Introduction to Aircraft Control System

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Week – 07

Lecture – 32

Feedback Control Problem in the Frequency Domain

In this lecture, we will be discussing how the closed loop control system can be analyzed in the frequency domain in the presence of disturbance and noise. So todays topic is feedback control problem in the frequency domain. We will revisit the closed loop control system, what we have done before for the aircraft attitude control problem. So let me draw the closed loop diagram.

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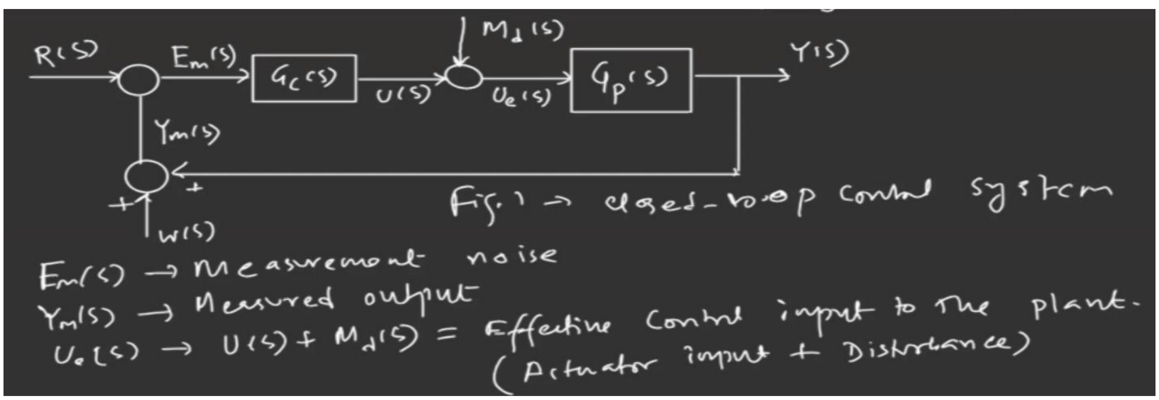

This is summing point where we are having this reference input to be tracked. And we have controller, it's giving the desired control input to the system, denoted by $G_c(s)$. And we are having another summing point where control input and disturbance coming into picture. Let's draw the disturbance signal, $M_d(s)$ which is given by the controller. And the sum control input and disturbance, which is denoted by $U_e(s)$ going to the plant to be controlled $G_p(s)$ and this the output from the plant Y(s). And this is the feedback signal. Here also we are assuming noise coming into the system, which is denoted by $W(s)$ and we are having feedback here. And the output from this summing point is denoted by $Y_m(s)$. And the error in the presence of measurement noise we can denote as $E_m(s)$.

This is basically measurement noise and $Y_m(s)$ is the measured output. And\

$$U_e(s) = U(s) + M_d(s)$$

which is nothing but effective control input to the plant. And this is if you say, if you have actuator, we can say actuator input plus disturbance. Now the measurement error we can write on the block diagram. From the figure 1, we can write

$$E_m(s) = R(s) - Y_m(s) = E(s) - W(s)$$

Where $E(s) = R(s) - Y(s)$ is basically tracking error. Now we'll find $U(s)$ from the block diagram from figure 1.

$$U(s) = G_c(s)[R(s) - Y(s) - W(s)]$$

$$Y(s) = G_p(s)U_e(s)$$

$$= G_p(s)G_c(s)[R(s) - Y(s) - W(s)] + G_p(s)M_d(s)$$

$$Y(s) = \frac{G_p(s)G_c(s)}{1 + G_p(s)G_c(s)}[R(s) - W(s)] + \frac{G_p(s)}{1 + G_p(s)G_c(s)}M_d(s) \ldots \ldots Eq(1)$$

Tracking error is given as

$$E(s) = R(s) - Y(s)$$

$$= \left[1 - \frac{G_p(s)G_c(s)}{1 + G_p(s)G_c(s)}\right]R(s) + \frac{G_p(s)G_c(s)}{1 + G_p(s)G_c(s)}W(s)$$
$$- \frac{G_p(s)}{1 + G_p(s)G_c(s)}M_d(s) \ldots \ldots Eq(2)$$

