Introduction to Aircraft Control System

Prof. Dipak Kumar Giri

Department of Aerospace Engineering

IIT Kanpur

Week – 10

Lecture – 47

Period, Time to Half or Double the Amplitude and Number of Cycles

Now we are going to start a new topic in the longitudinal motion dynamics. If you notice the approximations in the longitudinal motions, we had Δu and $\Delta\theta$ as design variables for the long period motion or approximations I can say and Δw or $\Delta\alpha$ and Δq are the state variables which involves in the short period motion. And this if you notice these variables actually arise in the dynamics due to the perturbations in the system and now we are going to study how these variables going to behave over time. So, basically we're going to study the dynamic response of the system with time in the longitudinal motion without the control in the system. So, here we are going to study few important terms through which we can study the behavior of the dynamic or dynamics of the system. So, these terms we can denote by period, time to half or double amplitude, number of cycles for doubling, or halving amplitude. So, using these terms we can study how the system going to behave over time. If you want to modify this behavior of the system response we can come up with the flight control or any other techniques we can change the behavior of the system. But first we need to study the natural behavior of these perturbations or perturbed variables Δu, $\Delta\theta$, $\Delta\alpha$, Δq and how we can achieve this kind of activities. Let's go step by step.

Longitudinal motion involves the pitching motion of an aircraft (rotation about the lateral axis), the period, time to half or double amplitude and the number of cycles or doubling or halving the amplitude are well connected or interconnected, interconnected through the dynamic response of the aircraft's pitching motion. And point in the context of longitudinal flight motions which is rotation about the lateral axis starting with the period, because these are very important terms to study the behavior of the system because this is very important to design the control system. If you don't know exactly what is happening in the response, how we can come up with that control? So, we need to study these kind of properties to design a suitable control algorithm for the system. The period, time to half or double amplitude and the number of cycles or doubling or halving the amplitude are essential here for several reasons. The first reason is dynamic response analysis. So, here understanding the dynamic response of an aircraft to longitudinal inputs. basically, we can say the inputs are basically δ_e and δ_t. This is crucial for assessing its stability and maneuverability. By starting these parameters engineers can evaluate how the aircraft behaves under various flight conditions and control input. This is very very important that engineers can come up with the suitable controls for adjusting the system behavior. The period, time to half or double amplitude and number of cycles or doubling or halving the amplitude provide a valuable insights into the aircraft natural frequency and damping characteristic. Because these are the very important part for this period and time to have a double amplitude number of cycles. So, these are the dominant parameters which are going be used to find the type of parameters like period, time to have a double amplitude, number of cycles of for double or having the amplitude in the response.

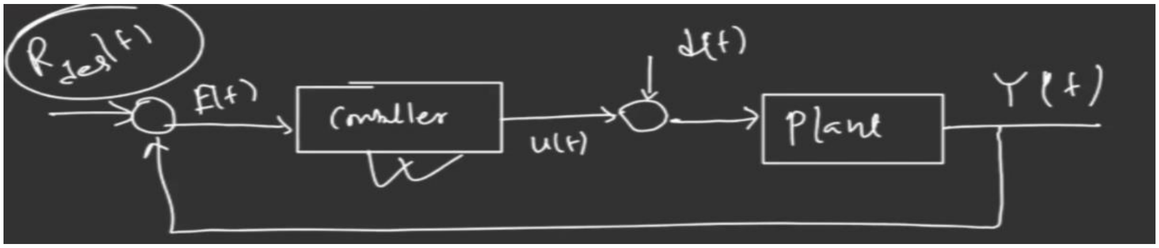
This information is fundamental for designing flight control system. If you remember, since in the attitude control problem of the aircraft, so we came up with the relation if you remember the PID control gains with the damping and damping natural frequency in the system. The same way you can do it here, so if you know this natural frequency and damping characteristic, you can come up with a suitable control design for the system. If you remember, in the beginning of this course we have considered the standard equation and it was

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$1 + G_0(s) = 0$$

So, this is the open-loop transfer function. So, based on the roots of this characteristic equation, we can comment the system is stable or not. So, those roots can be defined in terms of this damping ratio here and the natural frequency here. So, this is how we can do it. The period, time to half, and time to double the amplitude, helps the engineers assesses the stability characteristic of the aircraft and identify potential issues that may affect its flight performance. And final part is pilot training and safety. The pilot needs to understand how an aircraft responds to control input and external disturbances to operate it safely and effectively. So, it means we have to design the control system such that it can mitigate the external disturbance and how the pilot can track the desired values effectively.

(Refer Slide Time 19:24)



So, for example, if you have controller here and the controller is giving the output u(t) and also in the system there is a disturbance for example d(t) and the combined control input and disturbance going to the plant which is nothing but the database of the aircraft and this is the output and we have summing point and this is $R_{des}(t)$ and this is the error E(t). So, we have to design the controller in such that it can mitigate the external disturbance and Y(t) can track $R_{des}(t)$.

