Introduction to Aircraft Control System

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Lecture – 50

Lateral / Directional Approximations

Now, let us start how we can come up with the different approximations in the lateral directional motion. We will start with the equation what you had before the linearized equation motion.

$$\begin{bmatrix} \Delta\dot{v} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{bmatrix} = \begin{bmatrix} Y_v & Y_p & Y_r - u_0 & g\cos\theta_0 \\ L_v^* + \frac{I_{xz}}{I_x} N_v^* & L_p^* + \frac{I_{xz}}{I_x} N_p^* & L_r^* + \frac{I_{xz}}{I_x} N_r^* & Y_\phi + g\cos\theta_0 \\ N_v^* + \frac{I_{xz}}{I_x} L_v^* & N_p^* + \frac{I_{xz}}{I_x} L_p^* & N_r^* + \frac{I_{xz}}{I_x} L_r^* & 0 \\ 0 & 1 & \tan\theta_0 & 0 \end{bmatrix} \begin{bmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta\phi \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & Y_{\delta r} \\ L_{\delta a}^* + \frac{I_{xz}}{I_x} N_{\delta a}^* & L_{\delta r}^* + \frac{I_{xz}}{I_x} N_{\delta r}^* \\ N_{\delta a}^* + \frac{I_{xz}}{I_x} L_{\delta a}^* & N_{\delta r}^* + \frac{I_{xz}}{I_x} L_{\delta r}^* \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{bmatrix} \dots\dots Eq(1)$$

Now, we'll go through some assumption, we'll reduce this equation further into simpler form so that we can do the analysis in an easy way. So, now if you assume $I_{xz} = 0$ (off diagonal terms assume to be 0) and the effect of $\frac{1}{m}\frac{\partial Y}{\partial \phi}$ is minimal, $Y_\phi \approx 0$

$$L_v^* = \frac{L_v}{\left[1 - \frac{I_{xz}^2}{I_x I_z}\right]}, \quad N_v^* = \frac{N_v}{\left[1 - \frac{I_{xz}^2}{I_x I_z}\right]} \dots\dots Eq(2)$$

With assumption of $I_{xz} = 0$, Eq. (2) yields

$L_v^* = L_v, \quad N_v^* = N_v$ and so on. Hence Eq. (1) reduces to

$$\begin{bmatrix} \Delta\dot{v} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{bmatrix} = \begin{bmatrix} Y_v & Y_p & Y_r - u_0 & g\cos\theta_0 \\ L_v & L_p & L_r & 0 \\ N_v & N_p & N_r & 0 \\ 0 & 1 & \tan\theta_0 & 0 \end{bmatrix} \begin{bmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta\phi \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & Y_{\delta r} \\ L_{\delta a} & L_{\delta r} \\ N_{\delta a} & N_{\delta r} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{bmatrix} \ldots\ldots Eq(3)$$

So our motivation is how we can bring $\Delta\beta$ in place of Δv because in the lateral motion sideslip angle is one of the important parameter but if you look the dynamics linearized model and side slip angle does not appear, so we have to take the sideslip angle into the system. If we write the side velocity Δv with the sideslip angle $\Delta\beta$ (same way what we have done relationship between $\Delta\alpha$ and Δw) we can write

$$\Delta\beta = \tan^{-1}\frac{\Delta v}{u_0} \approx \frac{\Delta v}{u_0}$$

$$\Delta v = \Delta\beta u_0 \ldots\ldots Eq(4)$$

Using Eq. (4), we have the following transformed stability derivatives

$$Y_v = \frac{1}{m}\frac{\partial Y}{\partial(\beta u_0)} = \frac{1}{u_0}\frac{1}{m}\frac{\partial Y}{\partial\beta} = \frac{Y_\beta}{u_0}$$

Similarly $L_v = \frac{L_\beta}{u_0}, N_v = \frac{N_\beta}{u_0}$. Now, Eq. (3) can be written as

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{bmatrix} = \begin{bmatrix} Y_\beta/u_0 & Y_p/u_0 & \frac{Y_r - 1}{u_0} & \frac{g\cos\theta_0}{u_0} \\ L_\beta & L_p & L_r & 0 \\ N_\beta & N_p & N_r & 0 \\ 0 & 1 & \tan\theta_0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\phi \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & Y_{\delta r}/u_0 \\ L_{\delta a} & L_{\delta r} \\ N_{\delta a} & N_{\delta r} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{bmatrix} \ldots\ldots Eq(5)$$

Now how can we approximate this equation based on the roots of the characteristic equation. To study the dynamic behavior of the system without the controlling we can take $\dot{X} = AX$, this is the natural linearized model. The characteristic equation can be obtained as

$$|\lambda I - A| = 0$$

Where dimension of A matrix is 4x4. so we'll have four roots and the characteristic equation will be fourth order equation in the following form

$$A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0 \dots \dots Eq(6)$$

Here, A,B,C, D will be the function of the stability derivatives, mass and inertia of the airplane. The roots of the characteristic equation composes two real roots and pair of complex roots based on the nature of the roots the response of the airplane, which can be summarized by the following motions. The first one will be a slowly convergent divergent motion which is called the spiral mode. We will discuss about the modes in more details in the coming lectures. Note that an unstable spiral mode results in a turning flight trajectory. The second mode can have a highly convergent motion which is called the rolling mode. This mode usually is highly damped and since it is highly damped it can reach the steady state in short period of time. Finally, a lightly damped oscillatory motion having low frequency is called dutch roll mode. Here, the combination of the yawing and rolling oscillation is involved.

So this is how we can come up with three different mode under lateral directional motion based on the roots of the characteristic equation. So how we can come up with the mathematical form based on the roots of the linearized model of the lateral directional motion, we'll discuss in detail in the subsequent lecture. So let's stop it here. Thank you.