Introduction to Aircraft Control System

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Week – 01

Lecture – 03

In this course, we'll be dealing with classical controls for the linear systems. Also, we have to assume the system is time invariant. To prove the linearity of the system, we have to follow the principle of superposition under which there are two important laws. One is the law of additivity, another is the law of homogeneity. And once the system follows both the rules, then we can say the system is linear. And also, we have to be satisfied with whether the system time is invariant or not.

Once the system is time invariant and linear, then we can say the system is a linear time invariant system. We'll have some examples to validate these properties, then we'll conclude the lecture. As you know, all physical systems are nonlinear in nature. But to design a control algorithm in this course, we need to consider the linear system.

This is my linear system $y = f(x)$. But how will you validate the system you have is linear or not? This answer can be given by the rule which is called the principle of superposition. So now, I'm going to validate whether the system is linear or not. And through the principle of superposition, we can validate the principle of superposition. Under these two conditions, we have to test, first is the additivity test, beta test and second is homogeneity test.

Let me discuss what is additivity test. Let's consider we have system here, system what you have from the physical system or nonlinear system $y = f(x)$. And I'm applying to this system two different inputs. Let me say this is plus I'm applying to the summing point x_1 and another x_2 .

And the output from this block we can say $x_1 + x_2$. And the response from this block we can say $y =$ or I can say $y = y'$ I can write $f(x_1 + x_2)$. And if you notice in this loop, we are actually adding the state or inputs and we are applying this summed input to the function. But instead of this, if you apply x_1 to the first to the system $y = f(x)$ and x_2, if we apply the same system $y = f(x)$ and the output from this block we can write $f(x_1)$ and from this block we can write $f(x_2)$. And if you sum both the output to the summing point, we have output from the summing point $f(x_1) + f(x_2)$.

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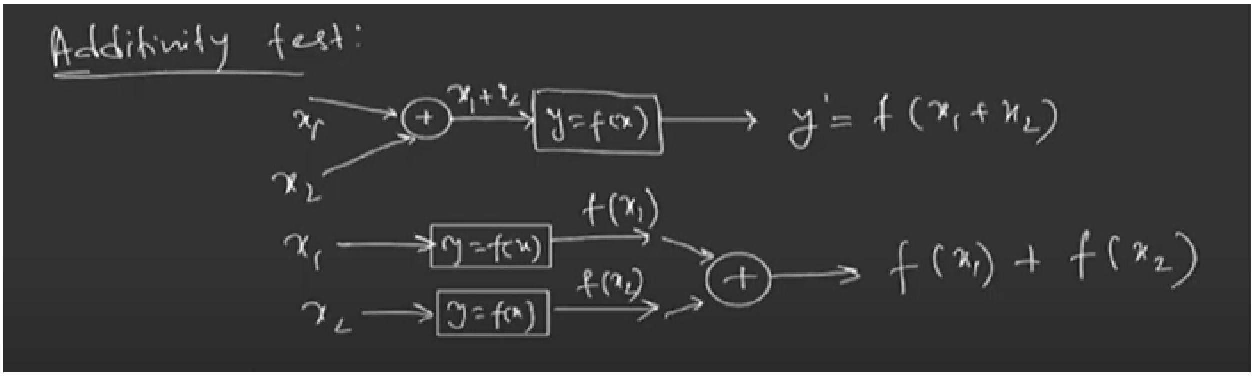

Now if y, let me write this y'' . If $y' = y''$, we can say the system follow the additivity rule. And we need to go to the second test which is the homogeneity test. In homogeneity, let's consider the same system $y = f(x)$, this is my system. And let's apply input which is $x(t)$.

