Introduction to Aircraft Control System

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Week – 09

Lecture – 44

Longitudinal Period Approximations

As of now we have discussed how we can come up with the linearized model of the longitudinal and lateral-directional motion of the aircraft in detail. Now we will study how we can come up with the different approximations in the lateral-directional and longitudinal motion. So we'll study the natural behavior of the system without having the control input in the system. So here in the longitudinal motion we'll have two approximations which are basically the short period and long period approximations and we'll study their dynamic analysis of the respective approximations based on the time period, natural frequency in the system and also the time to get the full or half amplitude of the response. The similar procedure also will follow for the lateral-directional motion of the aircraft. Here you're going to study the roots of the natural systems and based on the roots we'll come up with the three different modes which are basically the spiral mode, dutch roll and roll mode and we'll study their dynamic analysis and also we'll come up with the time period and natural frequency and time to get the full and half amplitude of the response and also we'll have some examples how we can validate these different approximations.

Now we are going to study how we can study the longitudinal motion approximation in linear regime. Under this there are two important approximations we're going to cover up. First is the long period approximation or the phugoid mode another is the short period motion. So in longitudinal motion approximation, here basically how the system going to behave without control input basically if you notice the longitudinal motion equation we had $\dot{X} = AX + BU$ form so here $U = 0$. So here we are going to study how the system going to behave naturally without the control input so how the system going to behave like $\dot{X} = AX$ over time without the control input. So we shall examine the longitudinal motion of the aircraft without control input. The longitudinal motion of the aircraft disturbed from the equilibrium flight condition can be characterized by two oscillatory modes of motion. So, these are one is the long period motion, another is the short period motion. The long period motion also sometimes we call phugoid motion, phugoid mode you can say. So, let's look how the long period motion going to be, how it look like and how the damping ratio and the oscillation going to affect the motion dynamics, the long period motion. So, what are the different states involves in the long period motions. The primary states involved first is the velocity v and second important element is the pitch angle θ or altitude h. So, these are the primary states involves in the long period motion. Why these are the variable involves here let me look. If you see the, how the change in altitude going to affect the aircraft speed in the long-term direction. So, if you write the change in altitude and this is the time axis.

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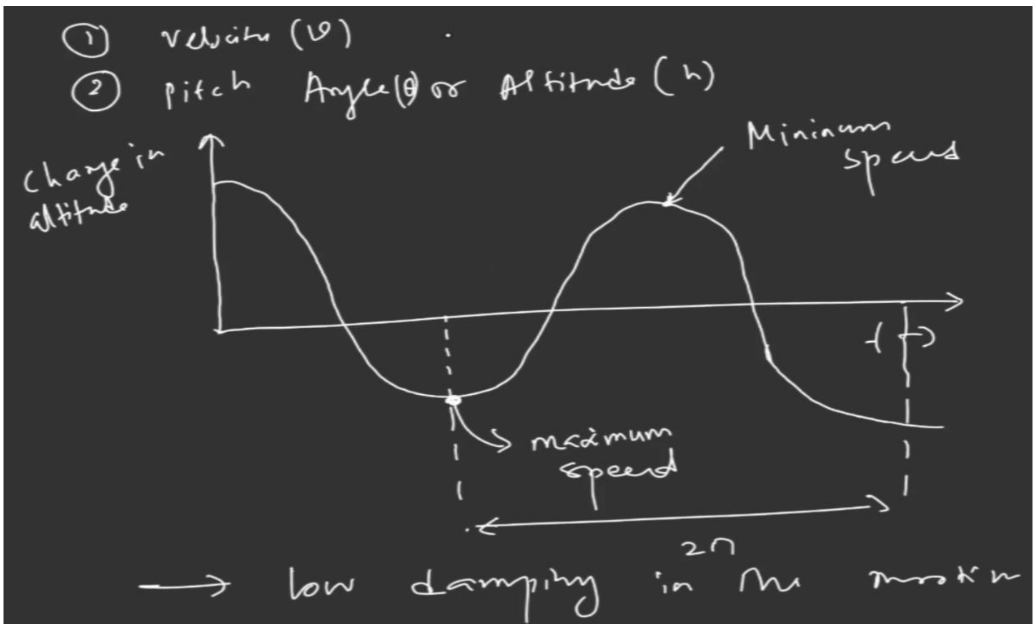

