Introduction to Aircraft Control System

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Lecture – 36

Review of Equation of Motion of the Aircraft

Now we are going to start on the most important topic in this course, which is basically how we can design the flight control system in linear regime. First, we'll come up with a list of equations which involves the rotational dynamics and translational motion of the aircraft followed by the kinematics. Then we'll divide the equation into two sets. One state will be representing the longitudinal motion, another state will be representing the lateral direction motion of the aircraft. We'll see the equations will be highly complex and non-linear, but to design the linear controls we should have the linear form of the respective equations of motion. Here we're going to use the small perturbation theory and how we can come up with the linear state stress model of the respective equation.

We'll discuss them in detail. Then we'll start different modes which are there in the longitudinal motions and lateral motions. We'll discuss in detail basically what are the approximation in the different motions. Here we're going to have go for more than seven lectures in this direction, in this content. Here we are going to start the second part in this course, the equation motion of the aircraft and how we can design the control for the same. So here we are going to consider the longitudinal motion of the aircraft and the lateral motion of the aircraft and how we can come up the linear model of these equations and also what are the different modes are there in the respective equation of motion, the longitudinal motion and lateral motion. So let's start with the equation of motion of the aircraft.

So here we are not going in details how the equations are being derived. We'll write the final form and from those equation we'll come up with the lateral motion and longitudinal motion and we'll go to the linearization technique and how can come up the linear model of the respective systems or equations and we can design the controls. So let me write the note then we can write the equation motion. The rigid body equations of motion are obtained from Newton's second law which states that which states that the summation of all external force acting on a body is equal to the time rate of change of momentum of the body.

So this talk about the forces equation and if you consider the angular momentum equation and the summation of the external moment of momentum or we can say angular momentum the time rate of the change of linear and angular momentum or refer to the absolute or inertial reference frame. For many problems in the airplane dynamics an axis system fixed to the earth can be considered as an inertial reference frame. So let's consider the following figure where all the forces moment and axis system are defined.

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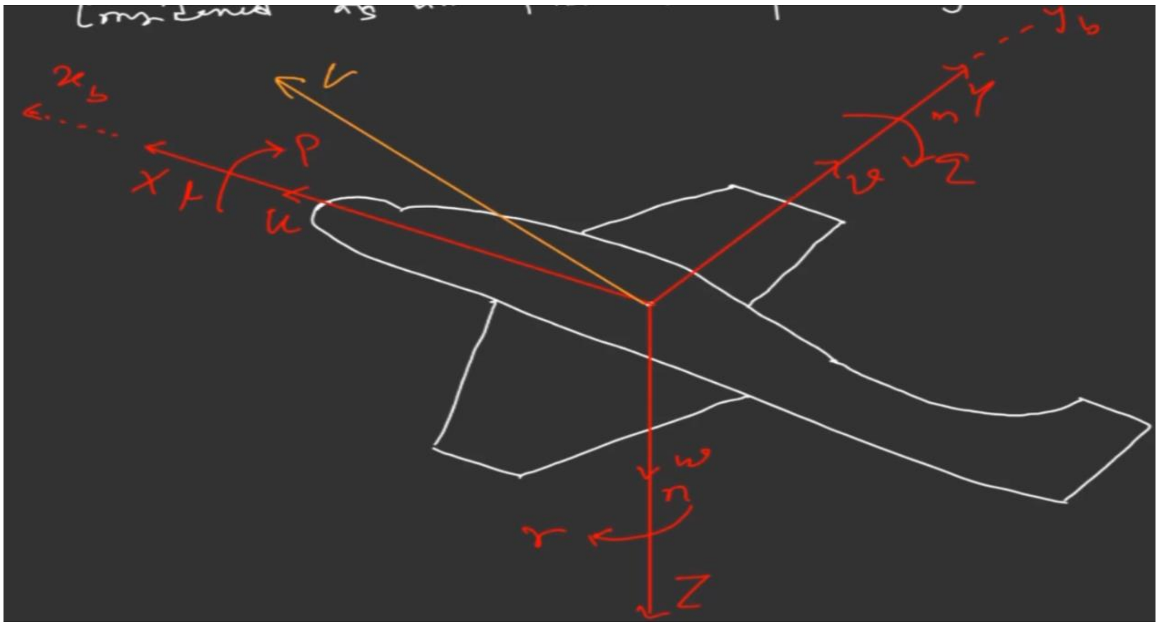

