Introduction to Aircraft Control System

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Week – 01

Lecture – 05

In this lecture, one of the most important points we will be discussing is the equilibrium point or trim point or fixed point, we say from a different perspective. Based on the equilibrium point, how we can define the stability of the system. So there are two types of stability we will be discussing in this lecture. One is static stability and another is dynamic stability. How the system is going to behave with respect to the equilibrium point. That is part of the static stability and dynamic stability, basically how the system behaves with time, the time history of the system.

And we will be discussing this part from the trajectory as well, how the system, it means the aircraft system evolves over time. And then we will conclude the lecture. Now let us discuss on the equilibrium point.

Let us consider we have system $x = f(x, u, t)$. Let us assume this is any dynamical system. And if this system is not affected by any control, u, or we can write $(u = 0)$. In that case, we can write $x = f(x, 0, t)$. And if you assume the solution of this equation, the solution of $x = f(x, 0, t)$, be $x^{\star}$, some constant value.

The point $x^{\star}$ is said to be an equilibrium point,if it has the property that whenever the state of the system x, so x is basically changing with time, but $x^{\star}$ is very, basically here equilibrium point is constant value. But once the system evolves over time, the x will change. So now, whenever the state of the system x starts at $x^{\star}$, it will remain at $x^{\star}$ for all future time or future state. If this condition holds, then I can say the system is in equilibrium state. So this is basically desirable property when designing and implementing.

So I can say this is a very desirable property while implementing a control system. So in a control system, our main motivation is to maintain the system at the equilibrium point. Here, we'll assume, since we'll be assuming the system in this course is the LTI system. So we can assume the equilibrium point is always zero. We shall consider the LTI system for its equilibrium point at the origin.

So we can, if you look at this, let's assume we have system $\dot{x} = -4x + u$, and we have some kind of force or torque in the system, let's me assume u. So as for the definition, we have to consider u, the control or some force to be zero to find the equilibrium point. So we need to put $u = 0$ and $\dot{x} = -4x$. And in the equilibrium condition, we assume the $x^{\star}$ is a constant value, as you mentioned, so in that case, we can write the rate of change of the state to be zero.

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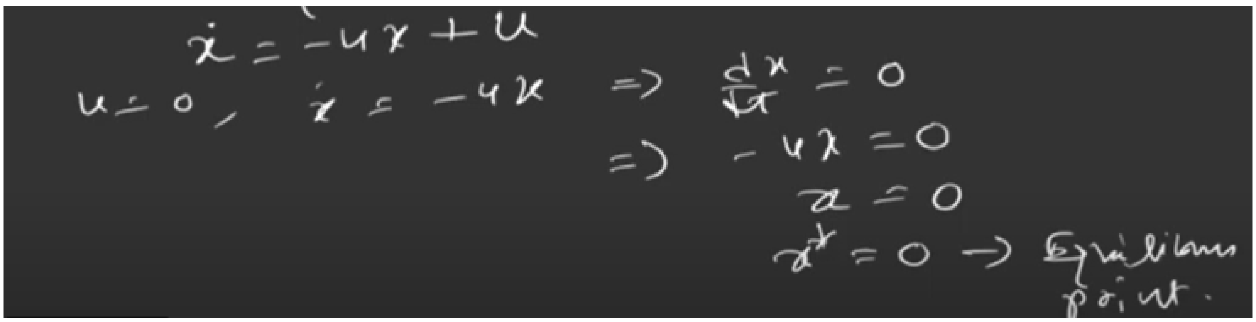

So here to find equilibrium point $\frac{dx}{dt} = 0$, we can make it zero. And from this $-4x = 0$, so $x = 0$. So in this case, $x^{\star}$ is zero, we can assume an equilibrium point. So in the LTI system, most of the LTI system or equilibrium point is assumed to be at origin $x^{\star} = 0$. So now we'll shift our attention, how we can look at this concept of equilibrium point for an aircraft.

Let me write, let's discuss the stability, because we'll define the stability of the system about the equilibrium point of the system. Air system basically aircraft with respect to, if an airplane remains in steady uniform flight. So it means the total force and moments acting on the system or in the aircraft is assumed to be balanced about cg. So in this condition, the resultant force and moment about cg, center of gravity must be equal to zero. So it means if the aircraft satisfies this condition, we can say that the aircraft is in the state of equilibrium or at trim condition.

