Introduction to Aircraft Control System

Prof. Dipak Kumar Giri

Department of Aerospace Engineering

IIT Kanpur

Week – 06

Lecture – 28

Effect of Addition of Poles and Zeros

If you notice in the earlier lecture, we have discussed about the locus, how the locus can be changed with the variation of the system gain. So, we have discussed the graphical representation of the movement of the poles of the closed loop, the control system with the different values of the system gain. If you notice here what you have done before, we have done this practice with the variation of the one single parameter. So, the root locus what you have done with the variation of one single parameter, which is basically K, we change the K and how the locus has been drawn. This is what we have looked at. But practically, this one single parameter cannot specify all the performance of the system.

So, within this variation, it is not possible to satisfy all the performance specifically using a single parameter. So, to solve this problem, we need to come up with some different kind of compensation which can be added to the control system to meet this performance or this specification for the designer has. So, we need to consider to add some form of, let us say, compensation to the control system to meet the design specifications. And this compensation we can add in form of electrical circuit, mechanical device or electromechanical compensator.

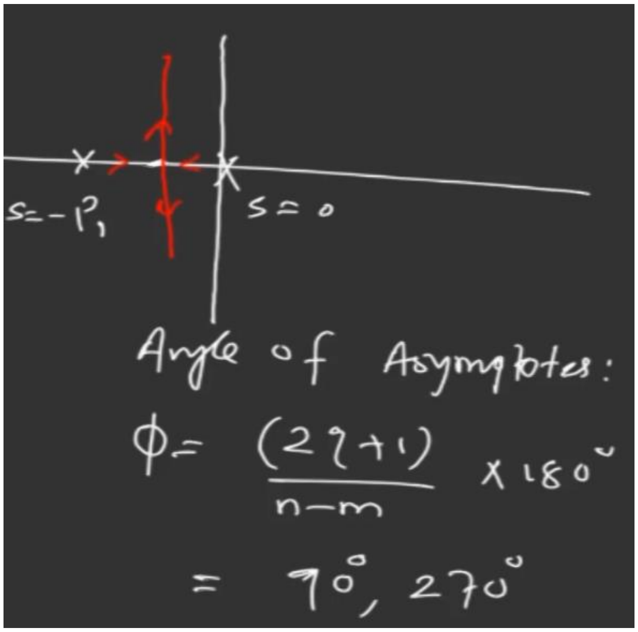
So, the compensator, maybe electrical circuit or mechanical device or electromechanical device to improve the system performance. And this compensator, if you look carefully, this compensator basically adding the poles or zeros to the system in forward or feedback path. So basically, the compensator has a transfer function which adds a pole or zeros to the system. So now, before we proceed to the control system, let's look how our root locus plot going to behave with the addition of poles and zeros to the system. So before we proceed, let's look the effect of addition of poles or zeros to the system.

So, let's go with a simple example, then it will be clear how the system going to be affected with the addition of poles and zeros. So, construct a root locus for the transfer function

$$G(s)H(s) = \frac{k}{s(s+p_1)}$$

So, there is no zeros in the system, only there are two poles, one pole at origin, another pole at S equal to minus p_1. And examine how the locus going to be affected by the addition of the following to the original transfer function. So let me go through this following what are the addition we have to do. The first addition is we'll add a simple pole and we'll see how root locus will be affected, simple pole, second case we'll consider multiple pole and third case we'll consider simple pole and zero. So we'll add this simple pole, multiple poles or simple zero to this transfer function. So we'll see solution. If you see these poles location in our s plane, we have

(Refer Slide Time 11:15)



And if you go back to our concept of asymptotes, the angle of asymptotes should be, it is clear that there should be our breakaway point will be somewhere here. That is quite obvious what you have done from the previous lectures. And this, once the breakaway happens, the locus will go to infinity since there is no zero in the system. And now let's look what angle this locus will depart from this breakaway point. The angle of asymptotes will be,

