Introduction to Aircraft Control System

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Week – 06

Lecture – 26

Root Locus

Hello everyone, this is the lecture number 26. In the previous lecture, we have discussed the concept of root locus, how the locus created between the poles. And in this lecture, we'll be discussing how we can construct the root locus based on some standard rules. So let's go first step by step the guidelines, how we can come up with the root locus plot. As we earlier told that we have to consider the open loop transfer function of the system and from the open loop transfer function we have to come up with the poles zero combination.

1. Identify the open loop transfer function

$$G_0 = G_p(s)G_c(s)$$
$$= k\frac{(s-z_1)(s-z_1)\ldots(s-z_m)}{(s-p_1)(s-p_2)\ldots(s-p_n)}$$

So here we'll assume H is equal to one unity feedback system. The numerator part gives us the open loop poles and the denominator part gives open loop zeros. Here $z_1, z_2 \ldots z_m$ are the zeros of OLTF and $p_1, p_2 \ldots p_n$ are the poles of OLTF.

2. Calculate the number of branches. The root locus will have as many branches as the number of poles. It is obvious the number of, locus will start from the poles, so if you have n number of poles, we'll have n number of branches. Each branch starts with at an open loop pole p i and when the root locus will start from open loop pole, when the gain is, when k equal to zero and at k equal to infinity it will end at pole. If there is no pole it will go to infinity, that we'll discuss in this lecture, k equal to zero and m branches out of n branches at an open loop zeros of z i. We can write m branches out of n branches end and at open loop zero z i when k tends to infinity. So it means we can write n minus m branches will go to infinity and m branches will go to zeros. So it means if there are n branches which are coming out from the poles out of m, m branches, m branches will go to zero, n zeros and rest of the branches will go to infinity.

3. The root locus includes all points on the real axis to the left of an odd number of open loop real poles and zeros.

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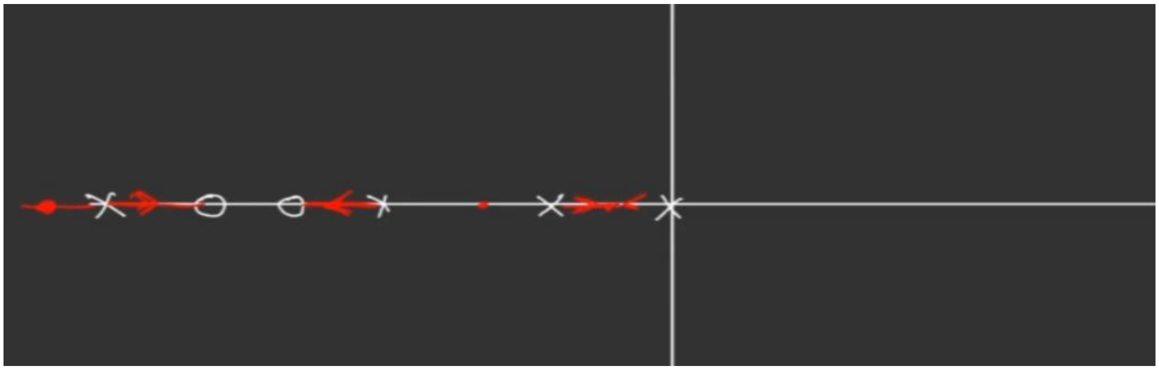

So this means suppose if we have poles and zeros on the real axis, for example we have one pole is here, another pole is here, another pole is here, another zero is here, another zero is here, and another pole is here for example. And the locus we can draw as, we'll take a test point here and we'll see the odd at this test point we will check how many poles and zeros on the right hand side whether they are odd or not. So in this case there is odd so the locus will be like this. We'll have another test point here so number of zeros and poles to the right as events the locus would be like this and of course there will be breakaway point we'll discuss later. We'll take another test point here we'll check the number of zeros and poles are to the right are odd so obviously there will be locus in this direction and for example here if you consider another test point is here and we'll check the number of poles and zeros to the right are event so the locus will be here the direction of the locus will be here so there will be locus here there will be locus here there will be locus here. So this is how we can come up some rough idea how the locus will start from the pole-zero combinations.

4. Determine the asymptotes which gives an indication of the direction in which the branches moves as gain increases. if you remember with the control aircraft control system we had the pole-zero combination like we had one pole was here another pole was here and if you remember as you increase the gain then the locus was there is a locus in between and if you increase the gain they came to some point minus half initially it was zero and minus one and at the minus one point there's a breakaway happens and the locus started to move like this.

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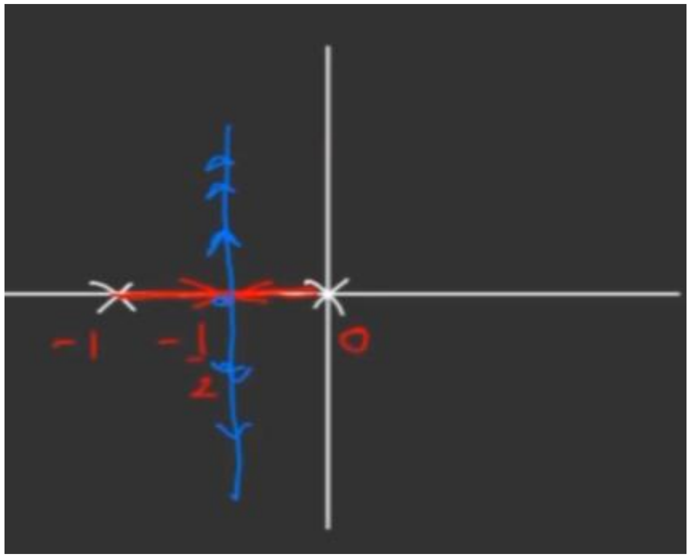

Now the thing is asymptotes will give the indicate indication how this locus will move with direction if you increase the gain it actually the locus will follow the asymptotes so asymptotes will give the indication how this locus will move on the s-plane if you increase the gain we'll discuss in detail for the example so this is basically asymptotes the advantage of having at asymptotes now how to find asymptotes let's work on it. As k goes to infinity, n - m branches approaches n - m asymptotes with angle measured counterclockwise from the positive real axis so we can if we denote the angle by ϕ and l denotes number of asymptotes

$$\phi_l = \frac{180° + 360°(l-1)}{n-m}$$

When l $= 1,2, \ldots$

$$\phi_l = \frac{(2k+1)180°}{n-m}$$

When $k = 0,1,2$ ...

