Introduction to Aircraft Control System

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Week – 11

Lecture – 52

Lateral / Directional Approximations (Cont.)

As we discussed in the last lecture, due to the difference in lift in the right and left wing, the restoring moment generated and due to which the equilibrium can be maintained. The roll mode approximation we are continuing from the last lecture. Let me write the equation, restoring moment, which basically stabilizing the aircraft orientation along the x-axis.

$$\sum rolling\ moment = I_x\ddot{\phi}$$

And the total moment we can write

$$\frac{\partial L}{\partial \delta_a}\Delta\delta_a + \frac{\partial L}{\partial p}\Delta p = I_x\ddot{\phi} \ldots \ldots Eq(1)$$

Here the roll rate contributes to the rolling moment due to the rotational inertia of the aircraft and $\frac{\partial L}{\partial \delta_a}\Delta\delta_a$represents roll moment due to deflection of the ailerons. $\frac{\partial L}{\partial p}$ represents roll damping moment L_p. And also we know that $\Delta p = \Delta\dot{\phi}, \Delta\dot{p} = \Delta\ddot{\phi}$. Now Eq. (1) becomes

$$\Delta\dot{p} = L_{\delta a}\Delta\delta_a + L_p\Delta p \ldots \ldots Eq(2)$$

Where $L_{\delta a} = \frac{1}{I_x}\frac{\partial L}{\partial \delta_a}$ and $L_p = \frac{1}{I_x}\frac{\partial L}{\partial p}$

$$\frac{\Delta\dot{p}}{L_p} - \Delta p = \frac{L_{\delta a}\Delta\delta_a}{L_p} \ldots \ldots Eq(3)$$

$$\tau\Delta\dot{p} + \Delta p = -\frac{L_{\delta a}\Delta\delta_a}{L_p} \ldots \ldots Eq(4)$$

Where $\tau = -\frac{1}{L_p}$ defines time constant of the system. This tell us how fast system approaches a new state after being disturbed. If τ is small, system will respond very rapidly and if τ is large, system will respond very slowly. Now let's see how it is happening. So, since we are doing the natural dynamics, we are not taking control into picture.

So, in equation number four we have to neglect R.H.S because this part talks about the control part. So, to study the natural behavior of the system, we have to consider the torque free motion. So, for roll approximation, control free response is

$$\tau \Delta \dot{p} + \Delta p = 0$$

$$\Delta \dot{p} + \frac{\Delta p}{\tau} = 0 \ldots \ldots Eq(5)$$

So, from this equation, we have only one eigenvalue and which is $\frac{1}{\tau}$. If you apply the Laplace transform, you can find the roots of the system and also from there also you can find the value of the eigenvalue of the roots.

$$(\tau s + 1)\Delta p(s) = 0$$

Hence, roots $s = -\frac{1}{\tau}$ or $\lambda_{roll} = \frac{-1}{\tau} = L_p$

Here L_p, when the aircraft is rolling, the angular momentum creates a moment opposing the change in roll rate. So, that rate is negative actually. Last lecture also we have discussed about the negativeness of L_p and this effect in the angular momentum is called, known as gyroscopic procession. This gyroscopic effect can influence the rolling moment. Eigen values corresponding to roll subsidence is negative and purely real root. It is also fairly large in magnitude and thus it is highly converging maneuver. So, this is how we can study the roll mode approximation in the lateral directional linearized model of the aircraft. Now, let's go to the last part of the approximation in the lateral direction which is called Dutch roll approximation. The Dutch roll is a type of laterally directional oscillation in aircraft dynamics which is characterized by a combination of yawing and rolling motions. And due to these two combined rolling and yawing motion, this Dutch roll happens in the aircraft dynamics. It occurs due to the coupling between the yawing (side to side) and rolling (tilting) motions of the aircraft.

When the aircraft experiences a disturbance, it can initiate a yawing motion. And due to this yawing motion, the aircraft also can lead to a rolling motion due to the interconnected nature of yaw and the roll.
Now, if you look pictorially how it is happening.