So, if it's starting from here and the motion can be characteristic that is by like this. So, here when the altitude is minimum or I can say the when the aircraft is descending so that this is the minimum value of the change in altitude and at this point the aircraft will attain maximum speed due to the kinetic energy of the system and this is the point where the aircraft will attain the minimum speed and if you this is we can characterize this is the total period of the system 2 pi and if you notice if you see practically this period first and more than 30 second or more. So, that's why this is basically long period taking more time to complete the one complete cycle. Also, the reason to have this high period is the the system basically in this condition generally has low damping in the motion and due to which the period is actually increasing. Once it is a high damping the system will automatically will be the oscillation will die out over time but in this case damping is very very low.

So, that's why the system has some sustained oscillation throughout the trajectory. Now, let's look how we can see the short period motion. Anyways, we'll have the mathematical description of the motion in a while. So, let's look the short period approximation but sometimes the long period also the angle of attack also comes into picture because if you see in case of phugoid motion even the pitch angle does not change dramatically the angle of attack factor as it influences the aircraft trajectory. So, sometime it is also effect but the dominant role of the angle of attack comes into the short period motion of the aircraft.

So, here we can write the short period motion approximation focuses on the aircraft pitch dynamics and altitude stability for a short time frame. Here basically short period motion the damping ratio in the motion dynamics is high and also it has high which is a high frequency in the natural dynamics. So, if the frequency is high the period will be low automatically and due to the damping is high in this case the oscillation will die out over time. If you check the how the angle of attack going to affect in the short period motion which is the y-axis represents the change in angle of attack is the time and if the angle is just starting from here over time it will die out. So, due to the damping is high and the high frequency in the system the short period here that the period is short and ends.

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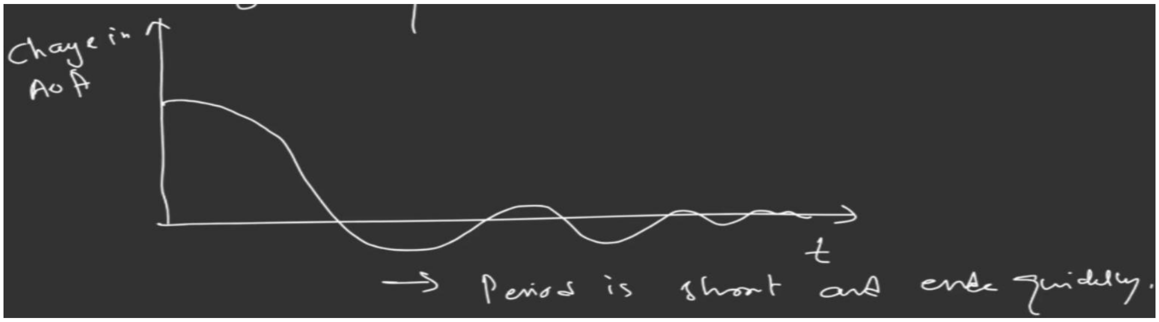

So, this is how we can characterize the short period motion of the aircraft. Now, here actually generally which state we will be looking in the short period motion the states will be looking one is the pitch rate q and second state will be looking angle of attack. So, these are the two state we will be looking in the short period motion and how the aircraft velocity and the pitch angle going to affect the rate of change of pitch angle how it is being driven we will be looking in the long period motion of the aircraft. Now, let us go step by step the mathematical part first let us start with the long period motion. So, here the main the extension of the two have the long period motion the forward mode is the interchanging of potential and kinetic energy about the equilibrium point.

