Introduction to Aircraft Control System

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Week – 06

Lecture – 30

Design of PID Controller

In this lecture, we will be designing PID controller for a plant to be controlled. We will validate the concept we discussed in the earlier lecture, the Ziegler-Nichols rules and root locus. So we will be using those methods to come up with the PID controller. And we validate this approach through the example. Consider the transfer function of PID controller and plant as

$$G_c(s) = k_p + \frac{k_i}{s} + k_d s$$

$$G_p(s) = \frac{4}{s(s+1)(s+2)}$$

So here k_p, k_i and k_d are the PID controller gains. The poles are at $s = 0, s = -1, s = -2$.There is no zero in the system. And if you look in closed loop control system and how this controller function and plant are arranged. The closed loop control system, we have reference signal to be tracked or to be regulated.

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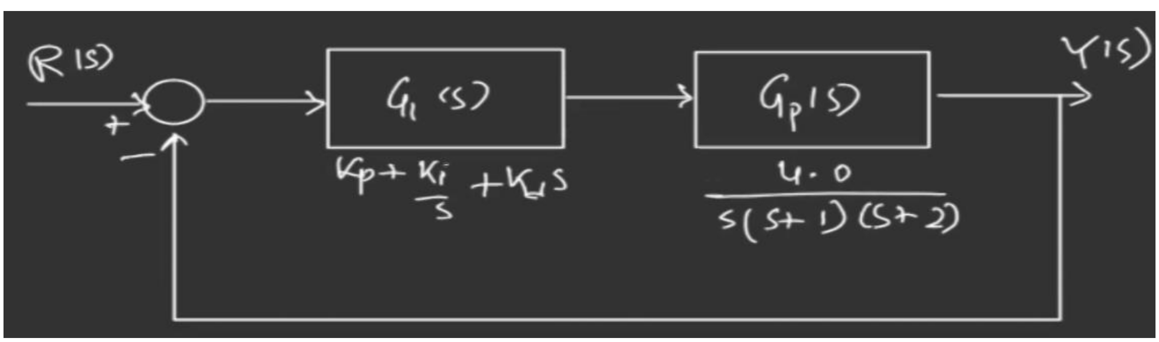

This is my PID controller. And the output of the controller goes to the plant. This is the controller, same plant also function output which is we can denote by Y(s) and this is the feedback, negative feedback. This is the problem we have been given and we have to design the PID controller. So the problem can be formulated as design a PID controller for the type of system using Ziegler-Nichlos rule. And find the overall closed loop transfer function and find closed loop system poles. And on stability of the control system based on pole locations. So this is basically autopilot design if you look. So this is closed loop control system we are designing PID controller here to control the system. Let's move to the solution part. So here let's consider the only control variable to be varied k_p the way we have done before. So here only control parameter for the root locus will be considered k_p which varies from zero to infinity and we assume $k_d = k_i = 0$. The open loop transfer function which is

$$G_o(s) = G_c(s)G_p(s)H(s) = k_p \frac{4}{s(s+1)(s+2)}$$

And the characteristic equation of the system,

$$1 + \frac{4k_p}{s(s+1)(s+2)} = 0$$

Now for $G_o(s)$ the number of zeros (m) is zero and number of poles (n) is three which is going to be n equal to three. And the location of the poles in the s plane

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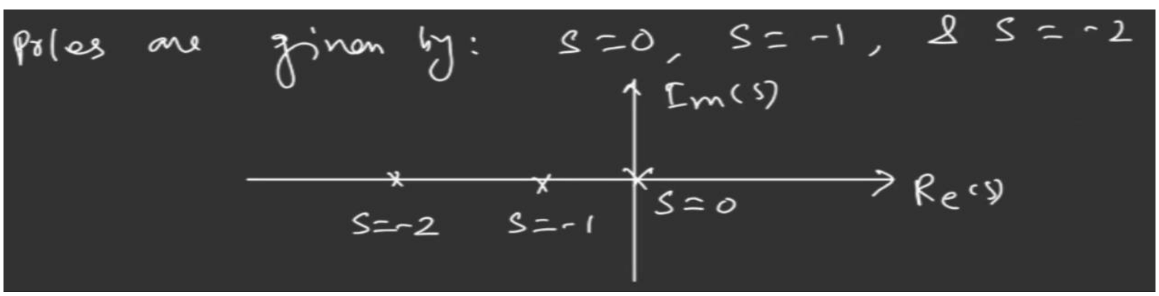

