Introduction to Aircraft Control System

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Week – 06

Lecture – 27

Root Locus (Cont.)

In this lecture, we will be taking some examples on root locus. The rules we have discussed in the previous lecture, those rules we will be using in these examples and how we can come up with the root locus plot. Let's see.

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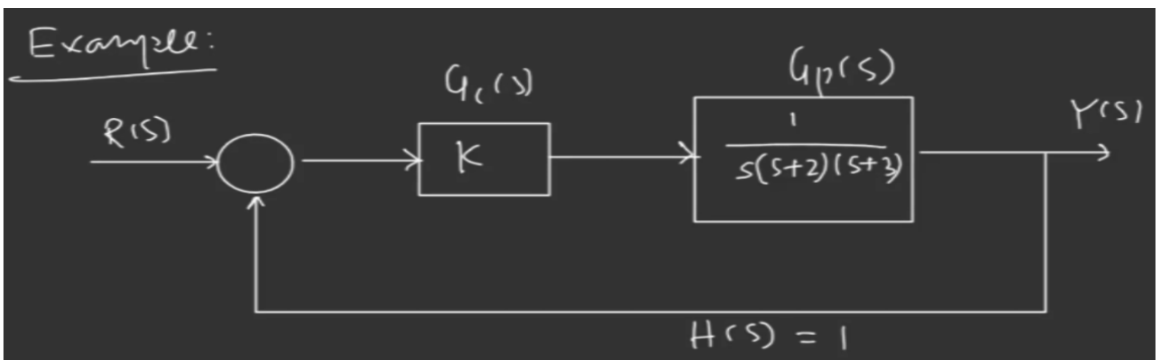

So fiR(s)t example, and we are having summing point here, giving the reference input R(s), and we are assuming there is one controller which is gain k, this is $G_c(s)$, and this output of this controller goes to the plant, and this is the output of this plant Y(s), and the output is feedback to the summing point through the unity feedback, and here the OLTF we can come up, the OLTF,

$$G_0(s) = \frac{k}{s(s+2)(s+3)}$$

We have poles of $G_0(s) = 0, -2, -3$ (n=3)

Zeros of $G_0(s) =$ no zeros (m=0)

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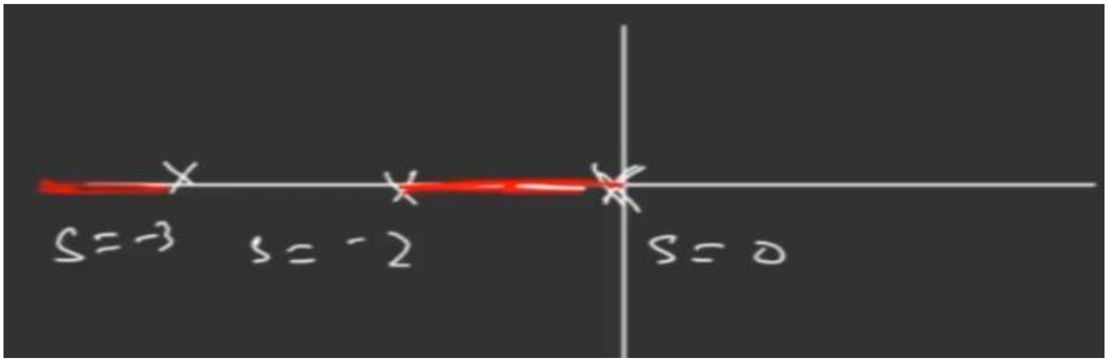

Now since there is locus in between them, there will be breakaway will happen here in between, so let's find the breakaway point. For breakaway point, we have to find $\frac{dk}{ds} = 0$

$$k = -s(s+2)(s+3)$$

C.E. will be $3s^2 + 10s + 6 = 0$

$$s_{1,2} = -0.785 \ or - 2.5$$

Now the breakaway point must lie on the root locus on real axis. Therefore the breakaway point is -0.785, which is between -2 and zero. Now we will find the number of asymptotes, number of asymptotes, so here number of asymptotes will be n-m which is 3. So there are three asymptotes and now let's find the angle of asymptotes,

$$\phi = \frac{(2q+1)}{n-m} * 180°$$

Where q $= 0,1,2$ ...

