Introduction to Aircraft Control System

Prof. Dipak Kumar Giri

Department of Aerospace Engineering

IIT Kanpur

Week – 09

Lecture – 43

State-Space Form of Lateral/Directional Equations

In this lecture, we will be deriving the state space model of the lateral directional motion of the aircraft. In the last lecture, we have discussed the linearized model of the velocity along the y axis of the side velocity going to happen due to the disturbance. And we have derived the linear model of Δv going to incorporate due to the disturbance in the system. And also we found the roll and yaw rates of the linearized model of those system. So, let me rewrite the equation what you have done in the previous lecture, the linearized model.

$$\Delta\dot{v} = Y_v\Delta v + Y_p\Delta p + (Y_r - u_0)\Delta r + \left(Y_\phi + \mathrm{g}\cos\theta_0\right)\Delta\phi + Y_{\delta r}\Delta\delta_r \dots\dots Eq(1)$$

$$\Delta\dot{p} = \left[L_v^* + \frac{I_{xz}}{I_x}N_v^*\right]\Delta v + \left[L_p^* + \frac{I_{xz}}{I_x}N_p^*\right]\Delta p + \left[L_r^* + \frac{I_{xz}}{I_x}N_r^*\right]\Delta r + \left[L_{\delta a}^* + \frac{I_{xz}}{I_x}N_{\delta a}^*\right]\Delta\delta_a + \left[L_{\delta r}^* + \frac{I_{xz}}{I_x}N_{\delta r}^*\right]\Delta\delta_r \dots\dots Eq(2)$$

$$\Delta\dot{r} = \left[N_v^* + \frac{I_{xz}}{I_x}L_v^*\right]\Delta v + \left[N_p^* + \frac{I_{xz}}{I_x}L_p^*\right]\Delta p + \left[N_r^* + \frac{I_{xz}}{I_x}L_r^*\right]\Delta r + \left[N_{\delta a}^* + \frac{I_{xz}}{I_x}L_{\delta a}^*\right]\Delta\delta_a + \left[N_{\delta r}^* + \frac{I_{xz}}{I_x}L_{\delta r}^*\right]\Delta\delta_r \dots\dots Eq(3)$$

Now we need one more variable the how the roll angle how it is going to change due to the lateral motion in the aircraft. So, final variable we have to find is $\Delta\dot{\phi}$ how it is going to change in the system in the lateral direction motion. This can be obtained from the kinematic relation as we can write

$$\dot{\phi} = p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \dots\dots Eq(4)$$

This equation non-linear equation basically because we have sine tan terms in the system and this is an also coupling term are there. So, system is equation is non-linear. So, we need to come up the linear form of this equation. So, we have to apply the small perturbation theory. So, here we will be incorporating

$$\phi = \phi_0 + \Delta\phi, p = p_0 + \Delta p, q = q_0 + \Delta q, \theta = \theta_0 + \Delta\theta$$

So, incorporating these variables in Eq.(4), we can write

$$\frac{d}{dt}(\phi_0 + \Delta\phi) = p_0 + \Delta p +$$

$$+(q_0 + \Delta q)\sin(\phi_0 + \Delta\phi)\tan(\theta_0 + \Delta\theta) + (r_0 + \Delta r)\cos(\phi_0 + \Delta\phi)\tan(\theta_0 + \Delta\theta)$$

Now we have find the linearized model of this equation using the definition of trim flight condition. This condition already we have done before, many times we have discussed in the lecture and small angle approximation. In small angle approximation, we get

$$\Delta\dot{\phi} = \Delta p + \Delta q \Delta\phi \tan(\theta_0 + \Delta\theta) + \Delta r \tan(\theta_0 + \Delta\theta) \ldots \ldots Eq(5)$$

Product of small perturbation are assumed to be zero. So, Eq. (5) can be written as

$$\Delta\dot{\phi} \approx \Delta p + \Delta r \tan(\theta_0 + \Delta\theta) \ldots \ldots Eq(6)$$

