Introduction to Aircraft Control System

Prof. Dipak Kumar Giri

Department of Aerospace Engineering

IIT Kanpur

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Lecture – 33

Closed-Loop Tracking Error

In this lecture, we will be studying how we can reduce the closed loop tracking error of the closed loop control system. Before we proceed, I would like to refer the combined error equation what I derived in the last lecture.

$$E(s) = \frac{1}{1+G_o(s)}R(s) + \frac{G_o(s)}{1+G_o(s)}W(s) - \frac{G_p(s)}{1+G_o(s)}M_d(s)$$

$$= E_r(s) + E_w(s) + E_{md}(s) \ldots\ldots Eq(1)$$

So, E_r basically the tracking error due to the reference signal, E_w is the tracking error with respect to the measurement noise, E_{md} is the tracking error due to the disturbance to the system. And also, as we mentioned in the last lecture, the reference signal and disturbance signal are slowly varying with time or you can say it has low frequency content.

And the measurement noise signal basically varies with time very fast, like the first varying signal we can say, first varying signal, or we can say it has high frequency content. Now, we look step by step how we can reduce the tracking error with respect to reference signal, tracking error with respect to measurement noise, tracking error with respect to disturbance. So, first let's go with the reference signal tracking.

From equation one, we can write,

$$E_r(s) = \frac{1}{1+G_o(s)}R(s)$$

In frequency domain $s = j\omega$

$$|E_r(j\omega)| = \left|\frac{1}{1+G_o(j\omega)}\right| |R(j\omega)|$$

If you notice from this equation, if you want to reduce your $j\omega$, this term should be very low value. Once it is very low, then we can track the reference signal. So, we can write for good reference signal tracking, we should have as a control engineer, we should be very careful that the error in the system should be zero. That is our main motivation. And based on this motivation, we are doing all these practices. We should have

$$\left|\frac{1}{1+G_o(j\omega)}\right| \ll 1$$

To satisfy this condition, we should have for this, we need to have $|G_o(j\omega)| \gg 1$. Since G_0 is basically the open loop transfer function with the unit feedback, so we can write

$$G_0 = G_p(j\omega)G_c(j\omega)$$

And it is obvious that the G_p is basically the natural system. We can't change anything in this transfer function and only we can play here.
So, we can change G_c, the control algorithm to satisfy this condition. We should have

$$G_c(j\omega) \gg 1$$

So, once you satisfy this condition, we can track the reference signal. Now, let's look the disturbance rejection. How can we reject the disturbance into the system? From equation one, we can write

$$E_{md}(s) = \frac{-G_p(s)}{1+G_o(s)}M_d(s)$$

And already I have mentioned the disturbance slowly varying signal. If you write this equation in frequency domain, we can write

$$E_{md}(j\omega) = \frac{-G_p(j\omega)}{1+G_o(j\omega)}M_d(j\omega)$$

For good external disturbance rejection, we should have, if you want to minimize this disturbance, error between the disturbance, so

$$|E_{md}(j\omega)| = \left|\frac{G_p(j\omega)}{1+G_o(j\omega)}\right| |M_d(j\omega)| \ll 1$$

this part should be very very low value. So, this condition will be valid in low frequency range. So, we can write for good reference signaling, the reference tracking, we already taken

$$G_c(j\omega) \gg 1$$

Now, we can write

$$\left|\frac{G_p(j\omega)}{1+G_o(j\omega)}\right| = \left|\frac{G_p(j\omega)}{G_o(j\omega)}\right| = \frac{1}{G_c(j\omega)}$$

The above equation suggests that we need to have $G_c(j\omega)$ should be large in the low frequency range for good reference signal tracking and disturbance rejection. So, as you mentioned before that, the reference signal and disturbance signal are low frequency, they have the low frequency content and we can receive the error due to the reference signal and error due to the disturbance once we have the controller gain is very high. Now, we will look how we can attenuate the measurement noise. Measurement noise attenuation from equation one, the tracking error due to measurement noise is

$$E_w(s) = \frac{G_o(s)}{1+G_o(s)}W(s)$$

In frequency domain, in magnitude we can write

$$|E_w(j\omega)| = \left|\frac{G_o(j\omega)}{1+G_o(j\omega)}\right| |W(j\omega)|$$

For good measurement noise attenuation from this equation we should have this term

$$\left|\frac{G_o(j\omega)}{1+G_o(j\omega)}\right| |W(j\omega)| \ll 1$$

should be very low in magnitude in the high-frequency range. So, this is how we can tackle the measurement noise in the system. So, to accomplish this we have to accomplish this condition we should have $|G_o(j\omega)| \ll 1$ at high frequency range. So, this is how we can tackle the combined error in the system and we can fulfill our mission objective or we can track the desired reference signal. Now we'd like to take the total control effort in the closed loop control system. Let's rewrite the equation we had the total control effort in the closed loop system from the previous lecture