$$\phi = (2q+1)60°$$

$$\phi_1 = 60°, 180°, 300°, -60°$$

Now the point of intersection of the asymptotes with the real axis, we can find through the expression of centroid, so centroid of asymptotes, we can find

$$\alpha = \frac{\sum p - \sum z}{n-m} = -1.667$$

So this is the point where the asymptote will generate. Now we will find when the root locus will cross the $j\omega$ axis, ie. when the root locus crosses the imaginary axis. So we will put s equal to $j\omega$ in the characteristic equation,

$$1+\frac{k}{s(s+2)(s+3)}=0\Big|_{s=j\omega}$$

Solving we get

$$6\omega-\omega^3=0$$

$\omega=0$ and $\omega=\pm\sqrt{6}$

$\omega=0$ is not valid because this is the point at root locus starting, so $\omega=\pm\sqrt{6}$is the valid point value for ω. The crossover frequency when root locus crosses imaginary axis is $\sqrt{6}$. Also we will find the value of k at this point. So we can substitute this value in this expression we can write

$$k=5\omega^2=30$$

This is the critical value. So this value is very very important for designing the control algorithm or control log what we'll be do later. Now this value we are getting from this concept and now let's do the same let's go to the our stability criteria in through Routh stability criterion technique and let's check whether the same value we are getting through the Routh stability criterion or not. Now the value of k can be rechecked using Routh stability criterion. The characteristic equation we need to find, so characteristic equation is

$$s^3+5s^2+6s+k=0$$

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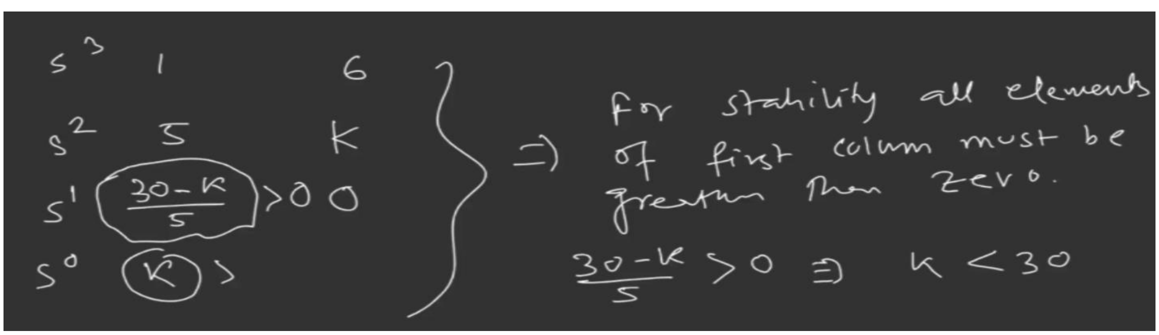

Now from this array we can come up with the condition for stability all elements of first column must be greater than 0 or positive we can say. So therefore this is the only part we have to check because here k should be greater than 0. Now we have to make this condition also greater than 0 or we can write k should be less than 30 and k should be greater than 0. So this is the condition for stability and as you if you remember if you substitute in this row if you substitute k equal to 30 then all the terms in this row will be 0.

So this is the condition for marginal stability or critical stability so now from this we can write hence the maximum value for which the system is stable this is the condition system is k less than 30 and k is greater than 0. Now the critical value value is k equal to 30. So the value obtained from the Routh stability criterion and we have value from the previous method it is same here the same value we are getting here. Now we are in the state for designing our locus plot so let's go step by step.

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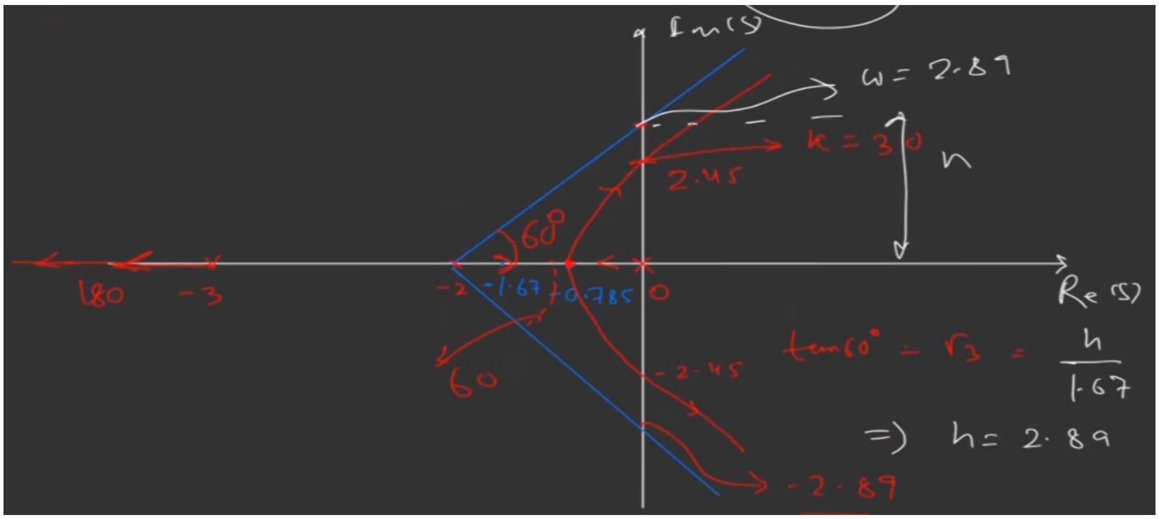