And the output we can write, and if you scale this output, sum scaling factor k, and we can get ky. This is, let me assume this is $y(t)$, for example, the same output, same input you are applying $x(t)$, which is going to the function $f(x)$, and we are getting the output $y(t)$. And if you multiply this $y(t)$ by scaling factor k, we have the output $ky(t)$. If you notice here, we are multiplying the scaling factor k to the output $y(t)$ after the system response. Now instead of this, if we multiply this scaling factor before we apply to the system, suppose this is my k, and the output from this after multiplication, I can write $kx(t)$.

And this, if we apply to the system, which is $y = f(x)$, and the output from the system, we can write y'. And this is, it is y'', and I write y'. If $y' = y''$, I can say the system follows the homogeneity rule. If it does not follow,then it does not satisfy the homogeneity. If the system, what you are getting from the physical system, follows both additivity and homogeneity, we can say that the system is linear.

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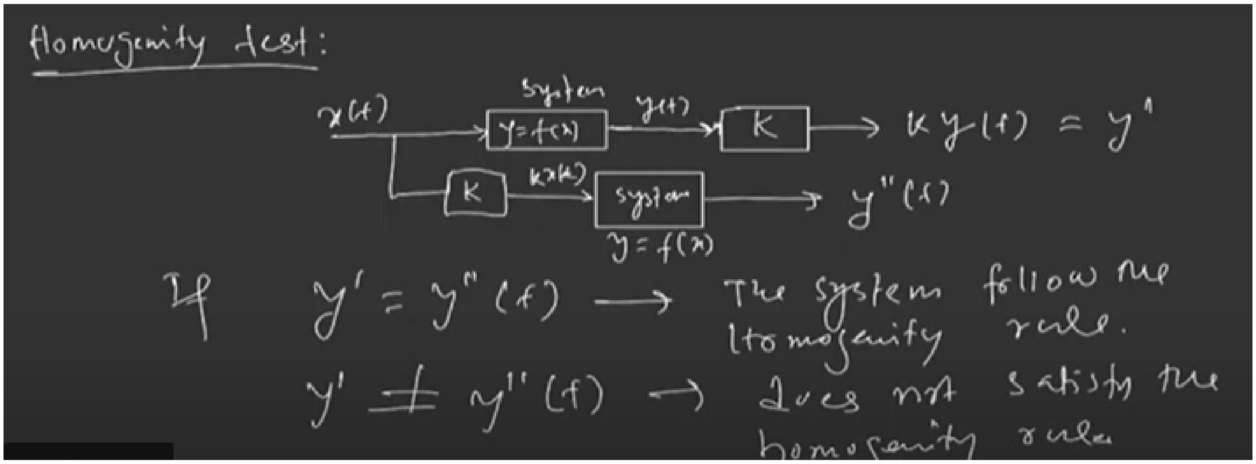

Let’s take an example, which can validate the above theory. So, $y = f(x)$, and the $f(x)$ function is $\frac{x^2}{16}$. We will say the system is linear or not. So, we will go two tests, one is the homogeneity test, another is the additivity test. First, let us go with the additivity test.

We will go the same steps what we have done, we have summing point, and we apply two inputs to the system x_1 and another input x_2 and sum the inputs and the output from this block, we can write $x_1 + x_2$. And this response will apply to the system $y = f(x)$, which is $\frac{x^2}{16}$. And the output from this block we can write $f(x_1 + x_2) = \frac{(x_1+x_2)^2}{16}$. And in the another case, we will apply the system individually to the system, so we have system $f(x)$, this is basically $y = f(x)$, I can write the same system $y = f(x)$, and we will apply here x_1 for this system, x_2, and the output from this we can write $f(x_1)$ on this block, $f(x_2)$. And if you sum both the output through the summing point, we can write $f(x_1) + f(x_2)$.