Lets simplify the expression $\tan(\theta_0 + \Delta\theta)$

$$\tan(\theta_0 + \Delta\theta) = \frac{\sin\theta_0 + \Delta\theta\cos\theta_0}{\cos\theta_0 - \Delta\theta\sin\theta_0} * \frac{\cos\theta_0 + \Delta\theta\sin\theta_0}{\cos\theta_0 + \Delta\theta\sin\theta_0}$$

$$= \tan\theta_0 + \frac{\Delta\theta}{\cos^2\theta_0} \ldots \ldots Eq(7)$$

Substituting Eq(7) in Eq(6) yields

$$\Delta\dot{\phi} \approx \Delta p + \Delta r \tan\theta_0$$

So, this is the another linearized equation in the lateral directional motion. Now, we will move to the how we can find the state space model of the system lateral direction motion of the aircraft. So, here we have to come up the form $\dot{X} = AX + BU$ here A will be the state matrix B will be the control matrix here X is the state of variable and U will be the control input. So, here basically X basically we can come up with the variable

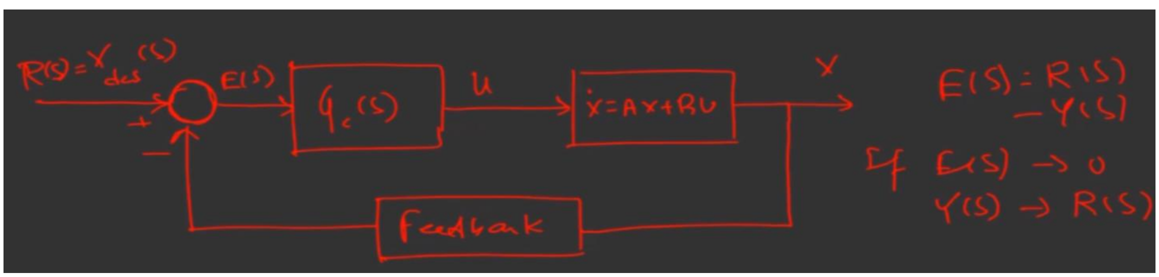
$$X = [\Delta U, \Delta p, \Delta r, \Delta\phi]$$

$$\dot{X} = \begin{bmatrix} \Delta\dot{v} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{bmatrix} = \begin{bmatrix} Y_v & Y_p & Y_r - u_0 & \\ L_v^* + \frac{I_{xz}}{I_x} N_v^* & L_p^* + \frac{I_{xz}}{I_x} N_p^* & L_r^* + \frac{I_{xz}}{I_x} N_r^* & Y_\phi + g\cos\theta_0 \\ & & & 0 \\ N_v^* + \frac{I_{xz}}{I_x} L_v^* & N_p^* + \frac{I_{xz}}{I_x} L_p^* & N_r^* + \frac{I_{xz}}{I_x} L_r^* & 0 \\ & & & 0 \\ 0 & 1 & \tan\theta_0 & \end{bmatrix} \begin{bmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta\phi \end{bmatrix} +$$

$$\begin{bmatrix} 0 & Y_{\delta r} \\ L_{\delta a}^* + \frac{I_{xz}}{I_x} N_{\delta a}^* & L_{\delta r}^* + \frac{I_{xz}}{I_x} N_{\delta r}^* \\ N_{\delta a}^* + \frac{I_{xz}}{I_x} L_{\delta a}^* & N_{\delta r}^* + \frac{I_{xz}}{I_x} L_{\delta r}^* \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{bmatrix}$$

Now we have to design the control algorithm I mean the control part and which is going to propagate our x dynamics so main purpose is how we can mitigate the perturbations $\Delta v, \Delta p, \Delta r, \Delta\phi$ so it's suitable control if you can come up with the algorithm is designed in such a way that it if it can handle such perturbations to zero then we can say this control will be stable control and if you want to design the closed loop control block diagram

(Refer Slide Time 26:23)



So this is the reference R(s) or you can write $X_{des}(s)$ in frequency domain we are writing and if this is the control G_c and this is the plant which is $\dot{X} = AX + BU$ and G_c is given the control digital control U the system and we have the actual output X and through the feedback element we can sense the actual output of the system and it can get back to the summing point and here we can find the error which needs to be minimized so if E(s) going to zero, Y(s) going to track R(s) because E(s)=R(s)-Y(s) so this is how we'll be moving in this course but before we proceed to the control part we need to study short period motion and another is the long period motion and under the lateral directional motion we'll be studying spiral, roll and dutch roll motions. So all this thing we'll be explaining in the next coming lecture so let's stop it here we'll continue from the next lecture. Thank you.