Introduction to Aircraft Control System

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Week – 11

Lecture – 55

Transfer Function Based Linear Control System

In the last lecture, we have done how we can design the controller for the plant which are basically the single input and single output. Basically, we designed the controller for the Dutch roll motion. And in today's lecture, here we are going to discuss how we can come up with the transfer function for the system which are basically represented in state space form. So, the topic is transfer function based linear control system. This is very very important topic because in this course basically we are designing the control system for the plant which are basically in the single input and single output form, SISO form. In the second half of this course, we have come up the system which is basically in state space form.

If you look at both longitudinal linearized equation and the lateral directional motion, we had a system $\dot{X} = AX + BU$ form. So, the system can be multiple input and multiple output form what you have seen before. But in the first half, we have discussed the aircraft attitude control using transfer function method. So, we had system $I\ddot{\theta} = u$. So, here θ is the basically output and u is the control input. So, now the question is how we can convert the system which are basically multiple input and multiple output system or state space form to the transfer function form. Once the system is transfer function form, we can apply the linear controls to design the linear control systems for the plant.

So, here this is the subject of study in this lecture, how we can come up the transfer function for the system which are represented in state space form. So, here we will have some assumptions. So, the first assumption is y as the output and u is the input. We are assuming both having dimension 1.

So, here what does it mean? So, in the system $\dot{X} = AX + BU$ and $Y = CX + DU$, the dimension of X is n number of states, $X \epsilon \mathbb{R}^n$ and the output dimension we are assuming, there are let us assume q number of output $Y \epsilon \mathbb{R}^q$. Similarly other dimensions are $A_{n\,x\,n}$ $B_{n\,x\,p}$ $C_{q\,x\,n}$ $D_{q\,x\,p}$. So, here D is assumed to be 0. In this case D is feedthrough or feed forward matrix. In cases where the system model does not have a direct feed through, or we can say feedforward and D is 0 matrix. So, the most of the system there is no feed forward or feed through. So, that is why we are will be assuming system $AX + BU$ and $Y = CX$. The output from the system basically X and which is of dimension n. So, here suppose we have

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

and if the C matrix is chosen to be

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

So, we will have only x_1

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1$$

and if the C matrix is chosen to be

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

So, we will have only x_2. Also we are assuming there is only one input in the system. So, for example, we have system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 4 \end{bmatrix} u$$

So, there is there is only one u acting on the whole system. So, this is the assumption we are considering to how we can get the transfer function from the state space model. Now, let us go back to our state space equation $\dot{X} = AX + BU$ and $Y = CX$. If you take Laplace transform with 0 initial condition we get

$$Y(s) = CX(s)$$

$$X(s) = C^{-1}Y(s)$$

$$sX(s) = AX(s) + BU(s)$$

$$\frac{Y(s)}{U(s)} = C(SI - A)^{-1}B$$

This is basically single input single output equation. Now, we will take a simple example. This is very powerful because most of the practical control system are SISO and the mathematical derivation we are getting in the state transform. Now, this is how we can come up with the state transfer function we can apply our linear control techniques what we have discussed in the first half of this course. So, example is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} u$$

$$\dot{x}_1 = x_1 + x_2 + 3u$$

$$\dot{x}_2 = 2x_2 + 4u$$

Let the output be the state variable $x_1(t)$

$$y(t) = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1(t)$$

Find the transfer function $\frac{Y(s)}{U(s)}$ and $\frac{X_1(s)}{U(s}$. So, here basically we will apply the systematic way of getting the transfer function via the formula just derived earlier. So, here we will apply

$$G(s) = \frac{Y(s)}{U(s)} = C(SI - A)^{-1}B$$

Here $Y(t) = x_1(t), C = [1 \quad 0]$

$$\frac{X_1(s)}{U(s} = [1 \quad 0] \begin{bmatrix} s-1 & -1 \\ 0 & s-2 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

Solving, we get

$$\frac{X_1(s)}{U(s} = \frac{3s-2}{s^2-3s+2}$$

So, this is how we can come up with the transfer function if the system is represented in the state space form. So, there is another method we can also apply this is called brute force method and another method is Cramer's rule. This is called brute force adhoc method. So, here we have system from the state space model we can get our system

$$\dot{x}_1 = x_1 + x_2 + 3u$$

$$\dot{x}_2 = 2x_2 + 4u$$

If you apply the Laplace transform with zero initial conditions we will have

$$\begin{bmatrix} s-1 & -1 \\ 0 & s-2 \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} U(s)$$

Dividing both sides by U(s) and then applying Cramers rule, we get

$$\frac{X_1(s)}{U(s} = \frac{\begin{bmatrix} 3 & -1 \\ 4 & s-2 \end{bmatrix}}{\begin{vmatrix} s-1 & -1 \\ 0 & s-2 \end{vmatrix}} = \frac{3s-2}{s^2-3s+2}$$

So, if you notice both the transfer functions are same. So, this is how we can find the transfer function from the state space model if the state space model assumed to be single input and single output equation. This is very important because once you have the transfer function we can apply the linear control techniques and techniques what you have done before.

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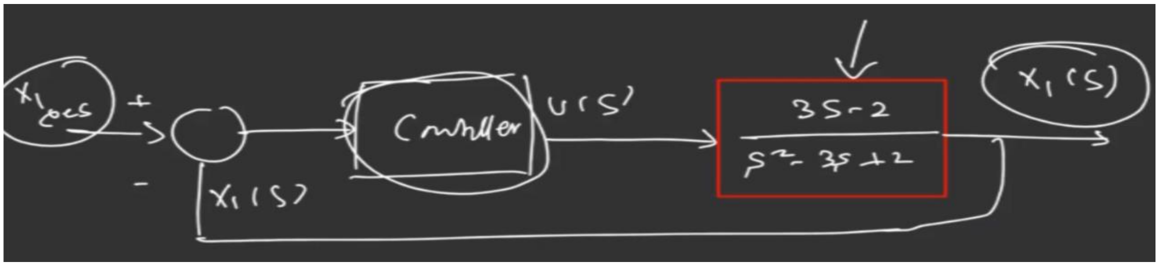

This is how we can design the controller here and this control will track help to track $X_1(s)$ to X_{des}. So, this part already have done in the first half of this course how we can design the control system if you are having the plant in transfer function form. So, let's stop it here we will continue from the next lecture we will come up with different system and how we can come up with the transfer function so that we can apply the linear control techniques. Thank you.