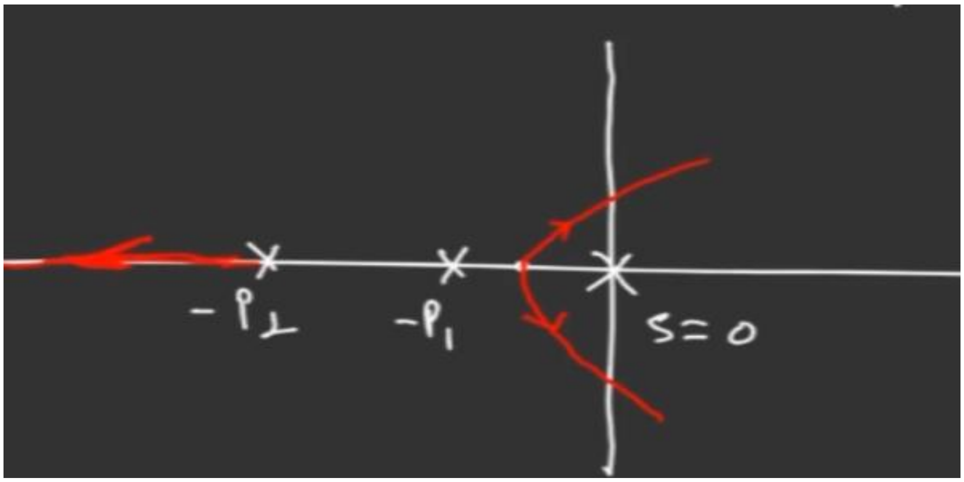
$$\phi = 90°, 270°$$

And since there is two poles, we can, there will be two angles, one is 90 degree and another 270 degree. So it is quite obvious the locus will be, will follow the asymptote. So locus will be something like this. So this is the original transfer function, how the locus will be plotted for the original transfer function. Now let's look at the case, first case, if we add simple poles to the system, how the locus will be affected. Let's look. Now if we add a simple pole, $s + p_2$ s to the G(s)H(s).

$$G(s)H(s) = \frac{k}{s(s+p_1)(s+p_2)}$$

So this will be the modified transfer function. And if you plot these poles in s plane, if you place these poles, we have one pole at origin, another pole is here,

(Refer Slide Time 13:25)



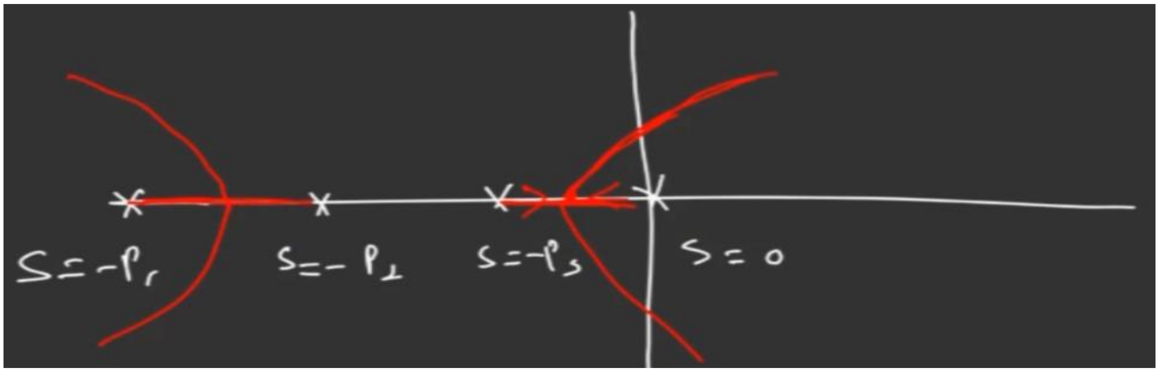
If you notice here, the angle of asymptote will change. So in this case, there will be three angles and one locus will, we can calculate the angles. So there should be breakaway happens between these two poles and locus will be going to like this. And there should be another locus will be going this direction like this. It means what is the takeaway from this addition of poles? We can write here, it will be interesting here. So it means the root locus will bend into the right half plane. So it means which limits the range of k for the system to be stable. But in the previous case, if you notice here, in the previous case here, for any values of k, the locus is always on the left hand side, the system is stable. But in the second case, if we add a pole to the system, so we have limits on the range of k. So system is going to be less stable for, we can't take any values for k.

So that's why system going to be, so there will be different part, one is stable region, another is unstable region for the different values of k. Now, let's look at another condition. If we add another pole, what will happen?

$$G(s)H(s) = \frac{k}{s(s+p_1)(s+p_2)(s+p_3)}$$

And if you place these poles in S plane, you have.