Now we first find the break away point. For finding the break away point we have to consider the characteristic equation. So we have characteristic equation

$$1 + \frac{4k_p}{s(s+1)(s+2)} = 0$$

Solving for k_p and then differentiating k_p with respect to s and equating to zero, we find two roots to be

$$s_{1,2} = -1.577, -0.4226$$

Here, $s = -1.577$ doe not lie on the root locus. Hence this is very clear because the center of CG of asymptotes will have one which I will show this after some time. So why it is not the desirable point for the root locus to be breakaway and hence the breakaway point exist at $s = -0.4226$. Now let's find the angle of asymptotes. So here number of angle will be n minus m which is three. So there will be three angles. So let's find those angles

$$\phi = \frac{(2q+1)}{3} * 180°$$

Where $q = 0,1,2 \ldots$

$$\phi_1 = 60°, \phi_2 = 180°, \phi_1 = 300°, \ldots$$

The centroid of asymptotes we can find as

$$\frac{\sum p - \sum z}{n - m} = -\frac{3}{3} = -1$$

So now I'll connect why we have taken the breakaway point this is our breakaway point. So here centroid is minus one that's why we can't take this value for the breakaway point. Now it is clear I hope why you have taken is equal to -0.4226 for the breakaway point. Now we will find the point where the root locus crosses the imaginary axis.

So for this we can come up with the characteristic equation and we can use the method of Routh stability criteria and we can find that value. To apply the Routh stability criteria we have to come up with the expression of the characteristic equation of the system. So characteristic equation of the closed loop transfer function is given by

$$s(s+1)(s+2) + 4k_p = 0$$

Routh array is formed in the same manner as discussed earlier.

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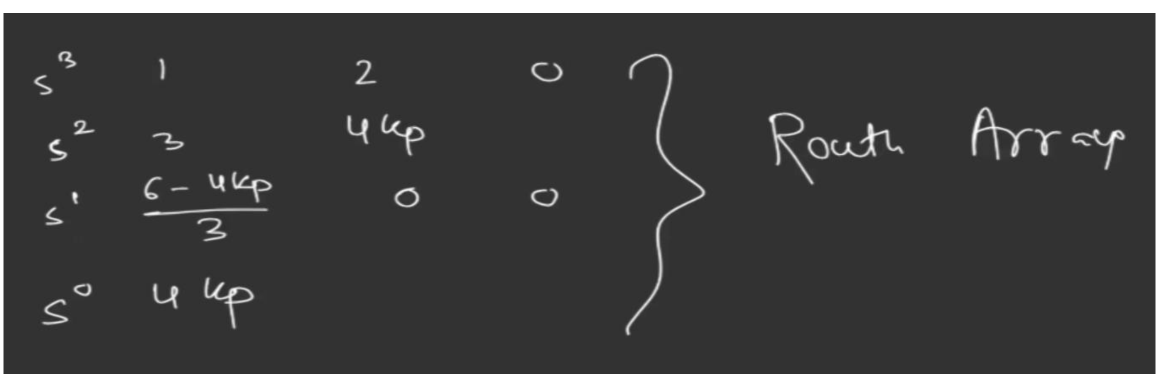

So from this Routh array will find the value of k_p at which the root locus causes the imaginary axis and now using sufficient condition for stability as you have done all the terms in the first column should be positive for that we need to this expression should be greater than zero we get

$$\frac{6-k_p}{3} > 0$$

$$k_p < \frac{3}{2}$$

Hence the limiting value of k_p for stability is 3/2, why because for this value the expression or the row for s^1 will be zero so this is the limited value so if k_p is greater than three by two then the system becomes unstable since root locus moves to right hand side. Now using $k_p = 3/2$ in the characteristic equation yields

$$s^3 + 3s^2 + 2s + 6 = 0$$

$$s = -3, \pm 1.41421j$$

Since we are interested in finding the values of s at which the root locus crosses the imaginary axis, hence