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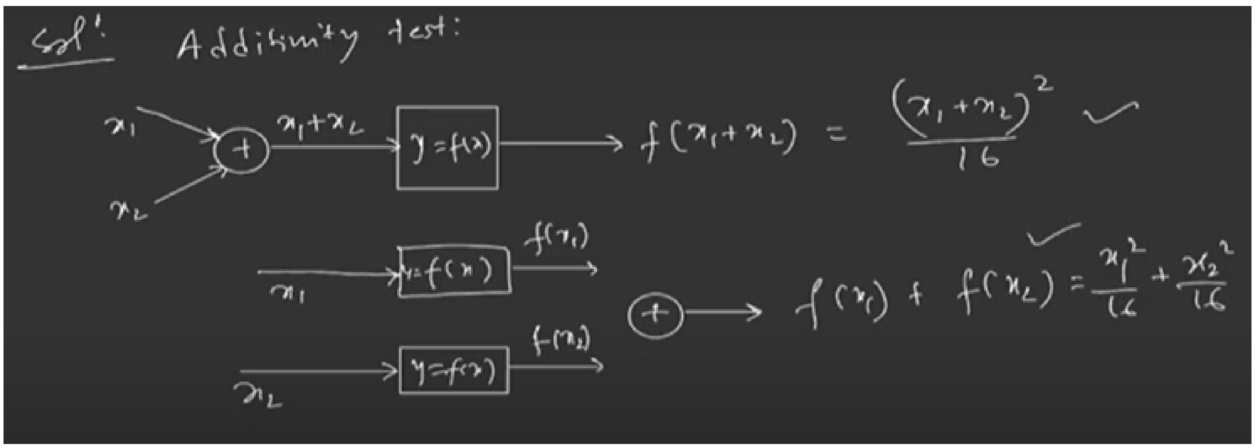

And we can write $\frac{x_1^2}{16} + \frac{x_2^2}{16}$. So if you notice, the response from this block and this block are different. So the system $y = f(x)$ does not does not satisfy the additivity test. With this condition also we can say the system is not linear, but still we can test the homogeneity test as well. Let's go to the homogeneity test.

To be system linear, we have to test both homogeneity test and additivity test. If both the conditions satisfy, then we can say the system is linear. But in this case, through additivity test system already is not linear. But still in homogeneity test, what we'll do is we'll, let's assume this is the $x(t)$ is my some reference input, and I am scaling this input by a scaling factor a, and the output from this summing point I can write ax .

And if we apply this ax response to the system $f(x)$, we can come up with the output $y = f(ax)$, or we can write $\frac{(ax)^2}{16}$, which is $\frac{a^2x^2}{16}$. Now, if you notice here, we are multiplying the reference input $x(t)$ by the scaling factor a before we apply to the system, basically pre-multiplication. Now, if you do, if we apply $x(t)$ as it is to the system $f(x)$, and the output from this block we can write $\frac{x^2}{16}$. And if you now multiply this output with a scaling factor a, and the output from this multiplication block we can come up with $\frac{(ax)^2}{16}$. Now, if you notice, the output from this line and this line are different.

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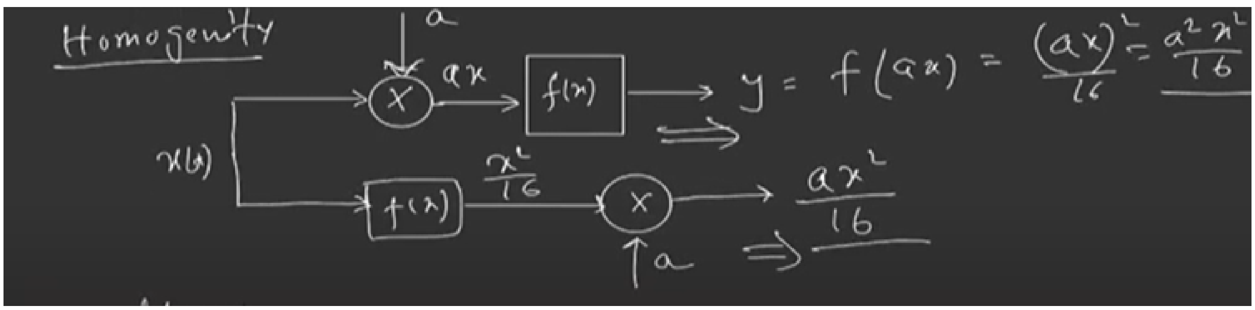

