Introduction to Aircraft Control System

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Lecture – 53

Lateral / Directional Approximations (Cont.)

Now we are going to study how we can study the dynamic analysis of the lateral directional motion and the different approximations what we have discussed in the previous lecture. So let's first come up with the example. Find the lateral eigen values of general aviation aircraft and compare the dynamic analysis results with lateral directional approximation. Given data

$$Y_v = -0.254\ (s^{-1}), L_v = -0.091\ (ft\ s^{-1}), Y_\beta = -45.72\ (ft\ s^{-2})$$

$$L_\beta = -16.02\ (\ s^{-2}), Y_p = 0, L_p = -8.4\ (s^{-1}), Y_r = 0, L_r = 2.19\ (s^{-1})$$

$$N_v = 0.025\ (ft\ s^{-1}), N_\beta = 4.49\ (\ s^{-2}), N_p = -0.35\ (s^{-1})$$

$$N_r = -0.76\ (s^{-1}), u_0 = 176\ ft\ s^{-1}$$

Assume $Y_\phi = 0$ and $\theta_0 = 0$

Lets rewrite linearized lateral directional motion

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{bmatrix} = \begin{bmatrix} Y_\beta/u_0 & Y_p/u_0 & \frac{Y_r - 1}{u_0} & \frac{g\cos\theta_0}{u_0} \\ L_\beta & L_p & L_r & 0 \\ N_\beta & N_p & N_r & 0 \\ 0 & 1 & \tan\theta_0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\phi \end{bmatrix}$$

If we substitute the given data in above equation, we get

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{bmatrix} = \begin{bmatrix} -0.254 & 0 & -1 & 0.182 \\ -16.02 & -8.4 & 2.19 & 0 \\ 4.488 & -0.350 & -0.760 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\phi \end{bmatrix} \dots\dots Eq(1)$$

The resulting C.E. is

$$|\lambda I - A| = 0$$

$$\lambda^4 + 9.417\lambda^3 + 13.982\lambda^2 + 48.102\lambda + 0.4205 = 0 \ldots\ldots Eq(2)$$

The solution of this C.E. yields Eigen values

$$\lambda_1 = -0.00877$$

$$\lambda_2 = -8.435$$

$$\lambda_{3,4} = -0.487 \pm i2.335$$

If you notice these eigenvalues, λ_1 is basically negative, so it is converging, but it is basically slowly converging. And if you have this kind of nature in the lateral directional motion, this basically talks about the spiral mode, and the second case λ_2, this is negative also, but high value, so it will be highly convergent motion, and this motion what we have discussed, this is basically a roll mode. And the third and fourth eigenvalues, these are actually complex conjugate, and due to the complex part in the eigenvalues, so it has some frequency, so the system will be decaying oscillatory behavior, but decaying. So, we can say real part talks about the damping, and imaginary part talks about how the system will be oscillatory. So, here we can say this highly damped oscillator, because the damping is quite high here. So, we can say this motion actually, dutch roll. So, this is how we can define the different modes based on the roots of the characteristic equation. Now, let's find the different eigenvalues for the respective approximations. So, let's go with the first spiral approximation. This is actually, these eigenvalues we obtained based on the complete state-space model.

Now, we will go with the individual model, and how we can see the roots of the system. First, we will go straight spiral approximation. If you remember, we had the eigenvalues for spiral mode

$$\lambda_{spiral} = \left[\frac{N_r L_\beta - N_\beta L_r}{L_\beta}\right] = -0.144$$

This is also not high value, this is the spiral approximation, and if you look at the roll approximation,, the eigenvalues we had

$$\lambda_{roll} = L_p = -8.4$$

The magnitude is quite high compared to the spiral, so it is very highly, high convergent mode. If you go to the dutch roll approximation, we had

$$\lambda^2 - \left(\frac{Y_\beta + u_0 N_r}{u_0}\right)\lambda + \frac{Y_\beta N_r - N_\beta Y_r + u_0 N_\beta}{u_0} = 0$$

$$\lambda^2 + 1.101\lambda + 4.71 = 0$$

$$\lambda_{DR} = -0.51 \pm i2.109$$

$$\omega_{n,DR} = 2.17\ rad/s$$

$$\xi_{DR} = 0.254$$

Now, we will find dynamic analysis of these motions. So, we will go step by step. So, first we will find the time period, time to get the half magnitude, full magnitude, or the number of cycles in the response. So, first we will go with the full linearized model, and based on the values or the roots, we will come up with the period, number of cycles, time to half magnitude. From the above example the exact values of $N_{1/2}, p, t_{1/2}$. For spiral

$$t_{1/2} = \frac{0.693}{|\eta|} = \frac{0.693}{0.00877} = 79.01$$

P and $N_{1/2}$ are undefined. For roll, we have

$$t_{1/2} = \frac{0.693}{|\eta|} = 0.082$$

P and $N_{1/2}$ are undefined. For dutch roll

$$t_{\frac{1}{2}} = \frac{0.693}{|\eta|} = 1.42$$

$$p = \frac{2\pi}{\omega} = 2.69$$

$$N_{1/2} = \frac{t_{\frac{1}{2}}}{p} = 0.527$$

Now, coming to approximations, for spiral we have

$$t_{1/2} = \frac{0.693}{|\eta|} = 4.81$$

P and $N_{1/2}$ are undefined. For roll, we have

$$t_{1/2} = \frac{0.693}{|\eta|} = 0.082$$

P and $N_{1/2}$ are undefined. For dutch roll

$$t_{1/2} = \frac{0.693}{|\eta|} = 1.35$$

$$p = \frac{2\pi}{\omega} = 2.89$$

$$N_{1/2} = \frac{t_{\frac{1}{2}}}{p} = 0.46$$

So now, if you compare the results of exact and the approximation solutions, other than spiral, the roll and Dutch roll are very, very close by. So, this is how we can do the analysis in the exact values or exact roots of the linearized model of the lateral directional motion and the different approximations in the lateral longitudinal motions. Now we can stop here because this is how we can study the dynamic analysis of the different system, basically natural system without control. Next part onwards, how we will be designing the control or how we can come up with the transfer function from the given state space model, we will discuss in detail. Thank you.