So, this thing can be done if you study the dynamic response of the system carefully and knowledge of the period, P and time to half $T_{1/2}$ and double amplitude T_2 and damping characteristic allows pilots to anticipate and mitigate potentially dangerous flight conditions. So it is basically enhancing the flight safety and again if we can come up another point in aircraft performance optimization by analyzing the parameters like the period or time to half or double amplitude number of cycles in the response in the specified time period by analyzing these parameters. Engineers can identify opportunities to improve the aircraft performance and efficiency adjustments aircraft design or flight control system (FCS) which can be made to optimize its dynamic response and enhance the overall performance so these are the points basically we need to tackle the dynamic response of the longitudinal motion of the aircraft. So now if you go back and the roots we come from the phugoid motion and the short period motion we had let me rewrite the equations, the roots pf C.E. for phugoid and short period are

$$\lambda_{1,2} = \frac{X_u \pm \sqrt{X_u^2 + 4\frac{Z_u g}{u_0}}}{2}$$

$$\lambda_{1,2} = -\xi_{SP}\omega_{n,SP} \pm i\omega_{n,SP}\sqrt{1 - \xi_{SP}^2}$$

Now we'll be using these roots to study a period, time to half or double amplitude number of cycles for doubling and halving amplitude, so if you can study these parameters using these roots then we can actually study the dynamic response of the individual approximations. Let's assume a root of the of the characteristic equation can be defined as

$$\lambda = \eta \pm i\omega$$

and this root actually tells us what type of response we have. Suppose you have characteristic equation

$$\lambda^2 + 2\lambda + 1 = 0$$

So this is basically an ODE equation

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 0$$

$$y(t) = e^{\lambda t}$$

$$\frac{dy}{dt} = \lambda e^{\lambda t}$$

$$\frac{d^2y}{dt^2} = \lambda^2 e^{\lambda t}$$

And if I substitute all parameters in above equation will have the characteristic equation

$$\lambda^2 + 2\lambda + 1 = 0$$

and the roots of this characteristic equation actually tells us the response of the system so using these roots we can come up with

$$x(t) = a_1 e^{()} + a_2 e^{()}$$

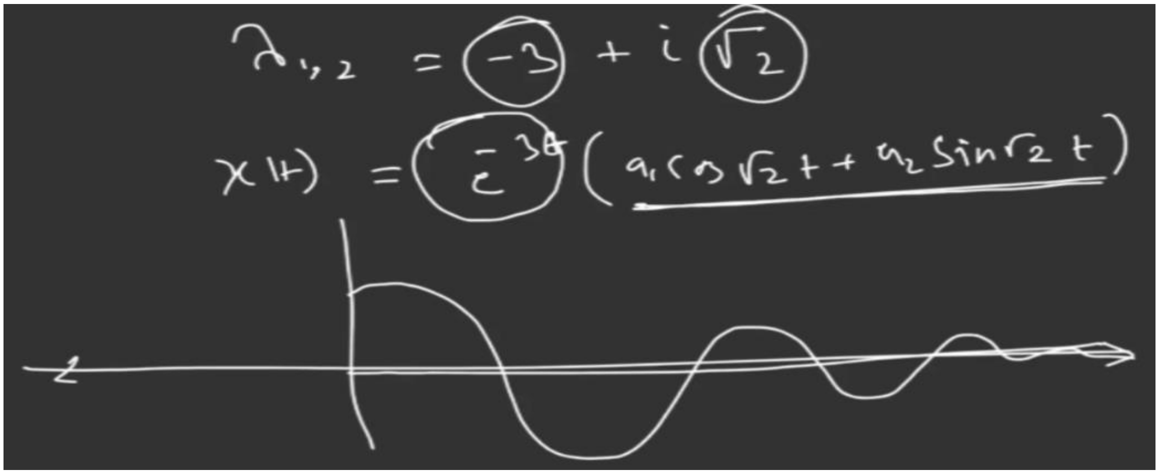
Or if it is complex then we have different structure, it will include $\cos \omega t + \sin \omega t$. If you notice here if the roots are complex so this basically talks about the real part and rest of the part actually talks about the how the response will be oscillating with some frequency so this part will tell us the amplitude of the response and this part will tell us how the system will oscillate with some frequency because the magnitude actually one here but the frequency will be different based on ω, so this part is very important how the system will be, what kind of amplitude the system will have, this talks about the real part of the root so that's why you can write the root of the system tells us what type of response the aircraft will have and if roots are real, response will be either pure divergence or subsidence depending on root is positive or negative, so what does it mean? Suppose we have

$$\lambda = -3 + 4i = \eta + \omega i$$

So here ω is four and η is minus three, so it means the response will be decaying over time as time proceed to infinity but if it is plus three, then it diverge from the equilibrium point. So that's why based on the polarity of the roots we can comment whether the response will be divergence or subsidence and also you can write if the roots are complex motion will be either damped or under damped sinusoidal oscillation so for example if the roots comes out to be

$$\lambda_{1,2} = -3 + i\sqrt{2}$$

(Refer Slide Time 34:31)



So we can write the response will be

$$x(t) = e^{-3t}\left(a_1 \cos\sqrt{2}\, t + a_2 \sin\sqrt{2}\, t\right)$$

So this part talks about the how the system will oscillate and this part how the system will be decaying, so if I plot and if I see this is my the reference line so it may start from here and over time it will be decaying because the exponential part is negative. So that's why you can comment how the response will look like, so let's stop it here in the next lecture we'll connect how we can study the period, time to half or double amplitude number of cycles for doubling or halving the amplitude in the longitudinal motion of the aircraft. So if you can understand that part very nicely then you can design the control system precisely for starting the dynamic analysis of the system with the control. Thank you.