Introduction to Aircraft Control System

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Week – 11

Lecture – 51

Lateral / Directional Approximations (Cont.)

In the last lecture we have discussed about the different modes in the lateral directional motion and based on the natural dynamics of the lateral directional motion, we came up with three modes. One is spiral approximation, second is roll mode approximation, and third, Dutch roll approximation. In todays lecture we will be discussing how we can come up with the spiral mode approximation. Let us have a small note before we proceed to the main mathematical part. The spiral approximation in lateral flight is a simplified analysis used to understand the behaviour of an aircraft during steady unaccelerated flight and subjected to small disturbance in its lateral direction.

So, if you remember basically we are starting the natural dynamics, the variable we are assuming the natural dynamics are the perturbed variables. So, here how the system going to behave, it means how the perturbed variable going to effect or going to propagate it in the presence of disturbance. So, here the spiral motion can be approximated by the following figure. Let's assume we have the initial flight path direction here.

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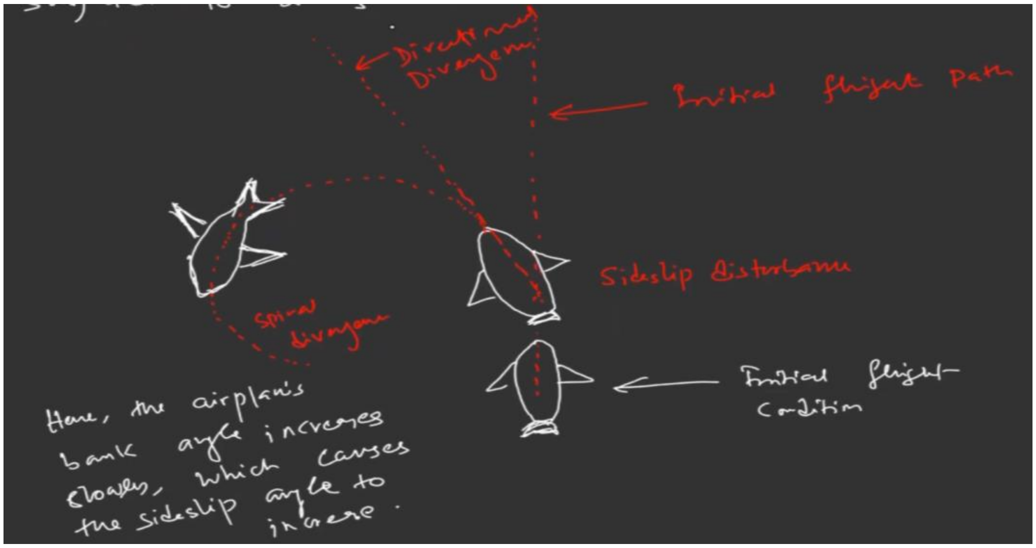

Let's denote this is the initial flight path, the initial flight is here somewhere. Due to the disturbance let's assume the flight path has been changed. Path is fixed but flight direction has been changed. Let's assume this is the current flight path direction and this direction we can say directional divergence and if the aircraft bank angle keep on increasing which causes the side slip angle to increase again. So the bank angle increases the motion like this. Let's assume it's the current stage of the flight and this kind of motion we can say spiral divergence. So, if you write here small note the airplanes bank angle increases slowly which causes the side slip angle to increase and due to which the motion look like the spiral nature. So, this is how the spiral motion occurs in the lateral direction of the flight motion.