So if the aircraft satisfies this condition, we can say the aircraft is in the state of equilibrium or flying at a trim condition. So these are the desirable properties for an aircraft. And in the other hand, if the resultant forces and moments are not summed to zero, then the airplane will go through some motion, which is basically translation motion or rotational motions. On the other hand, if the total forces and moment acting on the system are unbalanced, then the aircraft will be subjected to translational or rotational motions.

Based on this, we can come up with two important parts in stability, how the system evolves over time. And based on this definition, the stability of an aircraft can be divided into two parts.

Based on the, I mean, based on the equilibrium conditions. So, the stability of the aircraft can be divided into parts with respect to equilibrium point. The first is static stability and another is dynamic stability.

So let us start with the static stability part. Under static stability, we will see whether the system has an initial tendency to return to the equilibrium point after a disturbance is being applied. So let me define it. Static stability is the initial tendency of the aircraft to return to its equilibrium point after a disturbance. Let's look at some representation of how the static stability can be defined.

So let's assume we have a curved surface here. And let's assume this is the initial position of the ball on this curved surface. And if you apply some disturbance on this ball, and let's assume the balls after the disturbance has been shifted to here. Now after some time, the ball will come back to the equilibrium point because there is some restoring force acting on this ball after it is disturbed from the equilibrium conditions. This is the equilibrium conditions.

So if this kind of phenomena happens in the dynamical system and this kind of motion, we can say, this is the point or state of the system, we can say static or stable equilibrium point because the ball has initial tendency to back to the equilibrium conditions. So this is my equilibrium condition. So if the ball is disturbed from this equilibrium point, it will be coming back to the equilibrium point again due to some restoring force acting on the system. So this is what we call static stable equilibrium point or the system, we can say a statically stable system. If we consider another surface like this, and let's assume this is the equilibrium point.

And if you disturb the body or this point, and after the disturbance, let's assume the ball position is here. So if you see carefully, after the disturbance, the ball has no tendency to come back to this initial point. So if the system has this kind of phenomenon, we can say the system is statically unstable. So this is what we can say is statically unstable. And also we can say unstable equilibrium point.

And let's look at another condition. We have a surface. And let's assume the initial position of the vehicle is somewhere here. And after the disturbance, let's assume the vehicle position is here. And if you notice, even the body is part of or disturbed from this point to this point, this will be again it will stay in this point for all future time.

And if the system has this kind of property, we call the system is neutrally stable. And this property is called neutrally stable. And the equilibrium point is a neutrally stable equilibrium point. So from this example, we can conclude that if you are having a stable equilibrium point, we can say the vehicle must develop a restoring moment or force to bring the vehicle to the equilibrium point. Now let's extend this concept for our aircraft.

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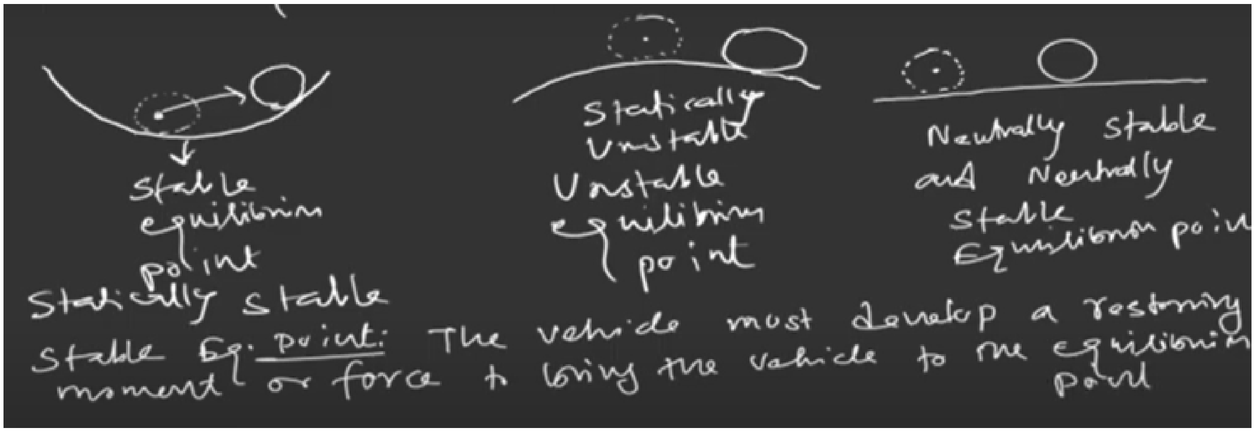

Let's assume the vehicle's initial position is here. And if you are having the system, if the system has initiated some turbulence, it goes through some loads down motion. But after some time, it comes back to the same trajectory that you have initially. And if the vehicle has this kind of property system properties, we can call this aircraft a statically stable aircraft. Now if we have another condition here, let's assume the vehicle initial position is here.

