Introduction to Aircraft Control System

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Lecture – 54

Lateral Flying Qualities of the Aircraft

In this lecture, we'll be taking the control into picture. How the control algorithm going to improve the flying qualities of the aircraft. Here we are going to take the example of dutch roll motion and how we can provide artificially the damping to the system. This is very very important in the flight control system. So before we go to the example, let me recap some important notes from the previous lecture. So previously we have done the natural system analysis which gives us the system is stable or not and this can be comment based on the eigenvalues of the system or the roots of the characteristic equation.

But this analysis it is difficult for the pilot to judge whether the airplane has acceptable flying qualities. In this direction, we'll take the example of the Dutch roll and how we can provide artificially the damping to the system so that the Dutch roll can be stabilized. The Dutch roll motion can be improved by increasing the yaw damper N_r which can be improved by increasing the vertical tail area of the wing. But increasing the vertical tail area will add additional drag to the airplane and it will also increase the directional stability of the system. So these are the problems if we increase the vertical tail area of the wing and which will actually give the necessary damping to the Dutch roll but this cannot be done because it will cause additional drag to the airplane and increase the directional stability of the airplane. Now this problem can be solved if we provide the yaw damping N_r artificially and this can be achieved by the use of the control. So here in this example we're going to study how we can take the control into picture and how we can improve the Dutch roll motion. Let me do some mathematical calculation for this problem.

$$\sum yawing\ moment = I_z\ddot{\psi} \ldots\ldots Eq(1)$$

N is the yawing moment and ψ is the yaw angle. In terms of reference and perturbed variable, N and ψ can be written as $N = N_0 + \Delta N$ and $\psi = \psi_0 + \Delta\psi$. This is already have done during the study of the small perturbation theory and here if you write the expression for ΔN

$$\Delta N = \frac{\partial N}{\partial \beta}\Delta\beta + \frac{\partial N}{\partial \dot{\beta}}\Delta\dot{\beta} + \frac{\partial N}{\partial r}\Delta r + \frac{\partial N}{\partial \delta r}\Delta\delta_r$$

$$\Delta\ddot{\psi} = \frac{1}{I_z}\frac{\partial N}{\partial \beta}\Delta\beta + \frac{1}{I_z}\frac{\partial N}{\partial \dot{\beta}}\Delta\dot{\beta} + \frac{1}{I_z}\frac{\partial N}{\partial r}\Delta r + \frac{1}{I_z}\frac{\partial N}{\partial \delta r}\Delta\delta_r$$

$$\Delta\ddot{\psi} = N_\beta\Delta\beta + N_{\dot{\beta}}\Delta\dot{\beta} + N_r\Delta r + N_{\delta r}\Delta\delta_r \dots\dots Eq(2)$$

Where $N_\beta = \frac{1}{I_z}\frac{\partial N}{\partial \beta}$ and so on.

Because the center of gravity is constrained by yaw angle, the yaw angle and sideslip angle are related by $\Delta\psi = -\Delta\beta, \Delta\dot{\psi} = -\Delta\dot{\beta}, \Delta\dot{\psi} = \Delta r$. Hence, Eq. (2) becomes

$$\Delta\ddot{\psi} - (N_r - N_{\dot{\beta}})\Delta\dot{\psi} + N_\beta\Delta\psi = N_{\delta r}\Delta\delta_r \dots\dots Eq(3)$$

For airplane, $N_{\dot{\beta}}$ is usually negative and can be eliminated from Eq. (3)

$$\Delta\ddot{\psi} - N_r\Delta\dot{\psi} + N_\beta\Delta\psi = N_{\delta r}\Delta\delta_r \dots\dots Eq(4)$$

Let us assume for a particular airplane the directional stability, yaw damping and control derivatives are

$$N_\beta = 1.71\ s^{-1}, N_r = -0.10\ s^{-1}, N_{\delta r} = -0.84\ \ s^{-1}$$

$$\Delta\ddot{\psi} + 0.1\Delta\dot{\psi} + 1.77\Delta\psi = -0.84\ \Delta\delta_r \dots\dots Eq(5)$$