So, basically the, this function and this function is different, the output from these blocks. So, we can say, the system does not satisfy the homogeneity, hence the system is not linear. So, both the conditions are not satisfied, I mean, the system does not follow both the homogeneity test and additivity test. So, hence the system is not linear. So, now we can test whether the system is linear or not, but there are other conditions that also need to be satisfied before you design control synthesis.

So, to the above test, we can say the system is linear or not. Now, the system which comes out to be linear, also needs to be time invariant. So, once, the system is linear, time invariant, or LTI, we can easily design the control because in this course, we'll be designing the control algorithm for the LTI system. So, through the above theorem, we can check if my system is linear or not. Now, also you have to check the system is also time varying or invariant, that is also needed to prove.

So, let's go how we can prove the system is time invariant or not. So, let me define the definition of time invariant or variant. Any delay applied in the input must be reflected in the output. So, how it can be defined, let's go to the blocks and it will be easy to understand. So, if system, we'll check if the system is time varying or not, and we are applying some reference input, which is $x(t)$, for example, and we are getting some output from the system.

Let me write, this is my $y(t)$ output from the system. Now, if you apply a delay to this output, let me write t_0, some delay t_0 to this output, we can write this, the output by $y(t - t_0)$. Now, instead of applying a delay to the output, if you apply delay to the input function, which is $x(t)$,and delay by t_0 , so we can come up with the output like $x(t - t_0)$. And if you apply this $x(t - t_0)$ to my system, and we can write, this is $y't$. So now, if $y(t - t_0) = y'(t)$, then I can say the system is time invariant.

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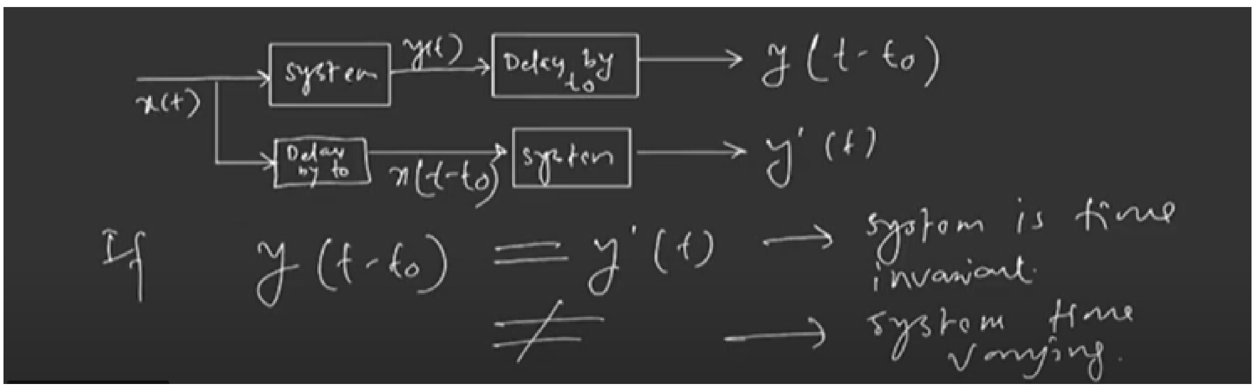

If it is not satisfied, then we can say the system is time varying. So, this is how we can check the system is time varying or not. Here, basically, most of the systems where the coefficients are assumed to be constant, basically, those kinds of systems are time invariant systems. Now, let us take an example that I think will probably be clear. Example, $y(t) = t\,x(t)$.

Let us check whether the system LTI or not. So, here also we will check the linearity, whether the system is linear or not, also the system is time invariant or not. So, let us go step by step. Let's assume the inputs are given inputs. I am not writing the block in blocks, just it is the same way you can write it mathematically.

