Introduction to Aircraft Control System

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Week – 06

Lecture – 29

Lead and Lag Compensator

The single gain in the root locus cannot specify all the performance specifications. To resolve these issues, we can come up with some kind of compensator in the control system. If you draw the closed loop control system,

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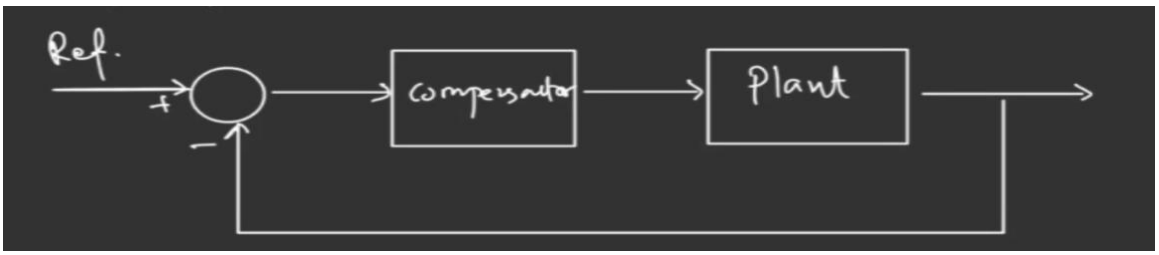

We have summing point here and we have reference input, we can add a compensator here. Compensator is nothing but a controller. If you add a compensator here, and which can provide additional poles or zeros to the plant through which we can modify the transient response or steady state response of the overall system. This is plant, this is output of the plant, this is the feedback. Now we will study what are the different kind of compensator we use in practice. So, let us start with the lead controller or lead compensator. The transfer function of lead controller, we can write

$$G_c(s) = k_{lead}\frac{s+\frac{1}{\tau}}{s+\frac{1}{\alpha\tau}}$$

Where $0 < \alpha, 1$ and τ is the time constant

And if you look at these poles and zeros of lead controller, since here alpha is between zero to one, so in this case zero is dominating. Zero is here and we have pole is here.

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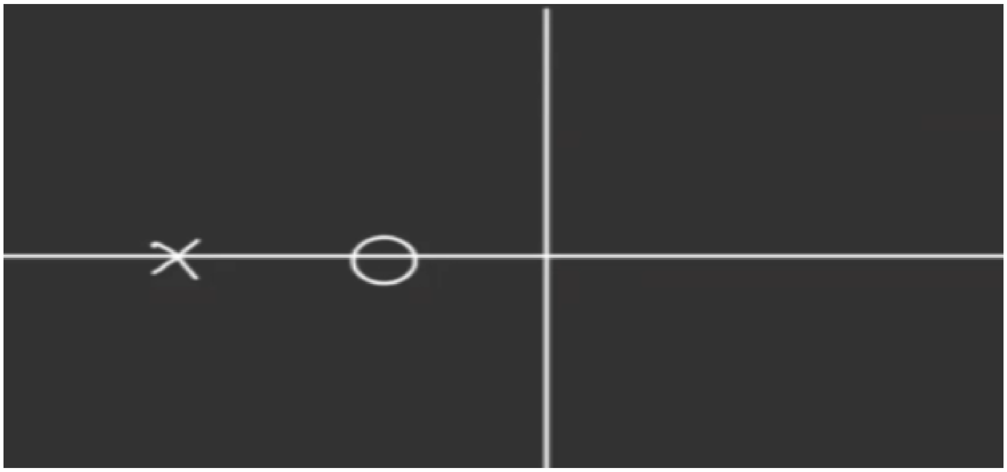

And in the system, if zero is dominating, let me write a note here, in a control system, if zero is dominating, dominating means if the zero or pole which are close to the imaginary axis, so those are actually dominating poles or zeros. In this case, since zero is dominating, the transient response can be improved. And in the system, if poles is dominating, if pole is dominating, we can improve the steady state response.

