Introduction to Aircraft Control System

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Week – 07

Lecture – 35

PID for Attitude Control Problem in Frequency Domain

Now let's design a PID control system for the aircraft attitude control problem in frequency domain. First we will set the problem then we will solve the design of PID control in frequency domain. Example, the aircraft attitude the equation in one axis can be written as

$$I\ddot{y} = u + M_d$$

Where $I = 1\ Kgm^2$

U is the control input and M_d is the disturbance. Now design the PID control in such a way that satisfy the following specification. The following specification basically then is an objective of the problem. So first specification is rise constant which is $t_r \leq 30\ s$. The system showed is to the desired value within 30 second and a maximum overshoot constant assumed to be less than 30% and settling time constraint t_s should be less than 100 second. So system should go to steady state value within 100 second and another condition the constant should be followed the steady state error for our step disturbance M_d must be zero. Now let's start solving the problem. We'll be designing the control system in frequency domain and the aircraft attitude motion in Laplace domain we can write

$$G_p(s) = \frac{1}{s^2}$$

this is basically plant transfer function this is already have derived while we are talking about the equation motion of the aircraft we have found that equation. Now we will be designing the PID control how can come up with the combination of PID and PI let's look. A PID control can be designed as a combined PD and PI controls. Let's consider the PI and PD are in the following form PD controller can be written as in the form of

$$G_{c,PD} = k_{PD}(T_{PD}s + 1)$$

If you look carefully in basically PD controller we can write

$$u = k_p e + k_d \dot{e}$$

$$U(s) = k_p E(s) + k_d s E(s)$$

$$= k_p \left(E(s) + \frac{k_d}{k_p} s E(s) \right)$$

And if you denote $k_p = k_{PD}$ and $\frac{k_d}{k_p} = T_{PD}$ as time constant so we can write the above expression

$$= k_{PD}(1 + T_{PD}s)E(s)$$

$$\frac{U(s)}{E(s)} = k_{PD}(1 + T_{PD}s)$$

This is the transfer function for the PD controller and can denote by this $G_{c,PD}$ this is how we can write in simplified form as here now let's consider similarly you can consider a PI on as in the form as the form

$$G_{c,PI} = k_{PI} \left(\frac{T_{PI}s + 1}{s} \right)$$

The combined PID controller:

$$G_c(s) = k(T_{PD}s + 1) \left(\frac{T_{PI}s + 1}{s} \right)$$

$$= k_P + s k_D + \frac{k_i}{s}$$

Where $k_P = k(T_{PD} + T_{PI}), k_D = kT_{PD}T_{PI}, k_i = k$

So this is how you can design the controller gain of the PID controller now we will find how can design the PID controller in frequency domain we'll start first let's look the overshoot and phase margin how they are related, let's consider the following figure which relates between phase margin in degree and maximum overshoot in percentage this already you have done the last in last before the three lectures.

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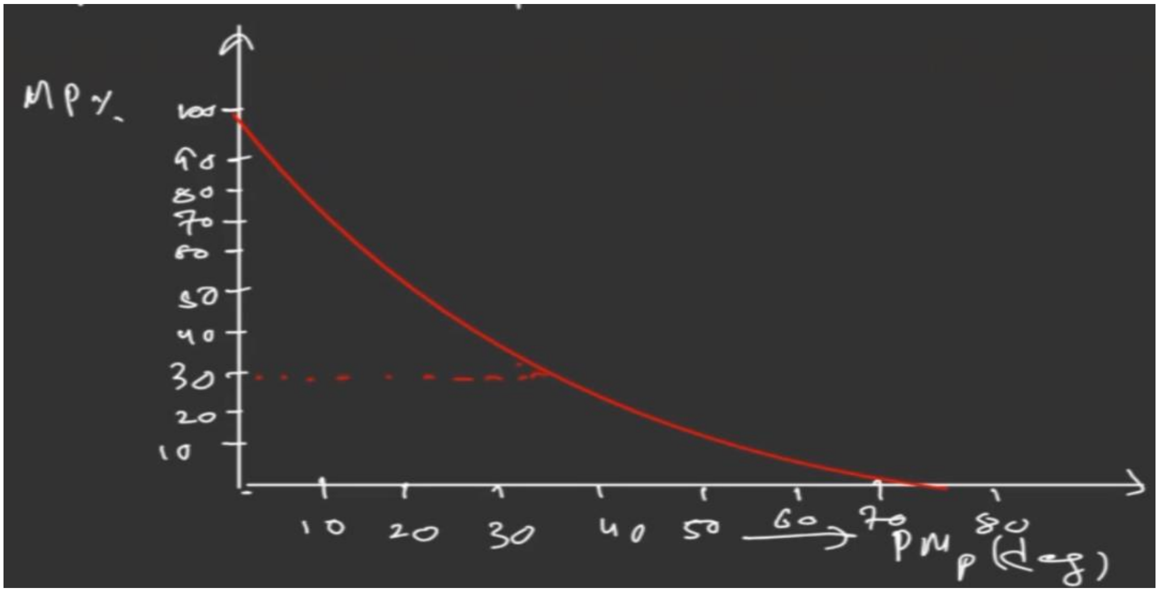