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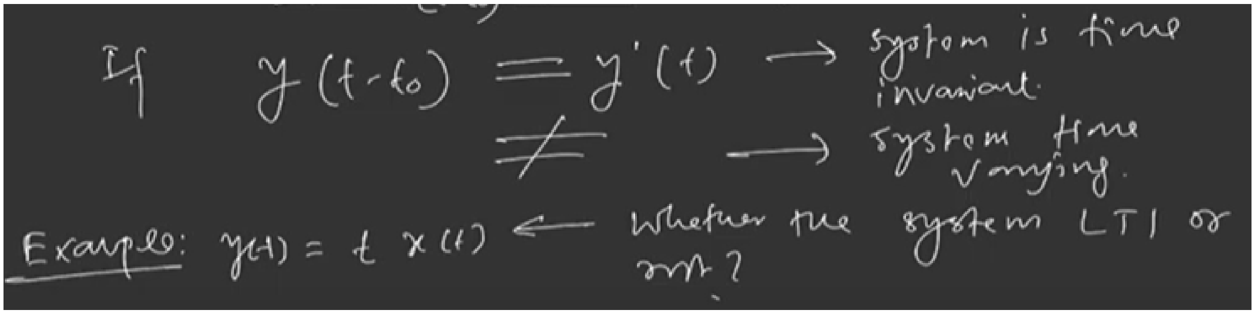

Given inputs are $x_1(t)$ and $x_2(t)$. Now, for the $x_1(t)$, the output I am getting through the system tx_1 (t) or I can write $y_1(t)$. And for the input $x_2(t)$, I am writing $t\,x_2(t)$ and output $y_2(t)$. We can write $y_1(t) + y_2(t) = t\,x_1(t) + t\,x_2(t)$. Now, if you now ,if we scale the inputs pi a and b, which yields we can write $ax_1 + bx_2$, we can write $t\,(ax_1 + bx_2) = a\,tx_1 + b\,tx_2$ And from this also you can write $a_1y_1(t) + by_2(t)$.

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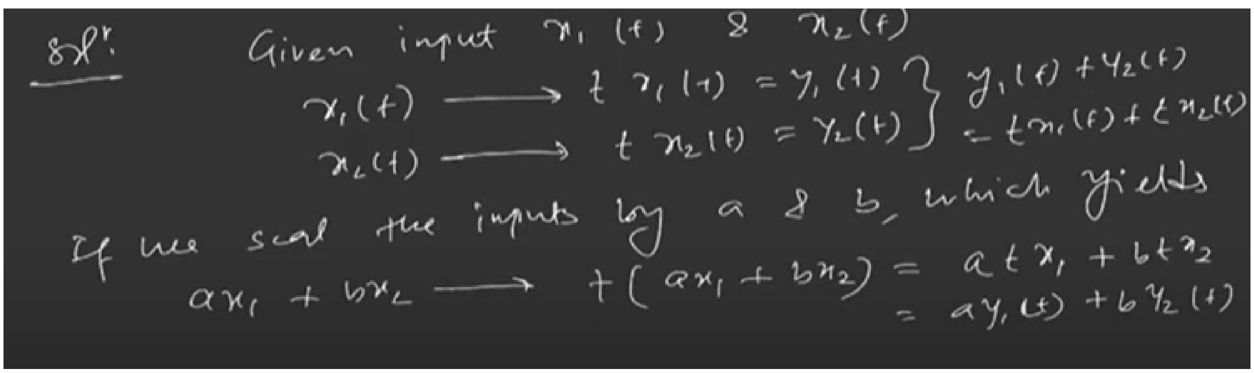

So, from this, I can say the system follows, hence, you can check through blocks, it will be also easy to understand, I am not doing it. So, hence the system follows both additivity and homogeneity. The system is linear. Now, we will go to the test of system is linear or not, sorry, linear system is time varying or not. So, for this, what we will do is, if we check time invariant or not.

