Introduction to Aircraft Control System

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Lecture – 34

Phase and Gain Margins in Frequency Domain

In this lecture, we will be discussing the concept of phase margin and gain margin and how these are important parameters for designing control algorithm in frequency domain. Let me write some notes, then we'll start the phase margin and gain margin in the frequency domain and how can we find from the magnitude and phase plot. Control system or control design in the frequency domain involves analyzing and designing control system based on their frequency response. The key important concept that play a crucial role in frequency response or in frequency domain analysis and design are phase margin and gain margin. Let's start with the phase margin and how we can come up with the phase margin from the frequency response.

Phase margin is a measure of how much the phase of the system lags behind minus 180 degree when the gain is unity or we can say zero decibel in the frequency response plot. The phase margin indicates the stability and robustness of the system. How can we interpret phase margin? Let me write interpretation. A large phase margin indicates better stability and robustness. Generally, they are positive in nature. Typically, a phase margin of 30 degree or more, basically positive values, more desirable for stability. And control design implementation is to increase phase margin in the system. So, the system, phase margin, adjustment to the controller or the system or system dynamics can be made. The lead or the lack compensator are often employed to achieve the desired phase margin.

We'll look all these parameters using the frequency response plot and it will be clear. So, this is how we can define the phase margin in the system and how we can increase the phase margin by introducing the compensator to the system. We can increase the phase margin. Now, let me explain the gain margin. Gain margin is the amount by which the gain of the system can be increased, before the system becomes unstable. Before the system, this is also positive value in nature and that we'll explain later. The larger gain margin implies greater stability and robustness. A positive gain margin indicates stability and control design implications are to increase gain margin which is crucial for robustness against uncertainties and disturbances. Next, a controller such as PID controller can be tuned to enhance gain margin. So, this is how we can explain the gain margin and phase margin in a control system and how they are very important for getting the desired stability and robustness of the closed loop control system. Now, let's go how we can come up with the gain margin and phase margin from the frequency plot. Let's consider the following figure.

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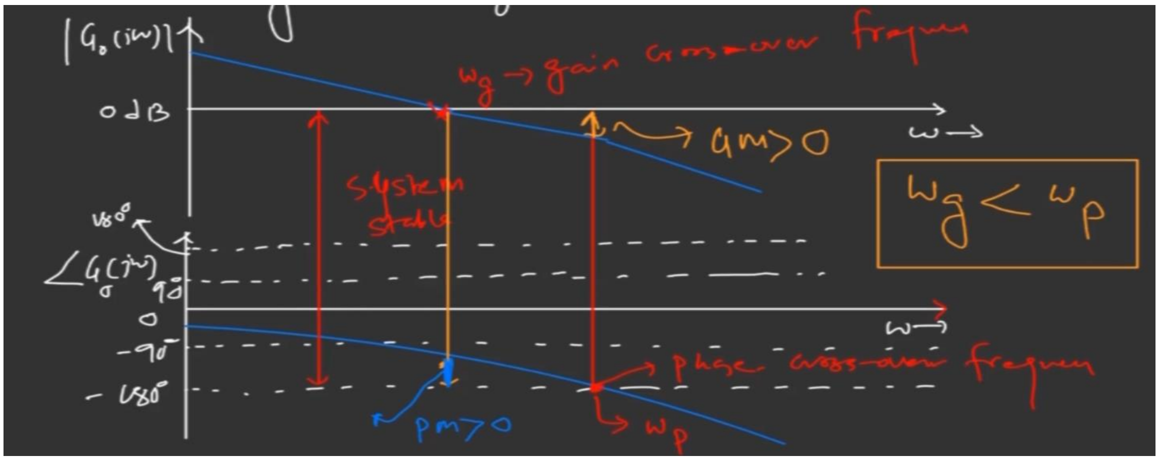

This is the magnitude plot and this is phase plot. So, here let's assume this is magnitude of $G_0(j\omega)$. The interesting thing from here is we'll plot the open loop control system, open loop graph function and we'll come up with the closed loop stability of the system. This is very beautiful thing from the frequency response and let's assume this is the 0 dB line, 0 dB and this is frequency line and let's assume this is phase of the open loop transfer function, $G_0(j\omega)$ and let's assume this is 0 degree and this is plus 180 degree line and we have let's say another line in between and this is 90 degree line and below 0 degree we have minus 90 degree and this is minus 180 degrees. What is 0 dB line? 0 dB line basically here we are the plot is in logarithm scale. 0 dB basically we can say that one magnitude of the transfer function open loop transfer is one. So, if you take the log it is basically zero.