Basically, when you are talking about the linear system we study the system how it is going to evolve about the equilibrium point. So, that's why it's about the equilibrium point this due to the exchange of this potential and kinetic energy we can study this long period motion of the aircraft about the equilibrium altitude and air speed. So, in this case when the long period motion is characterized by the changes in pitch altitude and velocity at nearly constant angle vector . So, here let me go through some basic stuff now let's find a relation between the the aircraft velocity along the x-axis and z-axis and how they are related to the total velocity of the aircraft. For example, this is the aircraft

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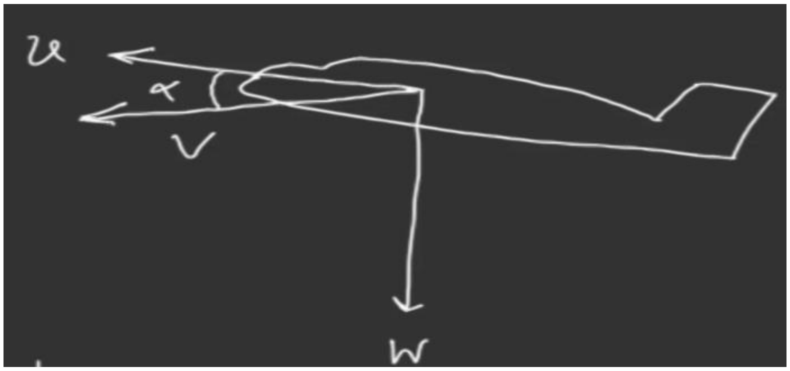

this indicates u and this is for example w the velocity along the z-axis and let's assume this is the total velocity of the aircraft which is V and let's assume this is alpha angle of attack. So, from this diagram we can write

$$\tan\alpha = \frac{w}{u} \Longrightarrow \alpha \approx w \Longrightarrow \dot{\alpha} = \dot{w}$$

u is the velocity along the x-axis is constant in this. So, this part will be using now and but this is the expression we are getting from this figure but as we mentioned here at nearly constant angle of attack so in this case we can write change in angle of attack is zero that is

$$\Delta\alpha = \frac{\Delta w}{u_0} \Longrightarrow \Delta\alpha = 0 \Longrightarrow \Delta w = 0$$

Here u_0 is the velocity which is assumed to be constant here, so this is what you can write now since only in pitch angle theta altitude and u_0 we can neglect we can easily neglect the pitching moment equation from the longitudinal state space model that is we can write $\Delta\dot{q} = 0$ so what does it mean so here we have longitudinal motion equation here so in this case we can ignore $\Delta\dot{q}$ term so the steady state value of pitch attitude we can write $\theta_0 = 0$. Again since $\Delta w = 0 \Longrightarrow \Delta\dot{w} = 0$. From the longitudinal state space model we can write

$$\Delta\dot{w} = Z_u\Delta u + Z_w\Delta w - u_0\Delta q - g\sin\theta_0\Delta\theta$$

Substituting the above assumptions we get

$$\Delta q = \frac{-Z_u \Delta u}{u_0} \ldots \ldots Eq(1)$$

We can write the state space form as

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & -g \\ \frac{-Z_u}{u_0} & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta\theta \end{bmatrix} \ldots \ldots Eq(2)$$

This is basically second order system so if the system is second order we'll have two eigenvalues because the matrix dimension is two so two eigenvalues $\lambda_{1,2}$ and based on which we will study the phugoid mode. Let's see how we can find eigenvalues from this state equation two so this is the characteristic equation

$$|\lambda I - A| = 0$$

$$\left| \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} X_u & -g \\ \frac{-Z_u}{u_0} & 0 \end{bmatrix} \right| = 0$$

$$\lambda^2 - X_u\lambda - \frac{Z_u g}{u_0} = 0 \ldots \ldots Eq(3)$$

$$\lambda_{1,2} = \frac{X_u \pm \sqrt{X_u^2 + 4\frac{Z_u g}{u_0}}}{2}$$

So these are the eigenvalues for this long period motion of the aircraft, now if you compare this characteristic equation with the standard characteristic equation

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\omega_{n,p} = \sqrt{-\frac{Z_u g}{u_0}}$$

$$\xi_p = -\frac{X_u}{2\omega_{n,p}}$$

Where $\omega_{n,p}$ and ξ_p represents phugoid natural frequency and damping ratio respectively. So now let's stop it here we'll continue from the next lecture how we can come up with more details about the long period motion of the aircraft. Thank you.