So, here if the input $x(t)$ is delayed by t_0 and also if we apply the same to the output like this, and if we apply to the system, we have $x(t - t_0)$, from this we can write $tx(t - t_0)$ or $y't$ and $y(t - t_0)$, from this I can write $(t - t_0)x(t - t_0)$. So, if you notice here, you can check this through the blocks. What I have done in the testing of system time invariant or not, you can check in this block. So, this is the exact same expression you will be getting. So, now from this, it is clear that both the outputs are not equal, hence the system is not time invariant. This is how we can check if the system is time varying or not.

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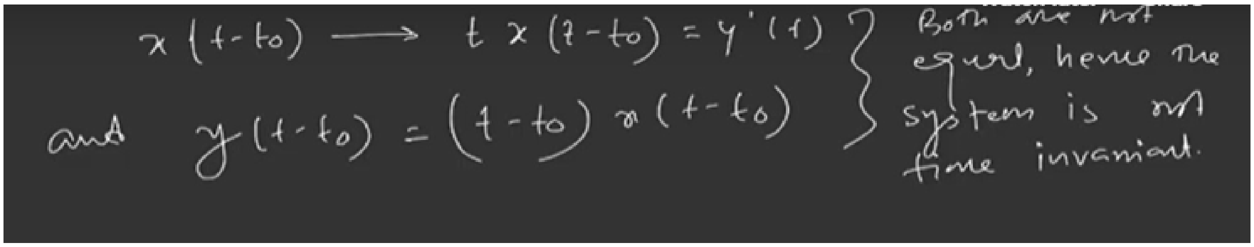

Let us take another example 2. We have a system $y(t) = x^2 t$. And let us check whether the system is LTI or not. Let's go step by step. First we will check if the system is linear or not. So, for this, we will first test the additivity test, then we will move to the LTI, homogeneity test.

So, through additivity test, if you consider there are two inputs x_1 and x_2 , we can write $\left(x_1 + x_2\right)^2 = x_1^2 + x_2^2 + 2x_1x_2 = y'$ and $y_1 + y_2 = x_1^2 + x_2^2 = y''$. So, through blocks also you will get the same kind of response. So, we can check $y' \neq y''$. So, it does not follow the additivity property.

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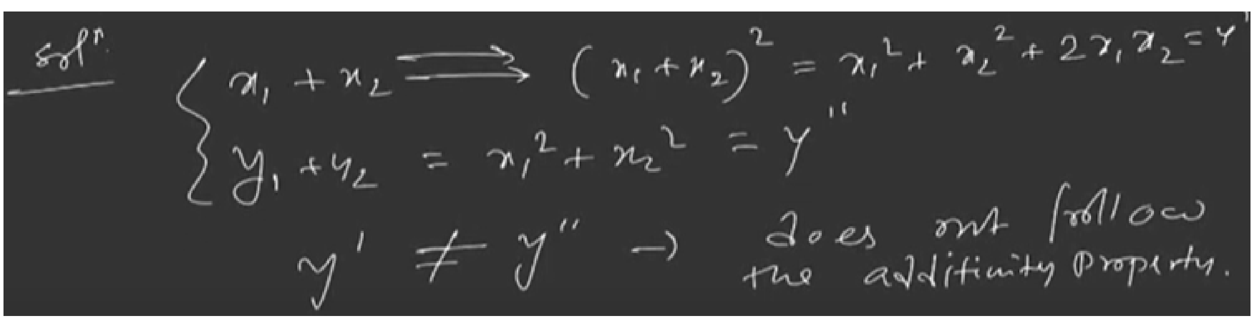

Let us see the homogeneity property. So, we have homogeneity, we have input $x(t)$, we are applying to the system and we are getting output which is scaled by k, and we are having $kx^2 t$ or I can write $y'(t)$. And similarly $x(t)$ will scale first by k and we are getting $k\,x(t)$ and we apply this $k\,x(t)$ to the system, and we will get $k^2 x^2(t) = y''(t)$. So, from this I can write $y'(t) \neq y''(t)$. The system does not follow the homogeneity properties.