If there is some turbulence acting on the system, and it deviates from the initial position, and it continues to diverge from the initial position, and the vehicle has this kind of tendency we call the system or the aircraft a statically unstable aircraft. Now we have another condition. If the vehicle is initially here, and due to some perturbations on the system, it comes to here again some perturbation system coming to here. And if you look carefully, the body continues to be in the same line. And if the vehicle has this kind of tendency, we can call the system is neutral stable.

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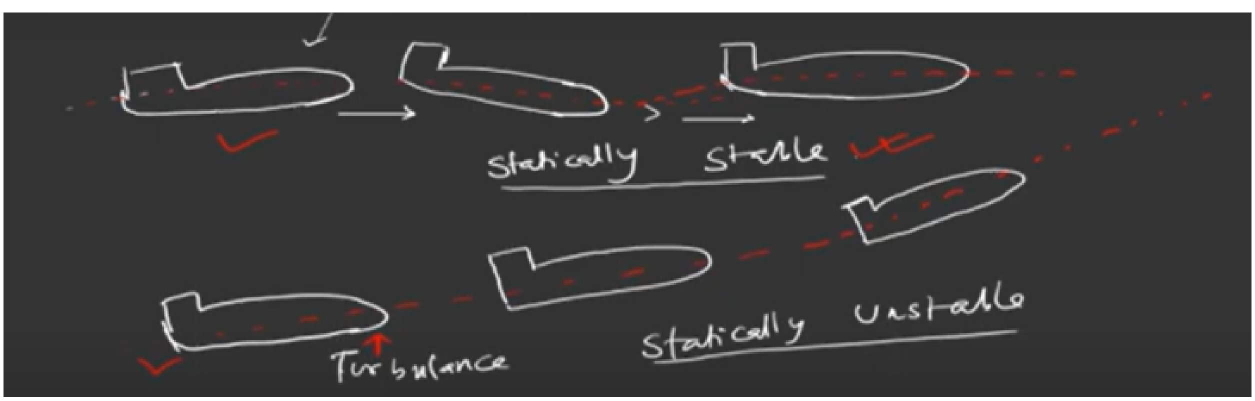

Now we'll extend the concept of dynamic stability, how we can define the dynamic stability of a system. So dynamic stability, basically, how the system behaves over time. So let me write dynamic stability. So here, mostly we are concerned with how the system behaves over time. Here we are concerned with the time history of the vehicle after it is disturbed from the equilibrium point.

So this is a very important property for designing control system, because designing control system will be looking at how the system is going to behave over time. So let's look some phenomena of how dynamic stability occurs. Let's assume we have the displacement of the aircraft, how it evolves over time. This is my time axis and this is the displacement of the aircraft.

Let's assume we are starting from here. If the vehicle over time follows this kind of response, then we can call the system is exponentially stable And if you have another kind of response, suppose this is the time axis and this is my displacement axis. If the system starts from here, for example, and if you have the system response over time, something like this, then we can say the system is diverging over time. And if you have another kind of response, this is my displacement of the aircraft, this is time.

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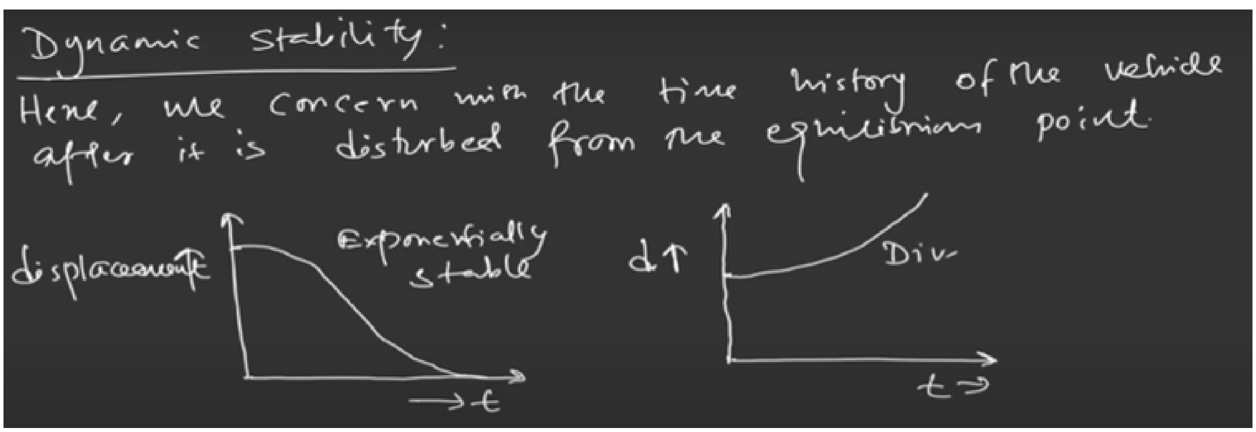