This is the s plane and we are having this is real of s this is imaginary of s and we are having two three poles at s equal to 0 at s equal to minus 2, 0 and s equal to minus 3. So centroid is minus 1.67 and the breakaway point we are getting minus 1.667 is the centroid of the value for the breakaway point would happen minus 0.785 so it will be minus 0.785. Now root on the asymptotes are angled 60 degree for example this is 60 degree this is also 60 degree mirror image will be there and from this pole there will be locus will be like this and breaker will be here at minus 0.785 and the root locus will be something like this, will follow the asymptotes so this is the value basically minus 2.45 and this point k equal to 30 and we are having this value we can find at this value what is the value of ω so this angle is basically 60 degree so we can find tan 60 degree equal to root 3 which is if you say the height is for example height is h, from this we can write h equal to 2.89 and this is also 60 degree and we have another asymptote which is 180 degrees so this is another asymptote from here which is 180 degree it is going to infinity and all the asymptotes are going to infinity and also the root locus also going to infinity because there is no zeros in the system so this is how we can draw the root locus plot so here also in the mirror image also this is minus 2.45 this value also minus 2.89 so this is how we can draw the root locus of this particular example let's have another example where we will be we will discuss the problem from the angle of departure and arrival point of view let's take another example.

Find the root locus for the given characteristic equation $s^2 + 2s + 2 + k(s+2) = 0$ s so in this example only the characteristic equation is given so we need to find the pole zero of the system solution we can find the OLTF open loop transfer function because we need to know to find the root locus so we know the transfer we know the characteristic equation in the form of open loop transfer function we can write

$$1 + G_0(s) = 0$$

$$1 + \frac{k(s+2)}{s^2 + 2s + 2} = 0$$

$$G_0(s) = \frac{k(s+2)}{s^2 + 2s + 2}$$

Zero is at $s = -2$ and poles are at $s = -1 - j$ and $-1 + j$. We have two complex conjugate poles

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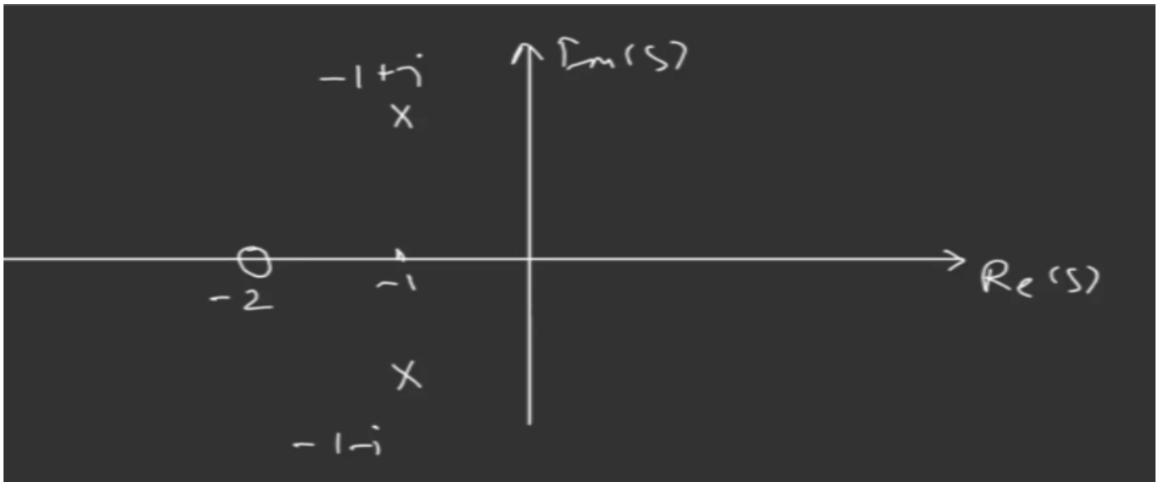

Now we can find number of asymptotes as $n - m = 2 - 1 = 1$. Now we can find the centroid of asymptotes which is

$$\alpha = \frac{\sum real\ part\ of\ poles - \sum real\ part\ of\ zeros}{n - m} = 0$$

