Introduction to Aircraft Control System

Prof. Dipak Kumar Giri

Department of Aerospace Engineering

IIT Kanpur

Week – 10

Lecture – 46

Short Period Approximations

In the last lecture, we have discussed about the short period approximations in the longitudinal motion of the aircraft and had the expression

$$M_{\alpha} = u_0 M_w$$

$$Z_w = \frac{1}{m}\frac{\partial Z}{\partial w} = u_0 Z_{\alpha}$$

$$M_w = \frac{1}{m}\frac{\partial M}{\partial w} = u_0 M_{\alpha}$$

Similarly we can find

$$Z_{\alpha} = u_0 Z_w$$

$$M_{\dot{w}} = \frac{1}{I_y}\frac{\partial M}{\partial \dot{w}}$$

$$M_{\dot{\alpha}} = u_0 M_{\dot{w}}$$

Let me rewrite again the short period motion in state space form

$$\begin{bmatrix}\Delta\dot{w}\\ \Delta\dot{q}\end{bmatrix} = \begin{bmatrix} Z_w & u_0 \\ M_q + M_{\dot{w}} Z_w & M_q + M_{\dot{w}} u_0\end{bmatrix}\begin{bmatrix}\Delta w\\ \Delta q\end{bmatrix} \dots\dots Eq(1)$$

Eq. (1), we will be writing in terms of α alpha. We know that

$$\Delta\alpha = \frac{\Delta w}{u_0}$$

Hence

$$\begin{bmatrix} \Delta\dot{\alpha}u_0 \\ \Delta\dot{q} \end{bmatrix} = \begin{bmatrix} Z_\alpha & u_0 \\ \frac{M_\alpha}{u_0} + \frac{M_{\dot{\alpha}}}{u_0}\frac{Z_\alpha}{u_0} & M_q + \frac{M_{\dot{\alpha}}}{u_0}u_0 \end{bmatrix} \begin{bmatrix} \Delta\alpha u_0 \\ \Delta q \end{bmatrix}$$

From this state-space model, we can get

$$\Delta\dot{\alpha} = \frac{Z_\alpha}{u_0}\Delta\alpha + \Delta q \ldots\ldots Eq(2)$$

$$\Delta\dot{q} = M_\alpha\Delta\alpha + \frac{M_{\dot{\alpha}}}{u_0}Z_\alpha\Delta\alpha + \left(M_q + M_{\dot{\alpha}}\right)\Delta q \ldots\ldots Eq(3)$$

From Eqs.(2) and (3)

$$\begin{bmatrix} \Delta\dot{\alpha} \\ \Delta\dot{q} \end{bmatrix} = \begin{bmatrix} \frac{Z_\alpha}{u_0} & 1 \\ M_\alpha + M_{\dot{\alpha}}\frac{Z_\alpha}{u_0} & M_q + M_{\dot{\alpha}} \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta q \end{bmatrix} \ldots\ldots Eq(4)$$

This equation actually is the state space model of short period motion and the motion variables are α and q so here we can find the roots of this characteristic equation from the system matrix, we can write

$$|\lambda I - A| = 0$$

Our main motivation is you have to find the roots of this equation, so here we'll compare this characteristic equation with the standard characteristic equation which is

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\begin{vmatrix} \lambda - \frac{Z_\alpha}{u_0} & -1 \\ -M_\alpha - M_{\dot{\alpha}}\frac{Z_\alpha}{u_0} & \lambda - \left(M_q + M_{\dot{\alpha}}\right) \end{vmatrix} = 0$$

Solving, we get

$$\lambda^2 - \left(M_q + M_{\dot{\alpha}} + \frac{Z_\alpha}{u_0}\right)\lambda + M_q\frac{Z_\alpha}{u_0} - M_\alpha = 0 \ldots\ldots Eq(5)$$

Comparing Eq. (5) with the standard characteristic equation, we get

$$\omega_{n,SP} = \sqrt{M_q\frac{Z_\alpha}{u_0} - M_\alpha}$$

$$\xi_{SP} = \frac{-\left(M_q + M_{\dot{\alpha}} + \frac{Z_\alpha}{u_0}\right)}{2\omega_{n,SP}}$$