So here let's assume this is maximum overshoot that's this MP percentage and this is phase margin in degree. Since we are given the overshoot which is basically a two percent for the 30% overshoot let's assume there's a 30% overshoot to this plot for it that means we know the secret again something like this so for a 30% overshoot we can assume this is our phase margin so from the figure for 30% overshoot MP, phase margin is assumed to be its margin being it's greater than 38 degree we are achieving this value for this plot we are getting this value and for the safe side let's increase this phase margin a little higher so to give ourselves some margin let us choose is margin equal to 45 degree for in the safe side we are achieving this value now we'll find the value of damping ratio for the given phase margin so we can again refer the relation between the phase margin and damping ratio plot the relation now why you need to find the damping ratio because you're going to find the gain crossover frequency and there is a relation between the gain crossover frequency and rise time settling time and in those relations we had this damping ratio factor.

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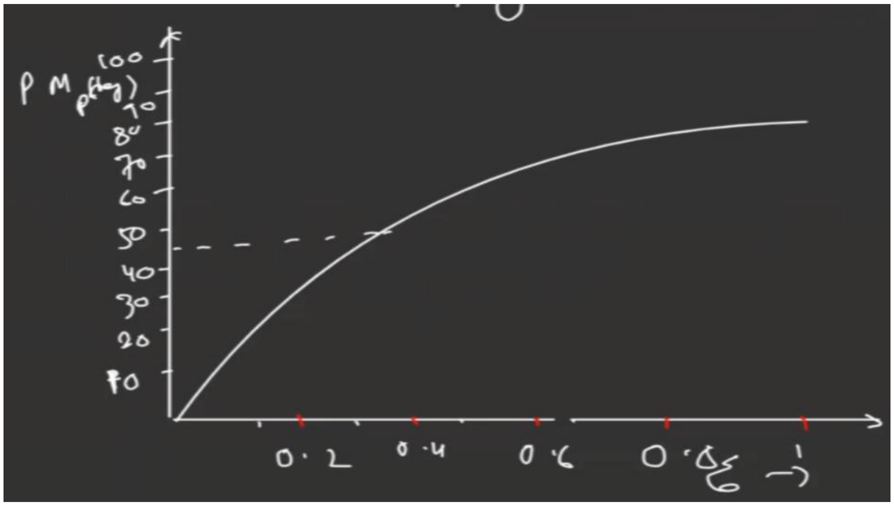

If you notice because we have already derived those expression the relation between damping ratio and phase margin is you can have the plot this is phase margin in degree and we had the plot something like this so and for the given phase margin 45 degree this is 45 degree at 45 degree we're assuming so we are assuming the 45 degree so some value here and there it's fine if it is plus minus 0. something that is okay so for 45 degree for 45 degree phase margin the damping ratio is found to be 0.42. I imagine we are assuming this value so this is just in the range it may be 0.41 or 0.43 so we are taking the strength value 0.42. Now let's find the gain cross over frequency from the relation between the rise time and gain cross over frequency and rise time settling time and gain cross over frequency so let's assume the following relation

$$\omega_g \geq \frac{\pi - \beta}{t_r\sqrt{(1-\xi^2)(4\xi^4+1)} - 2\xi^2}$$

substituting the value of $\xi = 0.42$ and $t_r = 30$

$$\omega_g \geq 0.0875\, rad/s$$

From relation between ω_g and t_s

$$\omega_g \geq \frac{-\ln\left(0.02\sqrt{1-\xi^2}\right)}{\xi t_s\left(\sqrt{4\xi^4+1} - 2\xi^2\right)^{1/2}}$$

