**Introduction to Aircraft Control System**

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**Week – 10**

**Lecture – 49**

# Examples of Approximations

Let us continue from the last lecture. We are taking the example of the longitudinal motion. So, we had the system $\dot{X} = AX$ where A is the matrix having basically 4 x 4 dimension. And after finding the characteristic equation, we had two pair of complex conjugate roots

$$\lambda_{1,2} = -0.071 \pm i(0.213)$$

$$\lambda_{3,4} = -2.5 \pm i(2.59)$$

And in $\lambda_{1,2}$ the damping ratio and natural frequency are low in magnitude, so you can say this is the phugoid mode and in $\lambda_{3,4}$ the damping ratio and natural frequency are quite large compared to the phugoid roots, so this is basically short period figured motion. So, now we will go with the approximation. So, from the phugoid approximation or you can say the long period approximation, we had the roots,

$$\lambda_{1,2} = \frac{X_u \pm \sqrt{X_u^2 + 4\frac{Z_u g}{u_0}}}{2}$$

$$\omega_{n,P} = \sqrt{\frac{-Z_u g}{u_0}} = \sqrt{2}\frac{g}{u_0}, \quad \xi_P = -\frac{X_u}{2\omega_{n,P}}$$

Basically, from the longitudinal linearized equation, we are getting these four roots and these four roots, we had divided into two parts and one pair yields the phugoid mode and another pair yields the short period mode. Now, we will check with the approximations, phugoid approximation and short period approximation with these results and how we can come up with the different properties in terms of the time period and number of cycles in the period and the time to reach the double or half in the amplitude.

And for the short period approximation, we have the eigenvalues or the roots of the characteristic equation, we can write

$$\lambda_{1,2} = -\xi_{SP}\omega_{n,SP} \pm i\omega_{n,SP}\sqrt{1-\xi_{SP}^2}$$

$$\omega_{n,SP} = \sqrt{M_q \frac{Z_\alpha}{u_0} - M_\alpha}$$

$$\xi_{SP} = \frac{-\left(M_q + M_{\dot{\alpha}} + \frac{Z_\alpha}{u_0}\right)}{2\omega_{n,SP}}$$

So let us find the values of time period and time to get the half and doubling amplitude and the number of cycle in the time period. Let us find for both the approximations then we can compare what we have done in the last lecture, whatever the time period and approximate the values we have from this phugoid mode and short period mode we can compare them. So, let us start with the phugoid mode first, in phugoid approximation we had

$$\omega_{n,P} = \sqrt{2}\frac{g}{u_0} = 0.26\, rad/s$$

$$\xi_P = -\frac{X_u}{2\omega_{n,P}} = 0.087$$

$$\lambda_{1,2} = -0.023 \pm i\, 0.26$$

$$P = \frac{2\pi}{\omega} = 24.2\, s$$

$$t_{1/2} = \frac{0.693}{|\eta|} = 30\, s$$

$$N_{t/2} = 0.110\frac{|\omega|}{|\eta|} = 1.24\, cycles$$

And for short period approximations, we get

$$\omega_{n,SP} = \sqrt{M_q \frac{Z_\alpha}{u_0} - M_\alpha}$$

Recall that $Z_\alpha = u_0 Z_w, M_\alpha = u_0 M_w, M_{\dot{\alpha}} = u_0 M_{\dot{w}}$

$$\omega_{n,SP} = 3.6\, rad/s$$

$$\xi_{SP} = 0.69$$

And the Eigen values

$$\lambda_{1,2,SP} = -\xi_{SP}\omega_{n,SP} \pm i\omega_{n,SP}\sqrt{1-\xi_{SP}^2} = -2.48 \pm i\ 2.61$$

$$P = \frac{2\pi}{\omega} = 2.4\ s$$

$$t_{1/2} = \frac{0.693}{|\eta|} = 0.278\ s$$

$$N_{t/2} = 0.110\frac{|\omega|}{|\eta|} = 0.16\ cycles$$

So here if you notice, from the linearized longitudinal motion dynamics which is basically $\dot{X} = AX$ where A is the matrix and let me write in table format

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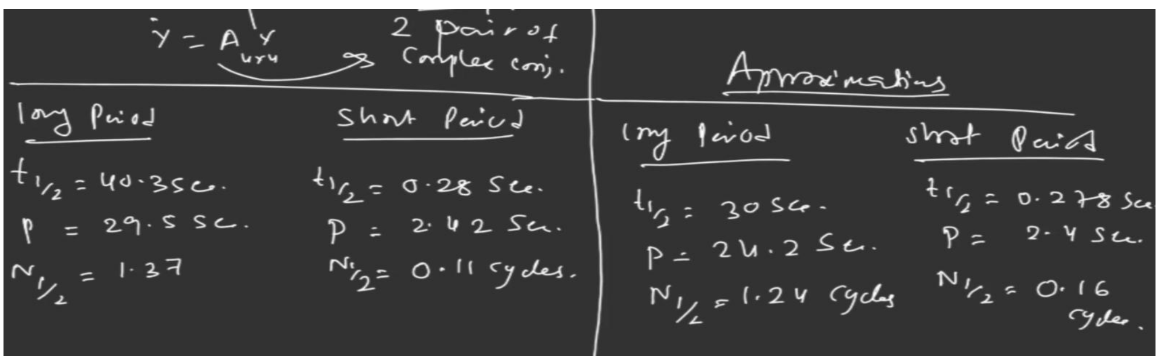


So here if you notice carefully the short period approximation is found to be in closure agreement with the exact solution, and the long period approximation actually differ from the exact solutions, hence in general the short period approximation is the more accurate. So these are the takeaway from the this analysis, so let's stop it here and so here actually we have completed the longitudinal motion dynamics with the different approximation. So this is the last lecture for the longitudinal motion, in the next class we'll be starting with the lateral directional motion and how we can come up with the different modes under the lateral directional motion. Thank you.