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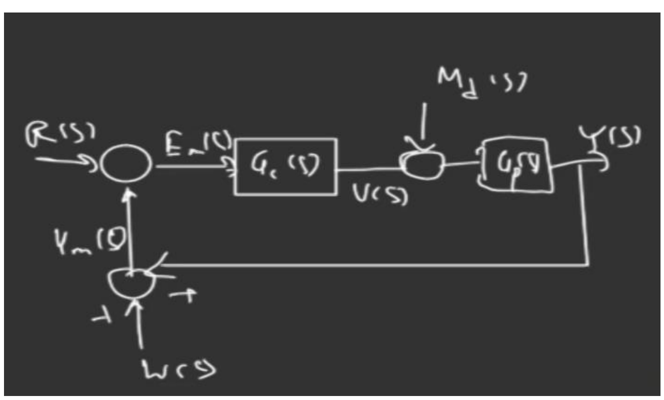

$$U(s) = G_c(s)E_m(s)$$

$$= \frac{G_c(s)}{1 + G_0(s)} R(s) + \left[\frac{-G_c(s)}{1 + G_0(s)}\right] W(s) + \left[\frac{-G_0(s)}{1 + G_0(s)}\right] M_d(s) \dots \dots Eq(2)$$

Let me write some note here, in the aircraft system we have many actuators in the system and based on the control input to the actuator, the actuator will respond to the system so any real actuator has limitation in the amount of control effort that can be applied to the system therefore we can write the controller should be designed in such a way that to avoid the control signal from becoming large, so this is very important point while designing the control system for the dynamical system to be controlled and also we have seen that for both reference signal tracking and disturbance rejection top loop gain should be very large that means $|G_0(j\omega)|$ should be very very large for low frequency and for measurement noise attenuation the open loop gain should be very small in high frequencies so we can write $|G_0(j\omega)|$ should be very small in high frequencies so based on this condition we'll be going to how we can look the control effort in the open loop system.

So if the open loop gain is very large which is $|G_0(j\omega)| \gg 1$ we find from equation so we are having three different component the control effort due to the reference signal control effort due to the measurement noise and control effort due to the disturbance to the system so we can write the control effort

$$|U_r(j\omega)| = \left|\frac{G_o(j\omega)}{1 + G_o(j\omega)}\right| |R(j\omega)|$$

$$= \frac{1}{|G_o(j\omega)|} |R(j\omega)| \dots \dots Eq(3)$$

And if you write the controller due to the disturbance

$$|U_{md}(j\omega)| = \left|\frac{G_o(j\omega)}{1 + G_o(j\omega)}\right| |M_d(j\omega)| \approx |M_d(j\omega)| \dots \dots Eq(4)$$

So this condition we are getting the final expression based on this condition basically for the large gain so these are the equation we are getting and if you write the total control effort due to the measurement noise

$$|U_w(j\omega)| = \left|\frac{G_c(j\omega)}{G_o(j\omega)}\right| |W(j\omega)| \approx \frac{1}{|G_p(j\omega)|} |W(j\omega)| \dots \dots Eq(5)$$

It is clear from equation three and five that the control effort can be very large if the plant is very small while the open loop gain $|G_o(j\omega)| \gg 1$. So therefore we should try to avoid the situation where the open loop bandwidth is significantly larger than the plant bandwidth so we should try to avoid the situation where the open loop bandwidth is significantly larger than the plant bandwidth so this we should keep in mind while designing the autopilot part. So this is the open loop bandwidth for the aircraft system this is very very important point now we'll look the small open loop gain so there are two condition we have discussed in the combined error tracking, one is high gain another is the low gain now we'll look how the small open loop gain going to be affected in total control effort, as you mentioned the measurement noise typically has primarily high frequency content and we try to make the open loop gain very small over those frequencies basically this is the condition we have already derived it should be very small for high frequencies so we can write, for small open loop gains ($|G_o(j\omega)| \ll 1$) should be very very small. So for the small gain let's look how the total control effort due to the reference signal due to the measurement noise and in the disturbance so we can write and in the disturbance so we can write

$$|U_r(j\omega)| = \left|\frac{G_c(j\omega)}{1 + G_o(j\omega)}\right| |R(j\omega)| \approx |G_c(j\omega)| |R(j\omega)| \ldots \ldots Eq(6)$$

Total control effort due to the measurement noise

$$|U_w(j\omega)| = \left|\frac{G_c(j\omega)}{1 + G_o(j\omega)}\right| |W(j\omega)| \approx |G_c(j\omega)| |W(j\omega)| \ldots \ldots Eq(7)$$

If you look that disturbance part

$$|U_{md}(j\omega)| = \left|\frac{G_0(j\omega)}{1 + G_o(j\omega)}\right| |M_d(j\omega)| \approx |G_o(j\omega)| |M_d(j\omega)| \ldots \ldots Eq(8)$$

So now we will look the that control effort due to the measurement noise. From equation seven it is clear that not only the open loop gain be very small where $|G_o(j\omega)| \ll 1$ at high frequencies but the control effort or the control gain should also be very small which is $|G_c(j\omega)| \ll$ to avoid unnecessary control chattering which resulting from the measurement noise so this second point is very very important how we should choose the gain in the system while designing the autopilot for the dynamical system. Let's stop it here basically these are the condition we have to follow while designing autopilot for the dynamical system, in the next lecture we'll be going through some basic design procedure in the frequency domain and before that we will come up some concept of phase margin and gain margin and how we can compute for those margins from the magnitude and phase plot, then we'll go to some example autopilot design for the aircraft system. Thank you very much.