Introduction to Aircraft Control System

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Week – 01

Lecture – 04

In this lecture, we will be starting with a convolutional sum, which plays an important role for filing the transfer function. Then we will discuss why LTI system we should consider for linear control synthesis and what are the different assumptions we have to consider for finding the transfer functions of an LTI system. Then we will discuss the lump parameter system, a system which you can write in ordinary differential equation form. And then we will conclude the lecture. As you have seen in the previous lecture, the system is called LTI system if it follows the properties of proposition principle, and linear time invariant. Now let's consider the system.

Let's consider the following LTI system where I'm applying impulse and we are having impulse response. Let me draw it. We have an LTI system. So this, what I'm going to do here, is valid for the LTI system only.

So now we are applying to this LTI system, $\delta(n)$ and impulse, and we are having some impulse response. Let me denote it as $h(n)$ impulse response. From this, we can further write, $\delta(n)$, which is passing through the LTI system and giving the response $h(n)$. Now, if you apply the time invariant property, if the system follows the time invariant property, we can write $\delta(n - k)$, if the time has been shifted by K, and it should also reflect in the output $(n - k)$. This is what we have seen in time invariant property.

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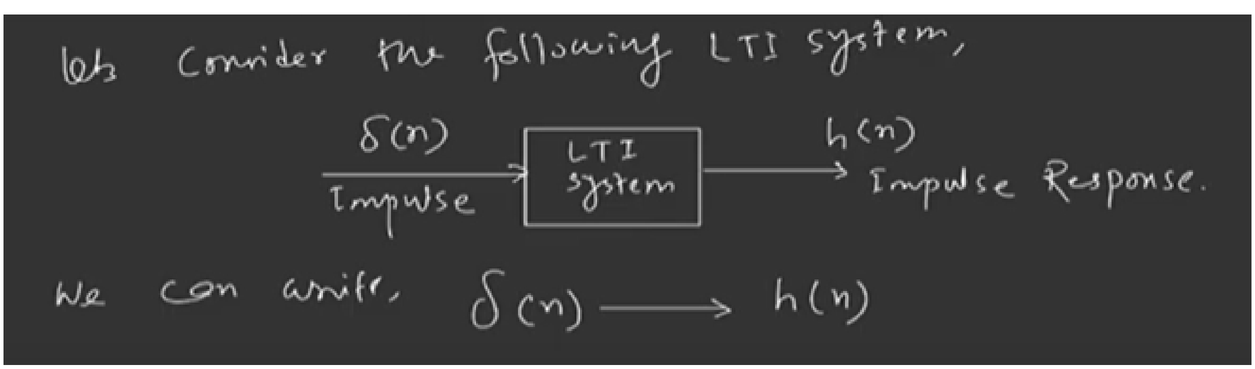

Now, if this system, let me write this equation number one. If the equation number one follows the rule of homogeneity, we can write

$$x(k)\delta(n - k) \to x(k)h(n - k)$$

This means the impulse scaled the input by $x(k)$, and also we are having the scaled output by $x(k)$. So I can write scaled the impulse, and scaled the impulse response. So from this, we can easily say, let me write this equation number two.

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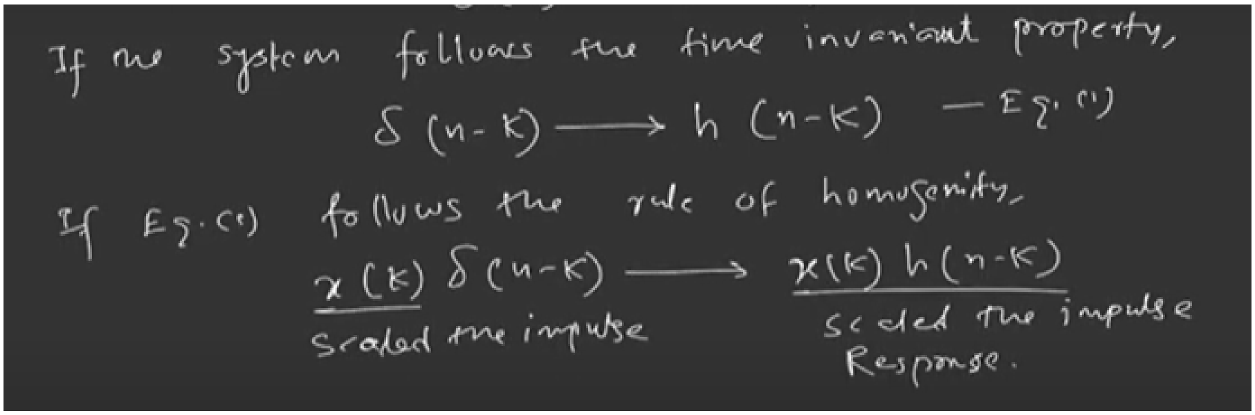