substituting the value of $\xi = 0.42$ and $t_s = 30$

$$\omega_g \geq 0.1135\, rad/s$$

Let's put some margin because we are getting the two different values for ω_g , let us choose $\omega_g = 0.12\, rad/s$ because if you notice it's all are both greater than because it is greater than actually that values it is also greater than that value so from these two condition we are choosing this again cross over frequency so now we'll move how to design the PID controller let us let us write the controller in the following form

$$G_c(s) = kG_{PD}(s)G_{PI}(s)$$

This is basically the controller transfer function and for the unity feedback system the open loop transfer function you can write

$$G_o(s) = kG_{PD}(s)G_{PI}(s)G_P(s)$$

Let's assume the PI controller adding five degree the phase lag $\angle G_{PI}(j\omega_g) \approx -5°$. Now

$$PM_P = 180° + \angle G_P(j\omega_g) + \angle G_{PD}(j\omega_g) + \angle G_{PI}(j\omega_g)$$

$$180° - 180° + \angle G_{PD}(j\omega_g) - 5°$$

$$\angle G_{PD}(j\omega_g) = PM_P + 5° = 50°$$

Now from the PD controller structure we can come up with again the phase of PD controller not at gain cross for any other frequency you can thus for the simple PD control system let's see how does the phase of the PD controller, so PD controller we know

$$G_{PD}(s) = k_{PD}(Ts + 1)$$

And the phase of the pd controller you can write

$$\angle G_{PD}(j\omega) = \tan^{-1} \omega T$$

Here basically we have to assume the omega should be positive value for positive omega the angle varies from of the PD controller

$$0 \le G_{PD}(j\omega) < 90°$$

For $\omega > 0$, by this PD controller we want to add some phase ϕ at a specified frequency (ω_c) by PD controller, so we can write from above equation

$$\phi = \tan^{-1} \omega_c T$$

$$T = \frac{\tan \phi}{\omega_c}$$

The controller gain k_{PD} can be chosen in such a way that to give desired gain crossover frequency. So for this purpose the pd controller design we can write

$$T_{PD} = \frac{\tan PM_P}{\omega_g} = 9.93\ sec$$

So this is the time constant for the PD controller now we'll see the PI controller which is basically giving the phase to the system now let's work on PI controller to find T_{PI} here let's assume the PI controller corner frequency to be sufficiently small, so what is the corner frequency for PI controller

$$G_{cPI} = k_{PI}T_{PI}\left(1 + \frac{1}{T_{PI}s}\right)$$

$$\frac{1}{T_{PI}} = 0.1\omega_g$$

We are assuming the corner frequency to be sufficiently small so that the resulting PI phase lag does not have significant effect on the crossover region of the open loop response, so that's why we are choosing very small value of the gain cross over frequency

$$T_{PI} = 83.3\ sec$$

Now we have found the time constant for the PD controller which is 9.93 and for PID controller we are having 83.3 second, okay now we'll find the total gain of the control system. To find the total gain cross over frequency we can take the magnitude of the open loop transfer function equal to one so we can find at $\omega_g = 0.12\ rad/s$ the open loop transfer function satisfy the following relation

$$\left|G_0(j\omega_g)\right| = 1$$

And from this if you substitute the open loop transfer function in this expression we'll have

$$k\left|G_P(j\omega_g)\right|\left|G_{cPD}(j\omega_g)\right|\left|G_c(j\omega_g)\right| = 1$$

$$k = 1.105 * 10^{-4}$$

Substituting k in the following controller gain

$$k_P = k(T_{PD} + T_{PI}), k_D = kT_{PD}T_{PI}, k_i = k$$

$$k_P = 0.0103, k_D = 0.0915, k_i = k = 1.105 * 10^{-4}$$

And now we have all the parameters for PID controller for the controller gains now we can form the PID controller we can write

$$u = k_P e - k_d \dot{y} + k_i \int_0^t e(\tau) d\tau$$

Here $e = r - y$, for a step signal, $\dot{r} = 0$

$$u = 0.0103\, e - 0.0915 \dot{y} + 1.105 * 10^{-4} \int_0^t e(\tau) d\tau$$

So this is how we can design the PID controller for the aircraft system in frequency domain but if you find from the simulation that the desired control system is not able to fulfill the mission objective then what you can do is we can change the phase margin and increase the gain cross over frequency and you can get our desired response so this is how we design the PID control in using frequency domain in classical controls now we'll wind off the classical control design for the aircraft attitude control problem and we'll move to our aircraft equation motion how we can come up with the 6 degree of motion of the aircraft and how can come up the lateral longitudinal motion of the aircraft and how we can design the control system for those dynamical system. Thank you.