So let's consider we have aircraft model and let's assume the CG of the aircraft and this is the x-axis and this is y-axis and this is z-axis and the body axis along this is the body axis x, p let's assume and this is yb and this is zb and the total force along x-axis the aerodynamic and propulsive force we are assuming along x axis X, along y axis, Y and along z is Z and the rates along x axis let's assume this is p along this u along this r and the linear velocity along x axis we are assuming u along y is v and along z is w and let's assume this is the total velocity of the aircraft which is along this x and we are assuming this is V. So now let's define what are the terms being used here let me write those things so here

V is the total velocity of the aircraft

u is the aircraft translational velocity in the xb axis

v is the aircraft translational velocity in the yb axis

w is the aircraft translational velocity in the zb axis p is the aircraft roll rate about xb axis

q is the aircraft pitch rate about yb axis

r is the aircraft yaw rate about zb axis

l is the aircraft rolling moment about xb axis

m is the aircraft pitching moment about yb axis

n is the aircraft yawing moment about zb axis

X,Y,Z are the aerodynamic and propulsive forces in xb, yb and zb axis respectively

Also, let F_x , F_y , F_z constitute the X,Y,Z (aerodynamic and propulsive forces and gravitational force components), then the force equations are

$$F_x = m[\dot{u} + qw - rv]$$

$$F_y = m[\dot{v} + ru - pw]$$

$$F_x = m[\dot{w} + pv - qu]$$

$$F_x = X + (F_x)_{gravity}$$

$$F_y = X + \left(F_y\right)_{gravity}$$

$$F_z = X + (F_z)_{gravity}$$

So now the gravity forces along the body axis are found to be

$$(F_x)_{gravity} = mg \sin\theta$$

$$\left(F_y\right)_{gravity} = mg \cos\theta \sin\phi$$

$$(F_z)_{gravity} = mg \cos\theta \cos\phi$$

Now if you combine all the force equation, then

$$X - mg \sin\theta = m[\dot{u} + qw - rv]$$

$$Y + mg \cos\theta \sin\phi = m[\dot{v} + ru - pw]$$

$$Z + mg \cos\theta \cos\phi = m[\dot{w} + pv - qu]$$

Now we'll write the rotational motion of the aircraft, basically in the first part of this course we have designed the classical control for the rotational motion of the aircraft how we can control the angle of the aircraft. Rotational motion of an aircraft can be written as

$$l = \dot{p}I_{xx} - \dot{r}I_{xz} + qr\left[I_{zz} - I_{yy}\right] - pqI_{xz}$$

$$m = \dot{q}I_{yy} + pr[I_{xx} - I_{zz}] + I_{xz}[p^2 - r^2]$$

$$n = -\dot{p}I_{xz} + \dot{r}I_{zz} + pq\left[I_{yy} - I_{xx}\right] + qrI_{xz}$$

and the rotational kinematic equation can be written as

$$\dot{\phi} = p + q\sin\phi\tan\theta + r\cos\phi\tan\theta$$

$$\dot{\theta} = q\cos\phi - r\sin\phi$$

$$\dot{\psi} = q\sec\theta\sin\phi + r\sec\theta\cos\phi$$

for the ease of the motion we can denote I_{xx}, I_{yy}, I_{zz} by I_x, I_y, I_z, . For the steady state condition or equilibrium point during level flight the following condition must be satisfied

$$v = 0, \phi = 0, p = q = r = 0$$

ie, side velocity is zero, wings level and angular rates are zero. So these are the condition needs to be satisfied for trim or steady level flight now we'll distinguish for the what are the equation are involved for the longitudinal equation of motion and what are the equation and involve for the lateral equation motion, so the longitudinal equation of motion defined by the following set of equation,

$$X - mg\sin\theta = m[\dot{u} + qw - rv]$$

$$Z + mg\cos\theta\cos\phi = m[\dot{w} + pv - qu]$$

$$m = \dot{q}I_{yy} + pr[I_{xx} - I_{zz}] + I_{xz}[p^2 - r^2]$$

The lateral directional equation of motion can be written as

$$Y + mg\cos\theta\sin\phi = m[\dot{v} + ru - pw]$$

$$l = \dot{p}I_{xx} - \dot{r}I_{xz} + qr\left[I_{zz} - I_{yy}\right] - pqI_{xz}$$

$$n = -\dot{p}I_{xz} + \dot{r}I_{zz} + pq\left[I_{yy} - I_{xx}\right] + qrI_{xz}$$

First we'll start with the longitudinal equation of motion and we will find the linear model of the longitudinal motion using the perturbation theory and once the system is a linear model we can design the control easily because we are in this course mainly we are discussing about the linear controls then we'll start with the lateral directional motion equation and also you'll be using the small perturbation theory here and we'll find the linear model of the lateral direction motion and you can design the linear control so let's stop it here. From the next lecture we'll be continuing how we can come up the linear model of the longitudinal motion using the perturbation theory.