Hence, according to the homogeneity property, this equation two also follows the homogeneity conditions,the homogeneity property. Now, we will see additivity to this equation number two. Now, if the LTI system here follows the rule of additivity, we can write

$$\sum_{k=-\infty}^{k=\infty} x(k)\delta(n - k) \to \sum_{k=-\infty}^{k=\infty} x(k)h(n - k)$$

So this follows the additivity rule, this is clear, we have done this before. Now, it is clear, let me write this is term one, and this is term two, and also this is equation three, and from this equation three, I can write term one, it's basically a linear combination of scaled and shifted versions of impulses. This is very clear, because this is the scaled and this is the impulse, and that linear combination, because this is the summation of the multiple terms. So we can say   linear combinations of scaled and shifted versions of impulses. And already we know that the signal, if I denote the term one as $x(n)$ and this is the combine, we will talk later, just in this moment. So now, hence, any signal $x(n)$ can be represented by a shifted and scaled version of the impulses.

So this is very clear , if we denote this is my $x(n)$, if term one is shifted and scaled version of impulses, then $x(n)$ also shifted and scaled version of impulses. Now, if you extend the same concept to the output impulse, the term two can be denoted by $y(n)$, similar to $x(n)$, and the same analogy also applied to term two as well. So what is the interpretation in this case? So we can write, if $x(n)$ is applied to an LTI system, we get the output $y(n)$ which is basically linear combination of scaled and shifted versions of the impulses or impulse response. So we can write, if you're having $x(n)$ as impulse, which is scaled and shifted version of the impulses, and if you apply to a LTI system and

$$y(n) = \sum_{k=-\infty}^{k=\infty} x(k)h(n - k)$$

So this is actually, if you notice, this is basically convolution sum. So here, so if I write this is term three, here the term three is actually convolution sum. So here what is the takeaway from this part. So we can say that if you know the impulse response of LTI system, we can calculate the output for any input. This is a very important part in impulse response and this concept we will be using for finding the transfer function.

Now consider why we have to look at the LTI system for designing control system. So let me write why LTI system we are assuming for designing the controls in this course. Let me write some points. First and foremost step is that these type of systems are easy to analyze . Second, a common mathematical framework can be used to analyze this type of system.

And also, the real life systems are not LTI in nature, but we can approximate those systems as LTI systems. And from these results, we can get the given inputs and can be used to analyze the properties of real life systems. The real life systems are not LTI systems approximations. The results we get for a given initial conditions and given inputs, we can analyze the properties of real life systems.

So that's why the LTI system is very important to analyze real life problems. The result we get, we can approximate the real life systems. Now before we proceed to our main content, we also need some basics. So what are the assumptions we have to consider in linear control systems? First, assumptions we have to consider.

The differential equation, because as you know, we need the differential equation to design control systems. In a control system, we need to have a mathematical model of the systems. And those mathematical models we can write in differential form. So hence, the equations or the differential equations we are using, that should be linear. The differential equations are assumed to be linear.

And also we'll confine our attention to single input and single output system. So here, what we will be doing, we are looking at the system, we shall assume we are having single input and single output. If it is $x(t)$, it is single input and single output or $u(t)$ some kind of thing. Instead of x, $u(t)$ can be an input and $y(t)$ can be an output. So a single input and single output system.

So also, sometimes we refer to it as SISO system, single input and single output system. And third part, we will also confine our attention to the lumped parameter system. So what is lumped parameter system? The system with those systems whose behavior can be analyzed or described by ordinary differential equations. And so these are the conditions we have to follow for designing control systems for the linear systems.