So, let's write before you go to mathematical part let's write some part of the variables should come into picture in the mathematical derivations. It can be noted that the spiral mode is generally characterized by the changes in the bank angle. Bank angle is nothing but the roll angle (ϕ) and heading angle is nothing but the ψ. However, though the side slip angle is quite small in this spiral approximation but cannot be neglected because the aerodynamic movements depends depend on β, p, r rather than ϕ and ψ. So, we can write further the aerodynamics contribution due to β and p are usually are on the same magnitude. So, it means we can neglect $\dot{\beta}$ and roll rate $\dot{\chi}$ from the linearized lateral directional equation of motion. So, let me relate to the linearized equation of motion of the lateral directional model without control torque.

$$\begin{bmatrix}\Delta\dot{\beta}\\ \Delta\dot{p}\\ \Delta\dot{r}\\ \Delta\dot{\phi}\end{bmatrix} = \begin{bmatrix} Y_\beta/u_0 & Y_p/u_0 & \frac{Y_r-1}{u_0} & \frac{g\cos\theta_0}{u_0} \\ L_\beta & L_p & L_r & 0 \\ N_\beta & N_p & N_r & 0 \\ 0 & 1 & \tan\theta_0 & 0 \end{bmatrix}\begin{bmatrix}\Delta\beta\\ \Delta p\\ \Delta r\\ \Delta\phi\end{bmatrix} \dots\dots Eq(1)$$

Now, if you remove $\Delta\dot{\beta}$ and $\Delta\dot{\phi}$ from the equation we will be left with $\Delta\dot{p}$ and $\Delta\dot{r}$. So, we can write

$$\Delta\dot{p} = L_\beta\Delta\beta + L_p\Delta p + L_r\Delta r$$

It should be noted that for steady increase in bank angle, $\Delta p = 0, \Delta\dot{p} = 0$. Hence, we can write

$$\Delta\beta = -\frac{L_r}{L_\beta}\Delta r \dots\dots Eq(2)$$

Also, from Eq. (1) and $\Delta p = 0$

$$\Delta\dot{r} = N_\beta\Delta\beta + N_p\Delta p + N_r\Delta r$$

$$\Delta\dot{r} = N_\beta\left(-\frac{L_r}{L_\beta}\Delta r\right) + N_r\Delta r$$

$$\Delta\dot{r} = -\left[\frac{N_\beta N_r - N_r L_\beta}{L_\beta}\right]\Delta r$$

$$\Delta\dot{r} + A\Delta r = 0$$

Where $A = \left[\frac{N_\beta L_r - N_r L_\beta}{L_\beta}\right]$

So, here this is the first order equation and only one variable Δr so the eigenvalues should be A here. So, characteristic equation we can write

$$\lambda = -A = \left[\frac{N_r L_\beta - N_\beta L_r}{L_\beta}\right]$$

Here, the stability derivative L_β(dihedral effect) and N_r (yaw rate damping) are usually negative quantities. On the other side, N_β (directional stability derivative) and L_r(roll moment due to yaw rate) generally are positive quantities. If we consider the derivatives have their usual sign here based on this condition, stable spiral model can be obtained based on the following inequality.

$$N_r L_\beta - N_\beta L_r > 0$$

$$N_r L_\beta > N_\beta L_r$$

Hence, increasing the dihedral effect (L_β) or the yaw damping (N_r) or both can make spiral mode stable. So, this is how we can study the spiral approximation in the lateral directional motion and we have done everything based on the linearized model of the lateral direction motion of the aircraft and now we can start the next approximation which is the roll mode approximation.

If you notice everything we are doing here based on the linearized model, based on the roots of the characteristic equation is a natural dynamics we are studying here then based on the system is stable or unstable, we can come up with the control because before designing the control we have to study the natural dynamics, how the system is going to behave and at what time. Now in the roll mode approximation, if the aircraft encounter disturbance in roll then the wing that dips down we will see an increase in angle of attack and due to this increase in angle of attack the lift will produced and opposite is true also for the other wing and this lift differential produces restoring moment which actually balances the aircraft. So it comes back to the original orientation again due to this differential lift. The original trim point or equilibrium point or we can say equilibrium condition is achieved. So this is how the roll mode approximation happens so that's why roll mode is always stable, we can say actually converging to the equilibrium condition. Hence, we can say from this that the motion can be approximated by single degree of freedom. Equation of motion of pure rolling motion is

$$\sum rolling\ moment = I_x \ddot{\phi}$$

Let's stop it here we will continue next lecture how we can come up the mathematical derivations based on this total rolling moment along the x-axis. Thank you.