One thing I need to say here why we are writing equations in terms of α, in Eq. (4) if you look at the state matrix all the terms in the state matrix are written in terms of alpha and q because here our state vector is $\Delta\alpha$ and Δq and how this state vector as represented in terms of the stability derivatives which are function of pitch rates, that's why we have changed the variable Δw to $\Delta\alpha$. So, here we are having the damping ratio and natural frequency in the short period motion. Now we're going to study how this natural frequency and short natural frequency and damping ratio of short period motion and long period motion going to affect the overall behavior of the system. So, basically here we are going to study the period how the period or the response of the individual approximation can be analyzed using these parameters. Now the roots of the characteristic equation defined by Eq. (5) are

$$\lambda_{1,2} = -\xi_{SP}\omega_{n,SP} \pm i\omega_{n,SP}\sqrt{1-\xi_{SP}^2} \dots\dots Eq(6)$$

equation six are lambda one two they are two roots so, omega n short period one minus zeta minus zeta short period omega n short period plus minus i this is the complex number omega n short period one minus zeta square short period. If you notice here the roots are defined in terms of the damping ratio and natural frequency. Now let's conclude this part short period approximation and long period approximation

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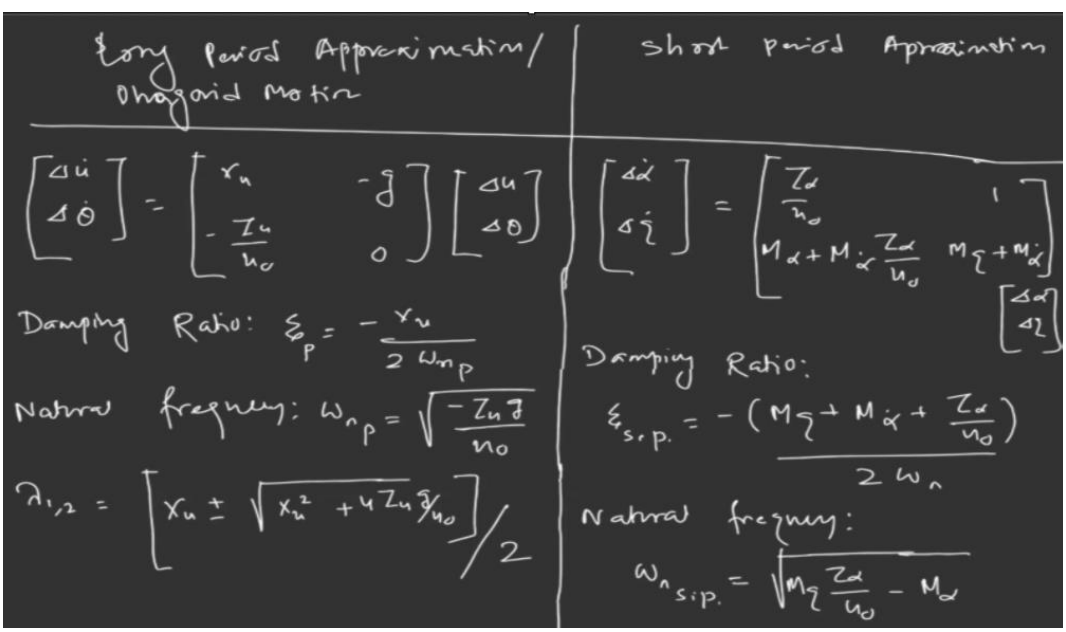

And the roots we can write in terms of Eq. (6). So this is how we can analyze the long period approximation and short period approximation and how we can come up with the damping ratio and natural frequency for both the approximations, in the next lecture we'll be discussing how this damping ratios and natural frequencies for both the approximations going to affect the overall response of the system and how we can analyze the behavior of the different motions which are basically the short period approximation and long period approximation, so we'll discuss them in detail. Thank you.