So let me assume we have a lumped parameter system. So here, we can describe the system with ordinary differential equations. And we are having input to the system by input $u(t)$. And we are having output from the system by $y(t)$. This is my output from the system.

Please, one thing here we are considering the natural system first, then we'll come up with the control. So here, let's assume a system, which is defined by our differential, ordinary differential equation. So let me write $\frac{d^4y(t)}{dt^4} + a_3\frac{d^3y(t)}{dt^3} + a_2\frac{d^2y(t)}{dt^2} + a_1\frac{dy(t)}{dt} + a_0y(t) = u(t)$ So this is, we can say this is the lump parameter system which is defined using the ordinary differential equation. This system, if you notice here, we have some parameter or the coefficient in the system, or I can have another coefficient in place of this.

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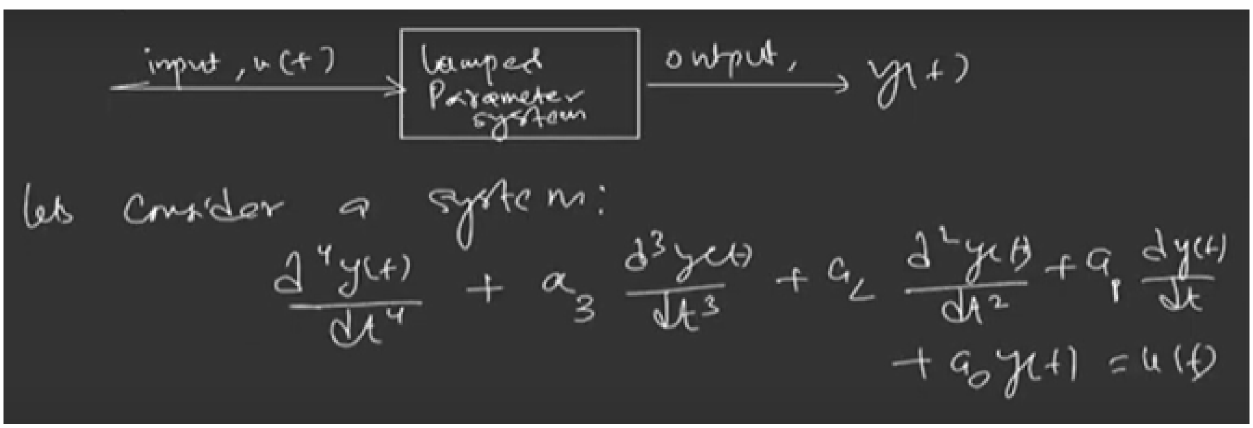

$$\frac{d^4y(t)}{dt^4} + a_3\frac{d^3y(t)}{dt^3} + a_2\frac{d^2y(t)}{dt^2} + a_1\frac{dy(t)}{dt} + a_0y(t) = u(t)$$

Let me write some coefficient $b_0u(t)$. Let's assume this is $u(t)$. So here, if the system parameters, because this a_3, a_2, a_1, a_0, b_0, these are unknown parameters in the system. So if the system parameters, a_3, a_2, a_1, a_0,and b_0 are time-varying, first of all, this system, I can say linear system, you can appropriate, this system is linear system averaging, but it is not confirmed if the system is time-varying or not. Okay, a time-varying, if the parameters a_3, a_2, a_1, a_0,and b_0 are time-varying, while the system is still linear.

Here, a system with time-varying parameters is called time varying systems. This is very, very important. If the coefficients of this system are changing with time, then we can say the system is time varying system, while a system whose parameters are constant with time is called a time invariant system. So if you want to check, you can take an example, something like that, and you can verify with the concept we have discussed before. Now, from this system, let me write this equation number one.

In equation number one, for a given initial condition, the solution of equation one is a function of time. I guess this part you have done in mathematics, because if you solve like some simple ODE, for example, $\frac{dx}{dt} =- 4x$, and you have initial condition some $x(0) = 4$. So we can come up with a solution $x(t) = 4e^{-4t}$. So this is the solution basically, right, this is what you have done in mathematics. So if you look at this solution, it is basically time varying, right? It is, this $x(t)$ is a function of time.