(Refer Slide Time 16:33)



So it is quite obvious, there will be locus in between, and this is also there will be locus in between these poles. And in between these poles, there will be breakaway somewhere here, and there will be another locus, something will be there like this. So if you can do this experiment, if you take some example with some numerical values of p_1, p_2, p_3 you'll have locus something like this. So what is the takeaway from this part? The addition of another pole adds another branch that goes to zero infinity. But the system can still be unstable. If the system gain, k, exceeds a certain value. So in this case, again, the system will be unstable if you increase the k after this part, after the imaginary axis. And in this case also, there are limits on k on which the system will be stable.

Now let's look how we can get the root locus plot for the addition of the zero. Before we proceed to the addition of the zero, let's have a note here, interpretation for the above analysis. The root locus plot bend towards the right half-plane with the addition of pole to the OLTF. The addition of a simple pole tends to destabilize the closed loop control system, this is very, very important part. If we add the poles to the system, system will tend to the destabilization. So this is very, very important part. Now let's look how the root locus will look like if you add pole and zeros to the system. Let's consider the original system. Original system we have,

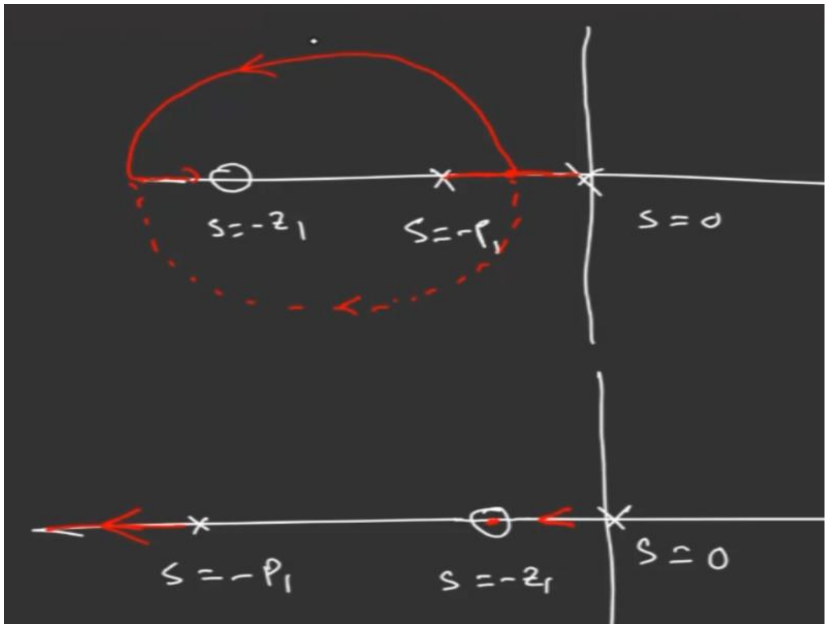
$$G(s)H(s) = \frac{k}{s(s+p_1)}$$

And let's add to this task of function a simple zero.

$$G(s)H(s) = \frac{k(s+z_1)}{s(s+p_1)}$$

And if you locate these poles and zeros in s plane, we have

(Refer Slide Time 22:53)



If you notice, there will be breakaway between these two poles. And break-in will happen here. This is quite sure. Now the root locus, it will be breakaway somewhere in between this point and it will go like this. So it means if we add zero to the system, it will, system will be more stabilized for any values of k, the locus always in the left hand side. And if you take another case, if you add a zero input in between these poles, let's consider another case. This is, for example, s equal to zero. If we add a zero here, s equal to minus z1. And if you have pole here, s equal $-p_1$ to minus p1. So in this case, the locus from this pole will end here and locus from this pole, s equal to $-p_1$, will go like this.

So in both the cases, in this case and in this case, locus actually stays in the, always in the left hand side. It is not going to the right half plane. So we can conclude from this part. We can write here with the addition of zeros to the system, root locus plot to bend further into left half plane. So hence we can write by adding a zero to the system in the open loop system, the closed loop system will be more stable than the original closed loop system. So what is the takeaway from this lecture is if you add zero to the system, system will be more stabilized. And if you add poles to the system, system will tend to the destabilization mode. And based on this motivation, we'll be, in the next lecture, we'll be discussing the concept of compensator to the system. How the compensator going to add a different stability criteria to the original system? Thank you.