So centroid is at the origin, now we'll find the angle of asymptotes

$$\phi = \frac{(2k+1)}{n-m} * 180°$$

Where $k = 0,1,2$ .... Since there is one asymptote we can calculate the angle only at k equal to 0 so we have

$$\phi = 180°$$

Now we'll find if there are complex conjugate poles so there should be angle of departure so when there are imaginary poles we need to find the angle of departure so this is the we can write

$$\theta_d = 180° - \left(\sum \theta_p - \sum \theta_z\right) = 135°$$

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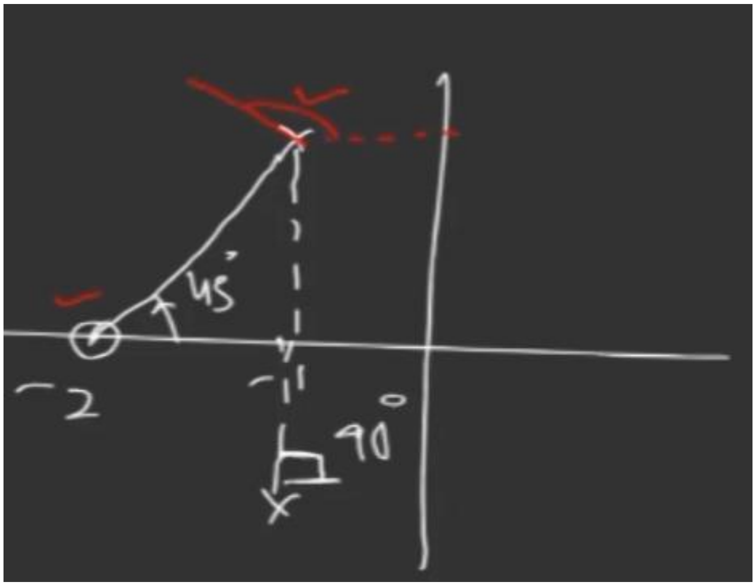

If you recheck the condition this is total angle from the remaining poles minus remaining zeros and from this we can write 180 degree minus this is if you notice the figure okay let me draw here so here these are the pole this is a pole which is at minus one and it is minus two so this is the zero and this is the another pole here and we have to find from the counterclockwise direction so this is the angle 45 degree this is 90 degree so we can write 90 degree minus 45 degree and from this we can the value is 135 degree so the 145 degree is the angle of departure so now if you notice we have zero here we have pole here and this that locus from this pole will end at zero and angle of departure if it is horizontal line so angle of departure something will be like this so this angle is this is the locus direction let me write something like this locus direction this is 135 degree we need to find the breakaway point, we can find so from the characteristic equation we can find the value of k expression

C.E. $s^2 + 2s + 2 + k(s + 2) = 0$

$$k = -\frac{(s^2 + 2s + 2)}{s + 2}$$

Breakaway will happen at $\frac{dk}{ds} = 0$. We will get

$$s_{1,2} = -0.585, -3.41$$

Breakaway point happens at the somewhere after minus 2 so let me draw the figure then it will be clear.

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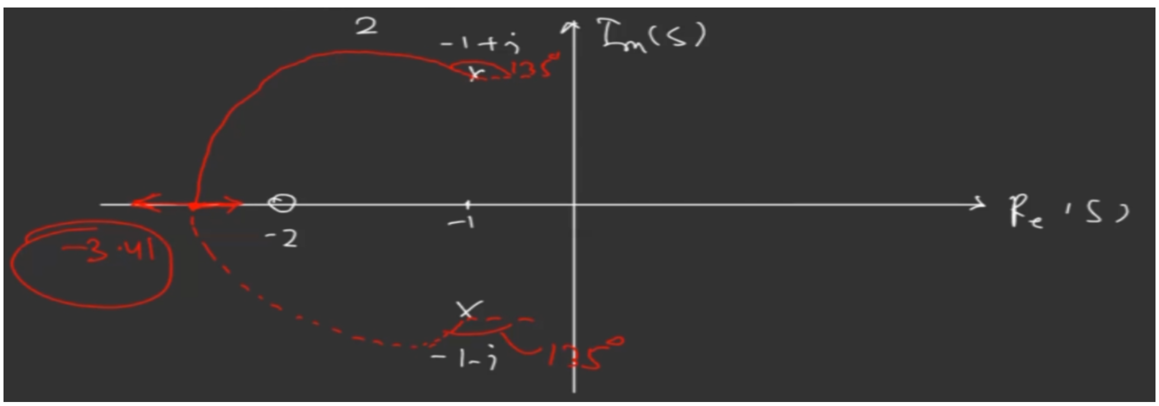

The breakaway point generally happens in the real axis now. So we are having two poles here which are minus one and minus j and minus one plus j and this is minus one and there is another zero is here which is at minus two so now this locus will start like this with 135 degree and it will go it will go like this similarly it will come like minus 35 degree it is 135 degree so the 135 degree here also 135 degree and this point actually breakaway point minus 3.41 and this is the way it will spread like this so this is how the break will happen here at minus 3.41 value so this is how we can solve the problems and if there are complex conjugate poles and how we can angle of departure the breakaway breaking points and we can solve the root locus plot in the next lecture we'll be using the concept of root locus method and how we can come up with some kind of compensating mechanism which will be very useful for aircraft applications. Thank you.