So, how we can do this? Let me go through this. Through lead controller, since the pole is left to the zero, the root locus will shift to the left and which improves the transient behavior of the system. So, this is how if we place lead controller in the control system, , we can improve the transient behavior of the response. Now, let us move to another kind of compensator which is lag controller. For lag compensator, a transfer function of the lag controller, we can write

$$G_c(s) = k_{lag}\frac{s+\frac{1}{\tau}}{s+\frac{1}{\beta\tau}}$$

Where $\beta > 1$

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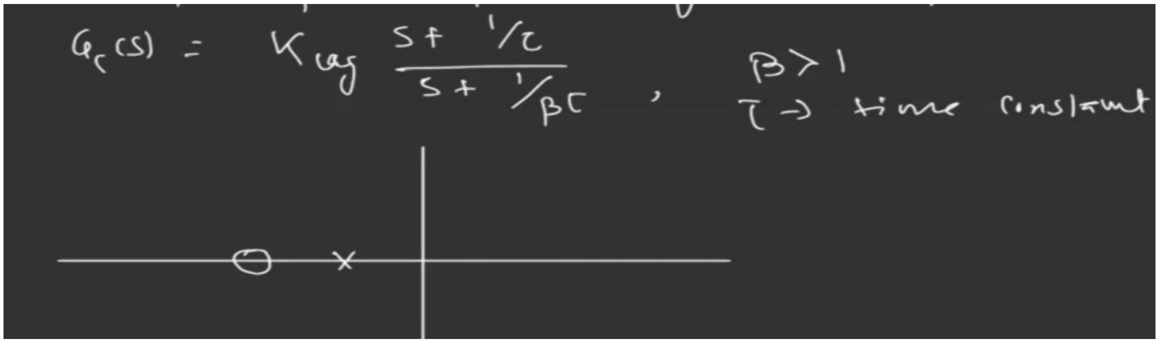

And if you place these poles and zeros of this controller in s plane, here pole is dominating and zero you can have here. So, since the pole dominating here, the steady state error can be reduced without affecting the transient response. So, this is how if you use lead lag controller in the control system, we can improve the steady state or transient response of the closed loop control system. Now, we will have another type of compensator which is motivated from the PID control system. So, let us look another compensator which is basically proportional and derivative.

These are basically standard control system proportional derivative control. This is already we have done. Now, I will connect how the proportional derivative control are close to the structure of lead controller. Let us look. The transfer function of, this is we can write

$$G_c(s) = k_{pd}\left(s + \frac{1}{\tau}\right)$$

In this case, the control system actually adding one zero to the plant. So, we can write the pd control adds a single zero to the loop locus, which decreases the number of asymptotes on the root locus. If it is decreased the number of asymptotes in the root locus, the angle will increase. So, how we can, if you remember the expression for angle

$$\phi = \frac{(2q+1)180°}{n-m}$$

So, if you are adding number of zero to the system, so angle will increase automatically. So, this is how we are increasing the number of zero in the system and simultaneously we are decreasing the number of asymptotes. So, we can also write the angle of the asymptote increases. If you now see closely in the limiting case of alpha, the limiting case of alpha, if alpha goes to zero, lead controller, the PD control acts lead controller. How it is happening? If you assume in the lead controller, if we assume alpha is close to zero, so in that case, pole goes to the negative infinity. So, we can ignore that pole which are very far to the negative infinity. So, in that, if alpha goes to zero, we can assume the PD control system acts like a lead controller. Since the PD controller adds a zero to the plant, the transient response can be improved. And let's look another type of control which is PI proportional and integral or we can say PI control. The transfer function of PI control, we can write

$$G_c(s) = k_{pi}\frac{\left(s+\frac{1}{\tau}\right)}{s}$$

Here, if we notice, the PI controller, PI controller adds a small open loop zero and an open loop pole at origin. So, here we are adding one pole at origin and another pole, another zero, we are adding to the system. Comparing now, comparing the PI controller with the with the lag controller, in the limiting case of beta, if beta is supposed to infinity, the PI controller acts like a lag compensator. And in this case, hence similar to the lag controller, the PI controller control reduces the steady state error which leaving the transient performance almost unchanged. So, this is how we can connect among the different compensators. So, mostly we use lead and lag compensator along with the another different control system to modify the transient and steady state response of the closed loop performance. And also, we have seen how the PI controller and PD control in the limiting case of alpha and beta can act as a lead and lag controller we have discussed here. Now, we'll have some examples and how we can design PID or PI or PD control system in the closed loop performance. Let's take an example. Through this example, how we can design a PID controller using root locus, we'll discuss them briefly.

