Introduction to Aircraft Control System

Prof. Dipak Kumar Giri

Department of Aerospace Engineering

IIT Kanpur

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Lecture – 45

Longitudinal Period Approximations (Cont.)

Now, let us go with the same motion, the long period motion what you have done in the last lecture. Let us rewrite the equations what you had in the last lecture.

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & -g \\ \frac{-Z_u}{u_0} & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta\theta \end{bmatrix} \dots\dots Eq(1)$$

$$\lambda^2 - X_u\lambda - \frac{Z_u g}{u_0} = 0 \dots\dots Eq(2)$$

$$\lambda_{1,2} = \frac{X_u \pm \sqrt{X_u^2 + 4\frac{Z_u g}{u_0}}}{2}$$

$$\omega_{n,p} = \sqrt{-\frac{Z_u g}{u_0}}$$

$$\xi_p = -\frac{X_u}{2\omega_{n,p}}$$

Now we'll see how the magnitude of the damping ratio is less in the phugoid motion that is we need to study how it is an important parameter for having the response sustained oscillation because in the short period motion as you have noticed that oscillations goes dies out with time but in the long period motion due to the long period the response goes to stay for longer duration and it has the sustained oscillation what we have discussed.

Now let's look how it is happening remember that remember that X_u and Z_u are the aerodynamics derivatives divided by the mass of the airplane that is we can write

$$X_u = \frac{1}{m}\frac{\partial X}{\partial u_0}$$

Here we substitute u in place of u_0 because the forward velocity u_0 is close to the total velocity, also you can write

$$Z_u = \frac{1}{m}\frac{\partial Z}{\partial u_0}$$

It should be noted that when we change forward velocity of the aircraft which is u_0 , drag changes in the opposite direction hence we can write or we can approximate X with the drag component in the x direction. Similarly Z can be approximated with your lift, further you can write

$$X_u = -\frac{\rho u_0 s C_D}{m}$$

$$Z_u == -\frac{\rho u_0 s C_L}{m}$$

Here we'll substitute the damping coefficient and natural frequency term, we already know that

$$\omega_{n,p} = \sqrt{-\frac{Z_u g}{u_0}} = \sqrt{\frac{\rho s C_L g}{m}}$$

For level trim flight

$$C_L = \frac{2mg}{\rho s u_0^2}$$

Hence

$$\omega_{n,p} = \sqrt{2}\frac{g}{u_0}$$

If you notice here carefully the aircraft which has higher velocity it has the lower natural frequency, hence the aircraft with lower velocity for example the gliders have high natural frequency, similarly you can do the same analysis for the damping ratio. We get

$$\xi_p = \frac{1}{\sqrt{2}}\frac{1}{L/D}$$

So it means if the aircraft has high L by D ratio it will have less damping ratio and if the damping ratio is less the system will take more time to settle down or the period of the oscillation will be quite large. Also we need C_L, C_D, M, u_0, ρ to approximate the phugoid without going into the system matrix A in Eq. (1). Hence, to improve damping of phugoid motion, designer would have to reduce L by D of the airplane. But this is unacceptable because it would degrade the performance of the aircraft so in this case we should have the automatic stabilization system on board to provide the proper damping to the system. So it means what you have whatever things we have done till now in this phugoid motion we have done everything in the natural motion dynamics but if you and for the in the natural dynamics it comes out of that the damping ratio is a function of L by D ratio but if you can add the control system to the aircraft, this low damping ratio can be improved and we can come up with some stable response without having the sustained oscillation in the response we can have the dying oscillation in the system so that's why you should have the automatic control system in the onboard the aircraft to improve this kind of phugoid motion.

Now let's start with the short period approximation on how we can look on the mathematical point of short period approximation. Let us write some note before it mathematical part, if the trimmed aircraft is subjected to step input of elevator or a vertical dust, it will enter a pitching motion. It means, until unless there is no external perturbation to the system, the system will not have some of the overshoot or undershoot in the response. But once the system is subjected to any step input or elevator, step input of the elevator or the vertical dust, it will go through the motion. But with time, that motion will be, dies out. I mean, the oscillation in the system will goes out and it will go to the stable motion. We can write the short period approximation as a motion with very little speed changes, but appreciable, angle of attack changes on relatively fast time scales. So here, the angle of attack will change over time. The response will be going out, oscillation will goes out, dies out very quickly. So this means the changes in the angle of attack is very fast. We'll see in mathematics how it's happening in a while. Now, here, in this condition, we can write, we assume $\Delta u = 0$, the change in the velocity assumed to be zero, And hence, we can drop X force, which generates Δu perturbations in the system of X force equation from the longitudinal equation, and we can write,

$$\begin{bmatrix} \Delta\dot{w} \\ \Delta\dot{q} \end{bmatrix} = \begin{bmatrix} Z_w & u_0 \\ M_q + M_{\dot{w}} Z_w & M_q + M_{\dot{w}} u_0 \end{bmatrix} \begin{bmatrix} \Delta w \\ \Delta q \end{bmatrix} \ldots\ldots Eq(3)$$

This equation can be written in terms of the angle of attack, because in the short period of motion, mostly we will be considering the angle of attack, how it changes very fast and gets back to the stable due to the high damping in the system. And by using the following relationship, we can write

$$\Delta\alpha = \frac{\Delta w}{u_0}$$

We can replace the derivatives w and $\dot{w}$ with derivatives due to α and $\dot{\alpha}$ using the following equations. The definition of the derivative M_α is

$$M_\alpha = \frac{1}{I_y}\frac{\partial M}{\partial \alpha} = u_0\left(\frac{1}{I_y}\frac{\partial M}{\partial w}\right) = u_0 M_w$$

Likewise $Z_w = \frac{1}{m}\frac{\partial Z}{\partial w}$

Okay, let's stop it here, we will continue from the next lecture because we need more mathematical analysis in this short period approximation. So we will wind up this part in the coming lecture. Thank you.