$$s = \pm 1.41421j$$

Now also we can find the ultimate gain k_{pu} from the magnitude condition as following

$$\left|\frac{4k_p}{s(s+1)(s+2)}\right| = 1 \ldots \ldots Eq(1)$$

Substituting $s = +1.41421j = \omega j$

$$|k_{pu}| = \left|\frac{s(s+1)(s+2)}{4}\right|_{s=1.41421j} = 1.5$$

Now if you draw the root locus for this particular system we have

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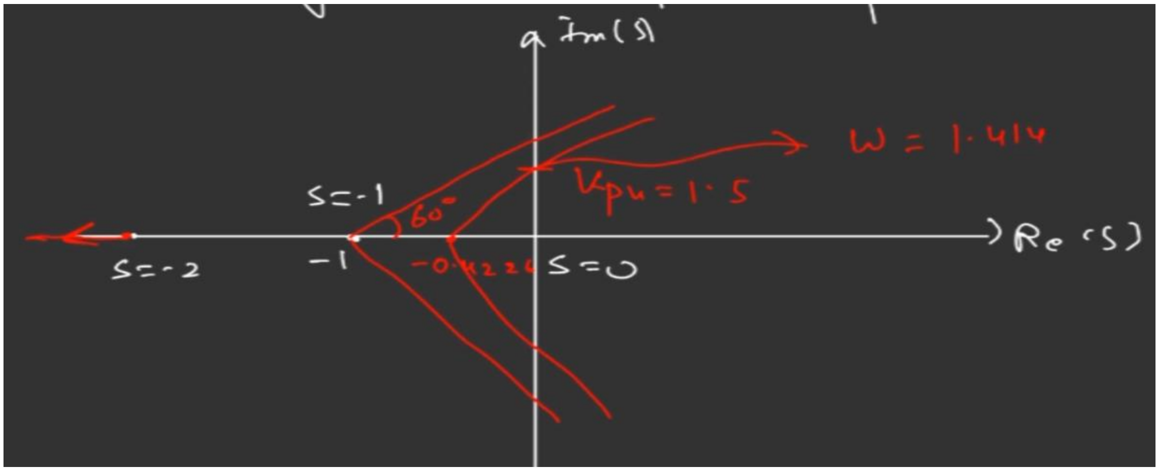

We are having the centroid of asymptote is minus one and which is basically so there are two two root locus that is equal to zero this is equal to minus one and is equal to minus two so we have the asymptote angles one is sixty degree and so here s equal to minus one is the pole but also the CG of the asymptote also minus one and we have another root locus here which is starting from s equal to minus two and goes to zero infinity and we have breakaway point somewhere which is minus 0.4226 this is our breakaway point and root locus goes like this and this is the point we are having at this point k_{pu} is 1.5 and $\omega = 1.414$,so this is the these two values are required for finding the PID gains of the PID controller so now we'll design the PID controller so for that we have to find the time period and what you have done in the previous lecture you can find

$$T_u = \frac{2\pi}{\omega} = 4.4429$$

Using Ziegler-Nichols rule the PID gains are

$$k_p = 0.6k_{pu} = 0.9$$

$$k_i = \frac{0.6k_{pu}}{0.5T_u} = 0.405$$

$$k_d = 0.6k_{pu} * 0.125T_u = 0.498$$

Hence the PID controller is designed as

$$G_c(s) = 0.9 + 0.498s + \frac{0.405}{s}$$

$$= \frac{0.498s^2 + 0.9s + 0.405}{s}$$

C.L.T.F is written as

$$\frac{Y(s)}{R(s)} = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)}$$

$$= \frac{4(0.498s^2 + 0.9s + 0.405)}{s^2(s+1)(s+2) + 4(0.498s^2 + 0.9s + 0.405)}$$

Now C.E. of C.L.T.F:

$$s^2(s+1)(s+2) + 4(0.498s^2 + 0.9s + 0.405) = 0$$

So if you solve this equation we can get the poles of the characteristic of the closed loop control system. So this is the task left to you to find the poles from this characteristic equation and you can comment on stability. So let's stop it here we'll continue from the next lecture. Thank you.