So once we have transfer function for the different output and input we can come up with the control what we have done before. This is the one of the most important topic how we can apply the classical control techniques to the state space-based model or system. So we have to convert the system into transfer function form for the individual input with the output and we can apply the control technique to the system.

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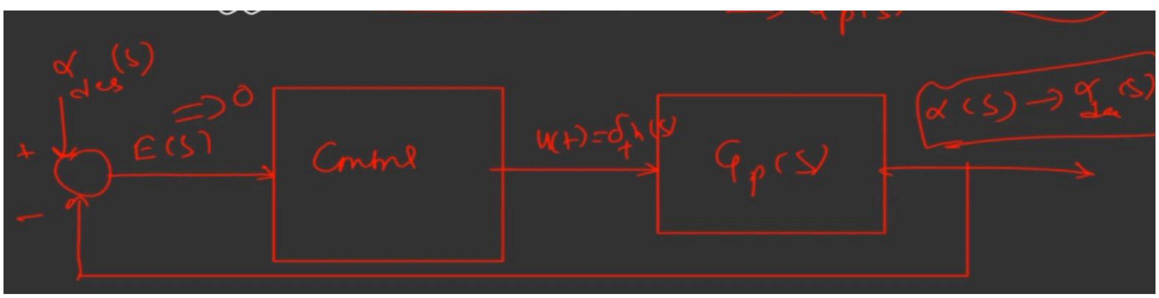

So for example if you are taking this system where the controller needs to be designed, so here we'll have the plant so this is basically our plant $G_p(s)$. We can come up with the control here suitable control block it can be any control PI, PD or PID. So we have summing point where we are having the output to be fedback here and this is the output where $\alpha(s)$ is the output and this is the control basically $u(t)$ or we can write $\delta_{th}(s)$ is the input going to the plant and now when our error is equals to zero then $\alpha = \alpha_{des}$,so we can track the desired values. So we can apply the same concept here and we can come up with the autopilot design for the given system. Thank you.