So, this is in logarithm scale. So, that's why magnitude generally we measure in decibel and here let's assume we are having that we are having the magnitude plot something like this and for example like this and for this something like this and the frequency or the angle plot something like this something like example here this is basically I can say this is this is the point while this crosses the 0 dB line this is basically we say gain crossover frequency and the phase angle of the open loop transfer function when it crosses minus 180 degree line this point this is called phase cross over phase frequency this is also we can see this is also ω because phase angle also you can write in terms of frequency so here and the stability the system will be stable from 0 dB to this is very very important the stability margin generally we defined in this range 0 dB to minus 180 degree so this is basically if the margin stays in between this then system is stable and how to find gain margin and phase margin let's look as for the definition the phase margin we generally define from the gain cross over frequency so what we'll do is we'll draw a line so we draw a line from the gain cross over frequency till minus 180 degree and this is actually this value this is actually case margin which is basically positive value here this point actually omega p is cross over frequency to find the gain margin we'll start from the phase cross over frequency draw a straight line with zero dB so this part is basically gain margin so this is gain here gain margin also positive value and if the margins lies between zero degree to minus 180 degree so we can say the system is stable and if the margins in between minus 180 degree to zero dB the margins are generally positive value another condition to come up with the positive gain margin and phase margin is $\omega_g < \omega_p$ so this is very important condition.

This is how we can come up with the gain margin and phase margin from the frequency plot let's take an example how to find gain margin and phase margin from this response let's assume ω_g be the crossover frequency at which the magnitude plot crosses zero dB line. The phase angle at ω_g is ϕ_c , Let's assume the phase margin we can calculate

$$PM = 180° + \phi_c$$

Assume $\omega_g = 2\, rad/sec$ and $\phi_c = -135°$ then phase margin you can calculate as 45° So this is how we can calculate the phase margin from the frequency response now let's look how we can find the gain margin okay let's assume ω_p be the crossover frequency at which the phase plot process minus 180 degree line let at ω_p the gain is -M, then the gain margin can be calculated as

$$GM = -\left(gain\ at\ crossover\ frequency\ \omega_p = +M\right)$$

If you take an example let ω_p my gain is -15 decibel then gain margin we can write

$$GM = -Gain\ at\ \omega_p = 15\ db$$

This is how we can find the gain margin of the system, now let's look why we use the magnitude in bode plot or the frequency plot in db This is because if bode actually where we plot the magnitude and phase of the open loop transfer function how they are varying with respect to frequency and this plot actually we call the bode plot okay frequency response in db The use of decibel simplifies the representation of the magnitude of a system also function over a wide range of frequencies so it means we can here basically every 10 times of frequency general we measured suppose if you are starting from suppose this is the zero db line and you are starting from zero ten the frequency will vary zero ten hundred thousand like that so this is how we denote this db so by wide range of frequency instead of zero one two three just we are multiple of ten and we can simplify the problem and another advantage is in the db scale, gains are additive which simplifies calculations. In the linear scale multiplication of gains correspond to addition of logarithmic values in db so basically what you do is suppose we have transfer function $G_0(j\omega)$

$$G_0(j\omega) = \frac{(j\omega + 3)(j\omega + 4)}{(j\omega + 8)(j\omega + 10)}$$

If we take the log of the transfer function it will be some addition and multiplication so if you take them and do basically we take the 20 log of both side and we can come up with some simple form of addition of the different poles and zeros so we'll take an example then if you see here so let's take an example, let's assume that

$$G_0(s) = kG_p(s)$$

here k is the controller gain and G_p is the plant and if ω_p be the phase crossover frequency then we can write

$$|G_0(j\omega)| = |kG_p(j\omega)|$$

and if you take the 20 log both side, we get

$$= 20\log k + 20\log|G_p(j\omega)|$$

It was obviously in multiplication form now it is in addition so we can easily visualize that individual component in the transfer function so this is quite easy to analyze the system and if you would like to solve the magnitude at phase cross over frequency or at ω_p we can write

$$20\log k + 20\log|G_0(j\omega_p)|$$

So this is how we can solve and generally find the gain margin of the system at ω_p phase cross over frequency so if you'd like to find the gain margin for this particular transfer function we can write

$$GM = -(20\log|G_0(j\omega_p)|)$$

We find $GM = -(20\log|G_p(j\omega_p)|)$ since the gain k does not change the phase cross over frequency when gain k is adjusted, so this is how we can find the gain margin log scale. Let's stop it here in the next lecture we will be taking an example how we can design control algorithm using phase margin and gain margin in frequency domain basically autopilot of the system. Thank you.