Then in next lecture, we'll have full example how we can go step by step root locus design and PID control. So, example is design a PID control. This is a very important example how we can design PID control for a closed loop control system because in the previous cases we designed the control based on one gain using root locus and here how we can design the full PID control using root locus concept. Design PID controller for the given control system. We have plant as a function which is given by

$$G_p(s) = \frac{0.2}{s(s+1)(s+1.5)}$$

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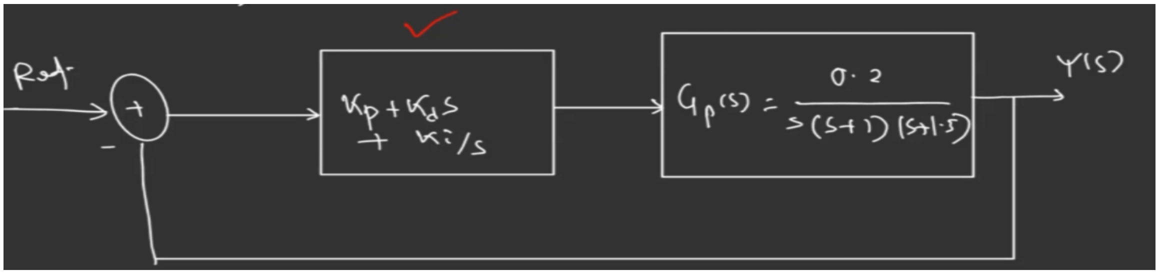

So, this is the closed loop control system. Now we are going to design this PID control We will design step by step. This problem we will be solving using the method of Ziegler-Nicholas method. This method we will be using in next few lectures.

How we can design PID control with the help of root locus and with the concept of Ziegler-Nicholas method. So, we'll solve this problem using the method of Ziegler-Nicholas. So, before we apply this method we need to find the root locus of the system and since as we have done in the root locus we can't consider multiple gain in the control system. We have to consider one gain at a time and we have to construct the root locus and based on that we'll design the rest of the control parameters. So, before applying the Ziegler-Nicholas method we should have the root locus plot. For designing root locus, let's assume the k_p is the control variable should be varied and k_i, k_d are assumed to be zero for finding the root locus plot. So, step one what we'll do is we'll vary k_p from zero to k_p to infinity. And step two find the root locus. I am not going to detail on it in the next lecture I will be defining how we can go step by step and here step three the gain k_{pu} this is we call ultimate gain. This is very important point ultimate gain of k_p can be found by finding the gain when root locus intersects the imaginary axis.

The process of finding this gain already we have done in our previous example. So, the root locus intersects the imaginary axis at $s = \pm 1.25j$ s which is the complex number. So, this part already you have done just here for the timing I am using this is the value this is the exact value which actually at which the root locus intersects the imaginary axis. I am not doing the full step you can try. In next lecture we'll have another example where I'll be doing the full things and finding at this let at this pole at this value of s we can find the k_{pu}. So, we know the closed loop characteristic equation, $1 + G_0(s) = 0$

$$1 + \frac{k_{pu} 0.2}{s(s+1)(s+1.5)} = 0$$

$$k_{pu} = 19.8$$

Step four we have to find the time period at the frequency 1.25. So, period at $s = \pm 1.25j$ we have

$$T_u = \frac{2\pi}{\omega} = 5.03$$

Now based on this data we'll apply the Ziegler-Nicholas method and how we can find the PID controller. So, step five we have to based on the Ziegler-Nicholas analysis the following rules for selecting gains. The gains can be calculated based on the obtained values of k_{pu} and T_u. So let me put everything in a table