And if the system behaves like this, then we can say this is neutral stable. And if you have something like this kind of properties, this is my displacement, this is the time axis. If the system starts from here and with time if it is oscillating and after some time if it is converging to the equilibrium point, in this case we are assuming the equilibrium point is zero, the displacement. And if you have such kind of response in the system, this kind of response we call damped oscillation. Because the system over time is due to the damping, it is oscillation in the system going out and it is converging to the equilibrium point.

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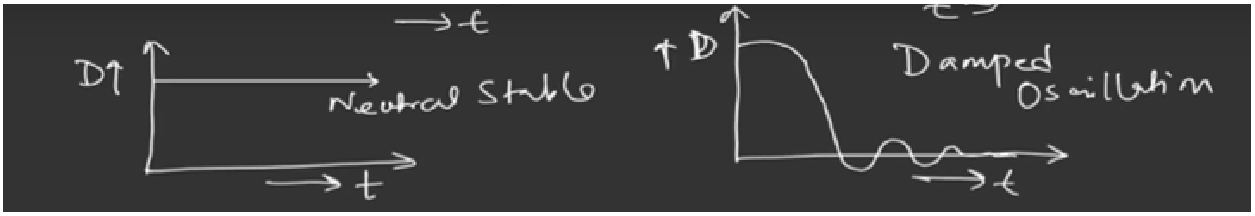

And if you're having the response something like this, this figure, this is my time axis, this is my displacement axis. And if the system starts from here and if it is over time, if it oscillating like this, and it is basically the system is going with the same magnitude of oscillation, then we can say this is undamped oscillations. Because if you look, the response remains to be the same over time, same magnitude of the same frequency of oscillations. So this kind of response we call undamped oscillations. And if you have some different kind of response with time, this is my displacement, this is the time axis.

And if you look at the response is like this, something like this, if you look carefully, the magnitude of oscillation is increasing with time. And if the system has this kind of behavior, we call divergent oscillations. So, this is how we can see how the dynamic stability in the system occurs. In this course, we'll be mostly talking about the dynamic stability of a system over time. And this is a very important part for designing the autopilot of a vehicle.

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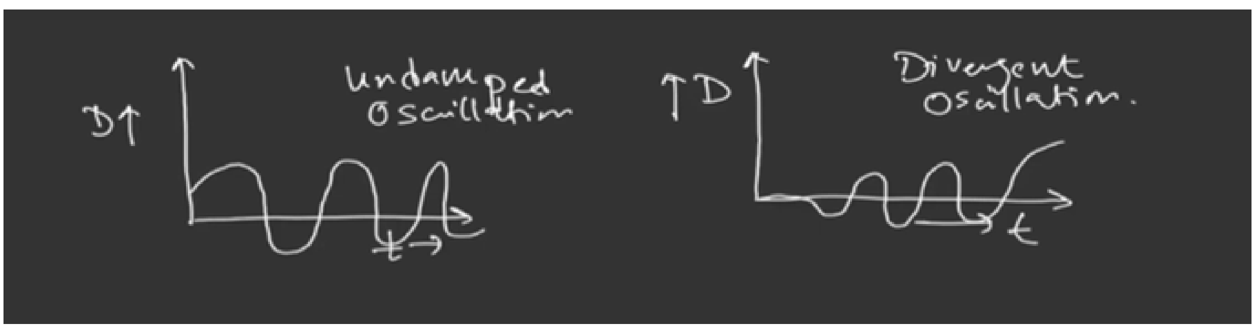

In the next lecture, we'll be talking about the dynamic stability of a nonlinear system. And in that lecture also we'll be discussing how we can find the equilibrium point of a nonlinear system. And based on the equilibrium point, we'll discuss the stability of the system. Thank you very much for attending.

We'll be continuing from the next lecture.