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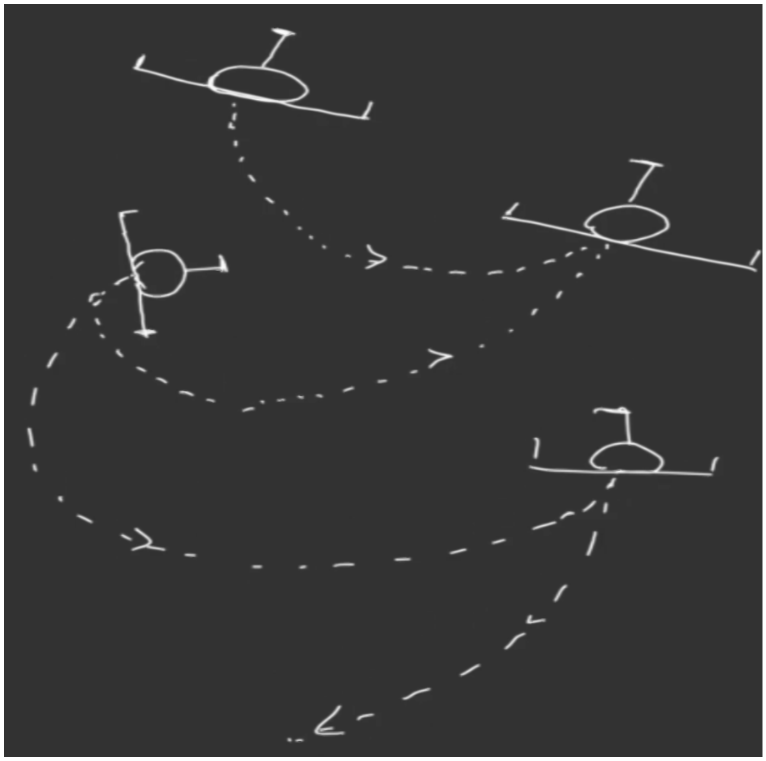

Let's assume we have aircraft something like this and due to this roll, it also goes like this. This is something due to this interconnected yawing motion and again it goes like this. So, due to this interconnected nature of the yaw roll, this kind of motion happens in the aircraft. So, that's why this when the yawing moment happened which lead to the rolling motion and due to the interconnected nature of the yaw roll, this kind of phenomena happens in the aircraft motion. Let's stop it here. We will continue from the last lecture how we can come up with the mathematical model of this dutch roll approximation in the lateral directional motion. The dutch roll approximation primarily consists of side slipping ($\Delta\beta$) and yaw motion (Δr).

So, how this variable going to affect the dutch roll motion, we will have a look and other than this parameter, we will neglect the rest of the state variable in the linearized model. With this assumption, the following state equation, let me rewrite the linearized equation motion of the lateral directional motion of the aircraft

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{bmatrix} = \begin{bmatrix} Y_\beta/u_0 & Y_p/u_0 & \frac{Y_r - 1}{u_0} & \frac{g\cos\theta_0}{u_0} \\ L_\beta & L_p & L_r & 0 \\ N_\beta & N_p & N_r & 0 \\ 0 & 1 & \tan\theta_0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\phi \end{bmatrix}$$

gets reduced to

$$\begin{bmatrix}\Delta\dot{\beta}\\ \Delta\dot{r}\end{bmatrix} = \begin{bmatrix}\frac{Y_\beta}{u_0} & \frac{Y_r}{u_0} - 1\\ N_\beta & N_r\end{bmatrix}\begin{bmatrix}\Delta\beta\\ \Delta r\end{bmatrix} \ldots\ldots Eq(6)$$

Once we have this state space model of the Dutch roll, we can come up with the characteristic equation and we can study how we can find the damping ratio and natural frequency in the Dutch roll approximation. To study the above equation, we need to find the roots of characteristic equation and how to find characteristic equation?

$$|\lambda I - A| = 0$$

After solving, we get

$$\lambda^2 - \left(\frac{Y_\beta + u_0 N_r}{u_0}\right)\lambda + \frac{Y_\beta N_r - N_\beta Y_r + u_0 N_\beta}{u_0} = 0 \ldots\ldots Eq(7)$$

If we compare above equation with $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$, the undamped natural frequency $\omega_{n,DR}$ and damping ratio ξ_{DR} can be obtained as

$$\omega_{n,DR} = \sqrt{\frac{Y_\beta N_r - N_\beta Y_r + u_0 N_\beta}{u_0}}$$

$$\xi_{DR} = -\frac{1}{2\omega_{n,DR}}\left(\frac{Y_\beta + u_0 N_r}{u_0}\right)$$

Let's stop it here we'll continue from the next lecture. In the next lecture how we can come up with the full linearized model $\dot{X} = AX + BU$,so here A dimension is 4x4 and from this we will have four roots and based on the four roots we will come up with three motions which are spiral, roll and dutch roll. Then we will find the period, time to get the half amplitude or full amplitude and again we will come up with the individual approximation separately and we will compare the results, then we'll wind up this part. Thank you.