If you notice here basically $-0.84\Delta\delta_r$ is acting as a control but while study of the approximation or the natural study of the dynamics we have not considered this control part so here this part has been added how we can improve the flying qualities of the Dutch roll. So this part is very important and where the control will be added to the system, if you compare after taking the laplace transform of Eq. (5), so this basically indicates the characteristic equation of the system now from the system if you compare $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$ then we can get $\xi = 0.037$ and $\omega_n = 1.33\ rad/s$. If you notice here the damping is very low and for this condition the response will be oscillatory and which will have poor damping, also large overshoot so we can say such airplane would be very difficult for the pilot to fly but if you come up with the closed loop control system and where if you apply the rudder deflection or the control deflection in the system, here the rudder deflection is proportional to the yaw rate, so we are designing a control proportional to the yaw rate where we can write

$$\Delta\delta_r = -k\,\Delta\dot{\psi} \ldots \ldots Eq(6)$$

and if you substitute this control law in the Eq. (4)

$$\Delta\ddot{\psi} - (N_r - kN_{\delta r})\Delta\dot{\psi} + N_\beta \Delta\psi = 0 \ldots \ldots Eq(7)$$

This is basically our closed loop control system because in the plant the system control has been incorporated so now we have to choose this control parameter k here such that it can provide some desired flying qualities to the pilot through which we can fly the aircraft. The flying qualities are specified as

$$\xi > 0.08, \quad \omega_n > 0.4\, rad/s$$

Based on these specifications $\xi = 0.2$ and $\omega_n = 1.33$ are considered by the pilot. Now, the standard C.E. becomes

$$s^2 + 2\xi\omega_n s + \omega_n^2 = s^2 + 2 * 0.2 * 1.33s + 1.33^2 \ldots \ldots Eq(8)$$

Comparing Eq. (8) and Eq. (7) yields $k = -0.514$. If you choose this value of k, ψ will be stable for this case and basically fulfill our desired specification. We will come up some modified control. This is basically proportional control. We can also modify the control.

Basically, control depends on the control system engineer, how we can choose the control suitably so that our mission objective can be fulfilled. So, here if you notice, here basically we are improving the damping. Why? Because if you look this expression, basically we have increased the damping here, but natural frequency we have kept as it is. But how can improve both damping and natural frequency together? So, here we will modify the control which will help us to improve both the damping and frequency of the system.

So, for Dutch roll, it is also possible to improve the damping and frequency using suitable control. So, for this our control can be modified by making the rudder deflection which is proportional to both the yaw rate and yaw angle.

$$\Delta\delta_r = -k_1\Delta\dot{\psi} - k_2\Delta\psi \ldots \ldots Eq(9)$$

Now Eq. (4) becomes

$$\Delta\ddot{\psi} - (N_r - k_1 N_{\delta r})\Delta\dot{\psi} + \left(N_\beta + k_2\right)\Delta\psi = 0 \ldots \ldots Eq(10)$$

So, this is basically closed loop control system. So, now here, so we have to choose k_1 and k_2 such that it can meet the desired damping and frequency. Comparing our standard CE function, we can get

$$k_1 = -0.514, \ \ k_2 = -0.0011$$

If you choose these control parameter values in the closed loop control system, it will fulfill the mission objective, which are defined by damping ratio and natural frequency system. So, we can write the overall control algorithm, which will improve the flying qualities of the dutch roll, can be written as

$$\Delta\delta_r = 0.514\Delta\dot{\psi} + 0.0011\Delta\psi$$

This is the control algorithm for the rudder deflection, which will give the desired deflection to the control system. So, this is how we can design the control algorithm in the flight equation, which will give us the desired flying qualities for the pilot. Thank you, we will continue in the next lecture. Thank you.