So now, however, there may exist special solutions which are constant. This is a very, very important point, the special solutions. And such constant solutions for an unforced system are called equilibrium points. This is a very, very important part in the control system. Until and unless you don't know the equilibrium point, we can't come up with the system in linear form from the physical systems.

So if you notice here, it is some term I have used here, unforced system. So it means if you look in our original system, this is our general system equation one. So with the effect of $u(t)$ here, my system is evolving. So here, $u(t)$ act as a force or input to the system and due to which the system propagates. I can say, if you want to find the solution of the system, we have to assume the force is zero.

So now you have to assume. So now what we will do is to solve this equation, you can write $\frac{d^4y(t)}{dt^4} + a_3\frac{d^3y(t)}{dt^3} + a_2\frac{d^2y(t)}{dt^2} + a_1\frac{dy}{dt} + a_0y(t) = 0$ So equal to zero we have to assume. And if you find the solution of this differential equation, there can be some special solution which we are calling the equilibrium point. In some cases, this equilibrium point also we can say, fixed point or trim points. So there are different terms we are using.

Let me write one point, why you are calling this fixed point or trim point? Because the system continues to be at rest when it is already at such points. It is very, very important for life. So at this point, the system will be remaining at its original or natural behavior properties. So this is how we can come up with the equilibrium point and how it is defined. So most of the control systems we design keeping the plant at this fixed point, because what you will do is the system, this is your system, for example, let me assume the pendulum system, this is my pendulum.

The pendulum, if it is the two positions, it can stay constant. If it is vertically up, this is the one position, and it can also stay this position. So there are two equilibrium points, this position and this position. So now, we have to design a control system in such a way that if in the system, I mean the pendulum, disturb, suppose from disturb to this point, for example, we have to design the control system in such a way that this pendulum should come back to the initial position. But in this case, this is basically stable, I can say, even if it is disturbed, it will come back automatically here.

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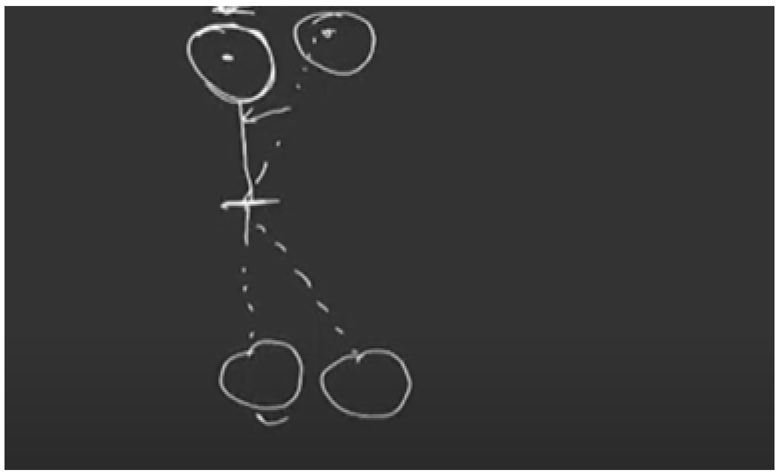

But this is basically my initial equilibrium point, if it is disturbed, the system should be able to come back to the equilibrium point, this is my equilibrium point. So we'll be talking in more detail after some time on this example, how it works. So our control system, we have to design in such a way that the plant at one of its equilibrium points should stay. And suppose it is the use of this kind of system or everywhere like missile, aircraft, like autonomous car, then this is how, I mean, we'll analyze or describe, or we need to design the control system around the equilibrium point. And so now we'll be talking on how we can find equilibrium point and how we'll check the system is stable or not.

This equilibrium point plays an important role in designing control system, since most of the practical system are nonlinear in nature. But since in this course, we'll be designing linear controls. So we need to study the equilibrium point. And how will make the system stable at the equilibrium point, even the system has unstable equilibrium point with the application of control, we can make the system stable. In the next lecture, we'll be talking about equilibrium point and how we'll study the stability of the equilibrium point of a system.

So thank you very much. We'll continue from the next lecture.