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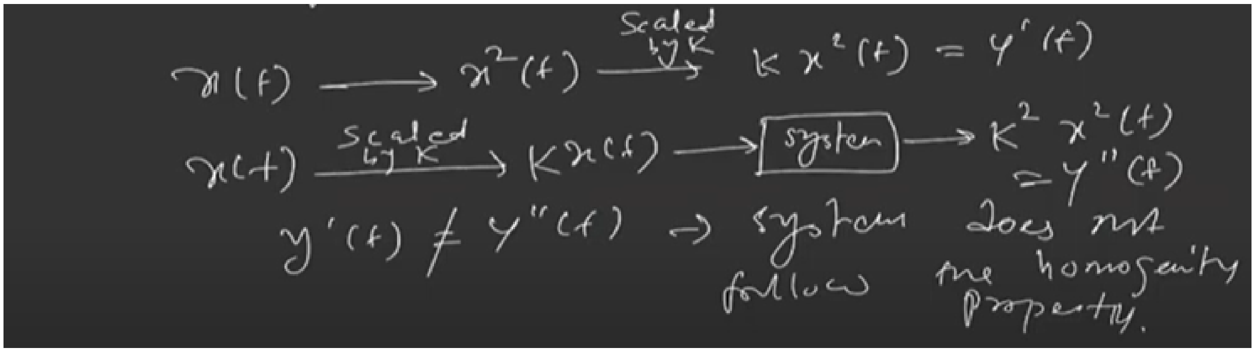

Hence the system is not linear. So, system does not follow both the additivity and homogeneity. So, hence the system is not linear. So, now we will go to the next part whether the system is time varying or not. So, we will test through the blocks. We have $x(t)$ as a reference input and I am applying to the system system and we are getting the output $x^2(t)$ and delay by t_0 and we are getting $x^2(t - t_0)$ or I can write $y\,(t - t_0)$.

And now we will apply the delay to the system input $x(t)$,then we will apply to the system. First we will apply delay here by t_0 and we apply this delay t to the system. This is my $x(t - t_0)$ system and we are getting the output $x^2(t - t_0)$ which is some y' . So, if you notice $y\,(t - t_0)$ is the same as y'.

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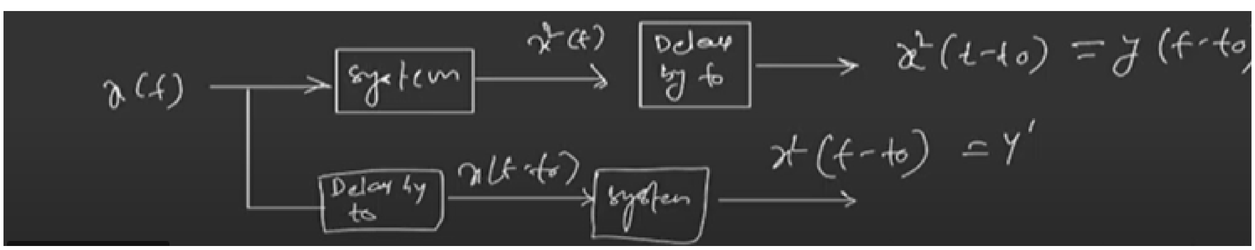

The system is time invariant. It is good that the system follows the time invariant properties but the system does not follow the linearity property. So, the system is not linear, but time invariant. So, this is how we can check if the system is LTI or not, system is time invariant or not, system is linear or not. So, this is how we will be doing. So, now in the next lecture we will be talking about why we have to consider the LTI system for designing control.

That is the very important part we will be talking about. So, let's stop it here. We will be continuing from the next lecture. Thank you.