If we denote $G_o(s) = G_p(s)G_c(s) \rightarrow OLTF$

Eq.(2) can be written as

$$E(s) = \frac{1}{1 + G_o(s)}R(s) + \frac{G_o(s)}{1 + G_o(s)}W(s) - \frac{G_p(s)}{1 + G_o(s)}M_d(s)$$

$$= E_r(s) + E_w(s) + E_{md}(s) \ldots \ldots Eq(3)$$

Where $E_r(s) = \frac{1}{1+G_o(s)}R(s), E_w(s) = \frac{G_o(s)}{1+G_o(s)}W(s), E_{md}(s) = -\frac{G_p(s)}{1+G_o(s)}M_d(s)$

Our main aim is $E_m(s) \rightarrow 0$ as $t \rightarrow \infty$. So once you achieve this condition, our control system can track the reference signal in the presence of disturbance and noise in the system.

$$E_m(s) = E(s) - W(s)$$

$$= \frac{1}{1 + G_o(s)} R(s) + \left[\frac{G_o(s)}{1 + G_o(s)} - 1\right] W(s) + \left[-\frac{G_p(s)}{1 + G_o(s)} M_d(s)\right] \dots \dots Eq(4)$$

Now we can find the overall control or the control input given by the controller we can write the control input, the control input given by the controller,

$$U(s) = G_c(s) E_m(s)$$

$$= \frac{G_c(s)}{1 + G_0(s)} R(s) + \left[\frac{-G_c(s)}{1 + G_0(s)}\right] W(s) + \left[\frac{-G_0(s)}{1 + G_0(s)}\right] M_d(s)$$

$$= U_r(s) + U_w(s) + U_{md}(s) \dots \dots Eq(5)$$

Where $U_r(s) = \frac{G_c(s)}{1+G_0(s)} R(s), U_w(s) = \left[\frac{-G_c(s)}{1+G_0(s)}\right] W(s), U_{md}(s) = \left[\frac{-G_0(s)}{1+G_0(s)}\right] M_d(s)$

So this is how we can find the control input and measurement error equation and our motivation is how we can neglect or mitigate the disturbance and measurement noise with designing suitable control algorithm in frequency domain. And before we proceed to that part, I would like to highlight few important points which are very useful for designing controller in frequency domain. As a control engineer, ideally we would like to have our objective is $E_r(s) = 0, E_{md}(s) = 0, E_w(s) = 0$

Once we achieve this condition, we can track $Y(s) \to R(s)$. It is quite difficult to achieve this objective. For this, if you would like to go to certain range or certain value close to this condition, it should be noted that, first one is the disturbance, it should be, this is very important points while designing the control algorithm in frequency domain, noted that the disturbance should be rejected. You can write disturbance should be rejected, vary relatively slowly with time. So it means the disturbance signal coming to the system, vary very slowly with time. So or in other words, you can write the disturbance signal have low frequency content.

While designing the control system, we need to consider this point, disturbance signal vary slowly, time and second point, the measurement noise typically varies with time. The $M_d(s)$ has high frequency content. This is very, very important to design the control system, how we can neglect the measurement noise with suitable design of control algorithm. Or also we have another point in addition to this, we need to consider $E_r(s)$ and $E_{md}(s)$ over low frequencies to ensure good reference signal tracking because our main motivation is how we can track the reference signal and disturbance rejection.

It means our reference signal also varies slowly with time. So that's why $E_r(s)$ and $E_{md}(s)$ over low frequencies to ensure good reference signal tracking and disturbance rejection. And while $E_w(s)$ only needs to be considered over high frequencies, we need to consider $E_r(s)$ to ensure good measurement noise attenuation. So this is important point while designing the control system. How can design the suitable control algorithms such that over low frequencies, it can reject the tracking error and the error due to the disturbance. Now let's stop it here. We'll continue from the next lecture how we can choose the control transfer function or controller in such a way that this condition can be achieved. Thank you.