Introduction to Aircraft Control System

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Lecture – 41

Linearization of Lateral/Directional Dynamics

In the last lecture, we have discussed about the linearized model of the longitudinal motion of the aircraft. And in today lecture, we will be discussing how we can come up with the linearized model of the lateral and directional equation of motion of the aircraft. So, this is lecture number 41. Let us rewrite the equation of motion of lateral and directional equation, of motion of lateral directional motion of the aircraft

$$Y + mg\cos\theta\sin\phi = m[\dot{v} + ru - pw]$$

$$l = \dot{p}I_{xx} - \dot{r}I_{xz} + qr\left[I_{zz} - I_{yy}\right] - pqI_{xz} \dots\dots Eq(1)$$

$$n = -\dot{p}I_{xz} + \dot{r}I_{zz} + pq\left[I_{yy} - I_{xx}\right] + qrI_{xz}$$

Our main motivation is how we can come up with the linear form of this non-linear equations. Before we proceed, first we have to consider the trim flight or the steady level flight for the lateral directional motion of the aircraft. So, we can write the steady state trim flight for lateral directional equation as, we already have discussed the trim values for the lateral directional motion are for the values of $p_0 = q_0 = r_0 = \phi_0 = 0$. For these conditions, the steady state flight equation for the lateral directional equation we can write

$$Y_0 = m\dot{v}_0$$

$$l_0 = 0 \dots\dots Eq(2)$$

$$n_0 = 0$$

Now, we will apply the perturbation to these equations and we will come up the linear model of the individual equation, force equation, moment equation. Let us start with the first force equation along y axis. Let us rewrite the force equation in y direction

$$Y + mg\cos\theta\sin\phi = m[\dot{v} + ru - pw] \dots\dots Eq(3)$$

Now, we will include the perturbation for this system. So, we can write the variable in this equation, if you consider there is two component in each variable, one is the reference value, another is the perturb variable. So, if we consider the perturbations we can write

$$Y = Y_0 + \Delta Y, \theta = \theta_0 + \Delta\theta, \phi = \phi_0 + \Delta\phi, u = u_0 + \Delta u, v = v_0 + \Delta v, w = w_0 + \Delta w$$

$$p = p_0 + \Delta p, r = r_0 + \Delta r$$

If we apply this perturbation variable to Eq. (3), we can write

$$Y_0 + \Delta Y + mg\cos(\theta_0 + \Delta\theta)\sin(\phi_0 + \Delta\phi) = m\left[\frac{d}{dt}(v_0 + \Delta v) + (r_0 + \Delta r)(u_0 + \Delta u) - (p_0 + \Delta p)(w_0 + \Delta w)\right] \ldots \ldots Eq(4)$$

For small perturbation of $\Delta\theta, \cos\Delta\theta \approx 1, \sin\Delta\theta \approx \Delta\theta,$ solving the above term $\cos(\theta_0 + \Delta\theta)\sin(\phi_0 + \Delta\phi)$, we get

$$\cos(\theta_0 + \Delta\theta)\sin(\phi_0 + \Delta\phi) = \cos\theta_0\sin\phi_0 + \Delta\phi\cos\theta_0\cos\phi_0 - \Delta\theta\sin\theta\sin\phi_0 - \Delta\theta\Delta\phi\sin\theta_0\cos\phi_0 \ldots \ldots Eq(5)$$

Again we can know that during the trim flight condition, we know that

$$v_0 = p_0 = q_0 = r_0 = \phi_0 = l_0 = n_0 = 0$$

and also neglecting the products of small disturbance, we can write Eq. (5) as

$$\cos(\theta_0 + \Delta\theta)\sin(\phi_0 + \Delta\phi) = \Delta\phi\cos\theta_0 \ldots \ldots Eq(6)$$

Using the trim flight condition $Y_0 = m\dot{v}_0$ and considering stability axis $(w_0 = 0)$ using Eq. (6), Eq. (4) can be written as

$$\Delta Y + mg\Delta\phi\cos\theta_0 = m[\Delta\dot{v} + u_0\Delta r] \ldots \ldots Eq(7)$$

Side force Y is basically a function of v, p, r, ϕ, δ_r. Also, aileron deflection has very little influence on the side force, hence it can be neglected. So, now using the Taylor series expansion, the force, the perturbed force can be written as

$$\Delta Y = \frac{\partial Y}{\partial v}\Delta v + \frac{\partial Y}{\partial p}\Delta p + \frac{\partial Y}{\partial r}\Delta r + \frac{\partial Y}{\partial \phi}\Delta\phi + \frac{\partial Y}{\partial \delta r}\Delta\delta_r \ldots \ldots Eq(8)$$

Using Eq.(8), Eq. (7) becomes

$$\Delta\dot{v} = Y_v\Delta v + Y_p\Delta p + Y_r\Delta r + Y_\phi\Delta\phi + Y_{\delta r}\Delta\delta_r \dots\dots Eq(9)$$

Where $Y_v = \frac{1}{m}\frac{\partial Y}{\partial v}$ and so on are the aerodynamic derivatives divided by airplanes mass. Eq. (9) can be further simplified as

$$\left[\frac{d}{dt} - Y_v\right]\Delta v - Y_p\Delta p - (Y_r - u_0)\Delta r - \left(Y_\phi + g\cos\theta_0\right)\Delta\phi = Y_{\delta r}\Delta\delta_r$$

So, this is the linear form of the force equation along y axis. If you notice this equation are linear because the coefficient of the all the terms are scalar component or not time varying or we can say the time invariant and this system also you can say linear time invariant system because all the coefficient of the equation are linear time invariant. If you notice this equation it will be very clear because this terms, this term, this term, this term and this term are basically we generally get from the wind tunnel testing for the given flight conditions. And now let's stop it here we will continue from the next lecture in the how we can come the linear form of the moment equation along x and z axis then we will finally come up the state space model of the lateral and directional motion of the aircraft then we can move to the control. Thank you.