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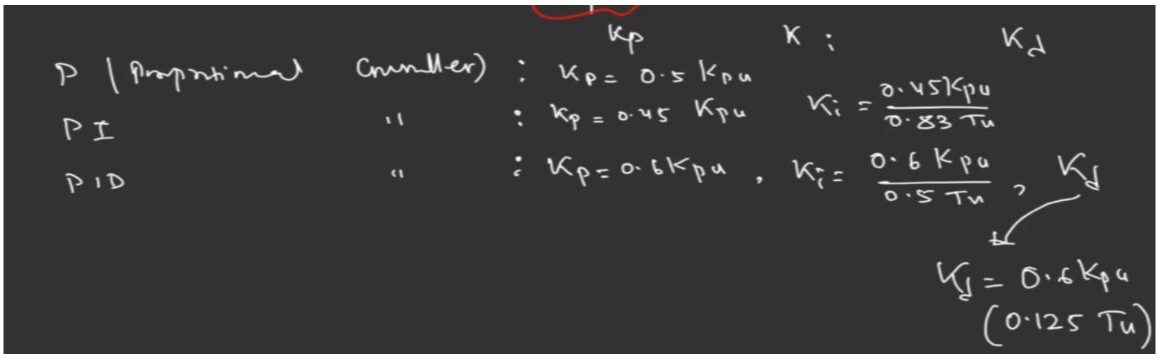

So first we'll find the proportional gain for proportional controller then we have PI controller then we have PID controller, for proportional controller we will find. If you are using proportional controller $k_p = 0.5k_{pu}$ so for proportional controller only we need k_p and this k_p can be found from the values of this k_{pu}. Now if you want to find PI controller so $k_p = 0.45k_{pu}$ and $k_i = \frac{0.45k_{pu}}{0.83T_u}$. If you want to design the PID controller so $k_p = 0.6k_{pu}$ and $k_i = \frac{0.6k_{pu}}{0.5T_u}$. So $k_d = 0.6k_{pu}(0.125T_u)$. So this is how we can form the Ziegler-Nicholas tables through which you can find the individual controller gains and if you substitute the values of k_{pu} and T_u in this expression

Step six gives the values of this controller gain, so knowing k_{pu} and T_u the controller gain can be obtained as

$$k_p = 0.6k_{pu} = 11.88$$

$$k_i = \frac{0.6k_{pu}}{0.5T_u} = 4.723$$

$$k_d = 0.6k_{pu}(0.125T_u) = 7.47$$

The PID controller we can design as

$$u = 11.88 + 4.723\frac{1}{s} + 7.47s$$

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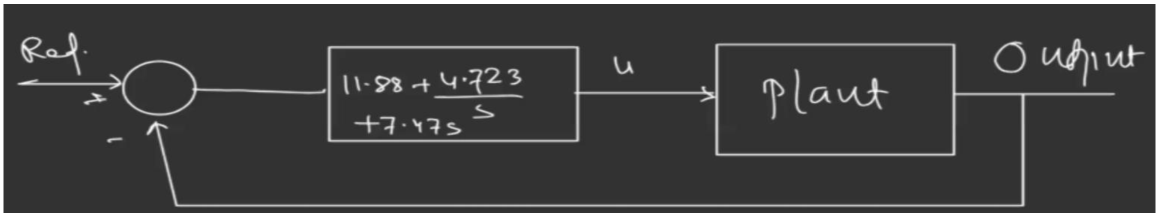

So this is the PID controller and if you place all the system in the closed loop control system so we have summing point here reference signal and this PID controller which giving the ideal control u which is going to the plant. This is the plant transfer function same as it is these are plant transfer function and we are having the negative feedback and if you design this control it will perfectly track the reference input so this is how we can design the PID control for the closed loop control system. In the next lecture we'll be dealing with another example how we can come up with the step by step the PID control for a system. So let's stop it here we'll continue from the next lecture. Thank you.