So let me write the centroid is the point where the asymptote will start, the centroid is the average location of the poles and zeros along the real axis and the center of the asymptotes is we can write α is the center we can write

$$\alpha = \frac{\sum p_i - \sum z_i}{n-m}$$

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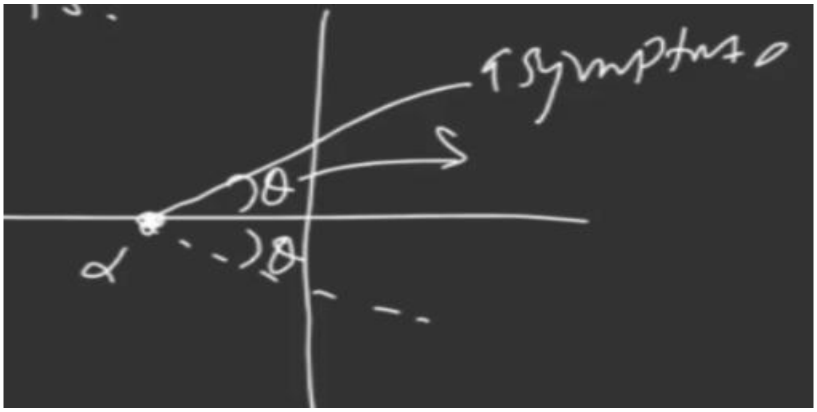

Suppose for example this is the point alpha and if you consider the asymptote angle by theta this is θ angle and since in root locus we should have another immediate image of this angle as well so this is basically θ also so this is how this is the asymptote this is the angle of asymptotes and this is the center we'll discuss all this talk through example then it will be clear how we can come up with these definitions .

5. The breakaway point and the breaking point. When the root locus when the root locus meet and split at the breaking point when the root locus root locus meet and joints so what does this mean here as you know breakaway points always happens at the poles and break in points always happens at the zeros. for example let's assume we have a pole zero location like this there is one pole is here another pole is here another zero is here another zero chair for example it is quite obvious there will be breakaway points will be here in between and there will be breaking point in between so the locus will start like this and this point we can say it will break in here and it will go to zeros and here the breakaway happens here it will come from the poles as you increase the gain it will go like this and it will insert the breaking so this is the breaking point and this is point we can say breakaway point and since in root locii there will be also mirror image we can have another site like this, this is the mirror image of the upper side so this is how breaking and breakaway points and how to find these points so the break in and breakaway points the breaking and breakaway points satisfy the following condition.

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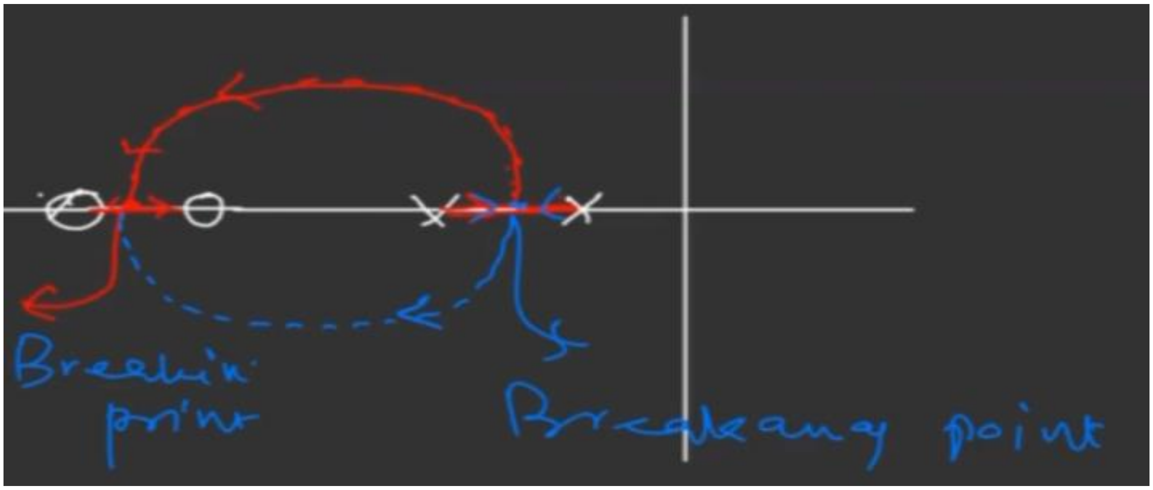

$$\frac{dk}{ds} = \frac{d}{ds}\left(-\frac{D(s)}{N(s)}\right) = 0$$

the solution of this equation yields points lie on the real axis. C.E is written as

$$1 + kG_p(s) = 0$$

$$k = -\frac{D(s)}{N(s)}$$

6. The angle of departure and the angle of arrival, at what angle the root locus will start from the breaking points, breaking points will be arrival and breakaway of departure. The angle of departure of the root locus from a pole of $G_0(s)$ can be found by the following expression
$$\phi_p = \pm 180°(2q + 1) + \phi$$
$$= \pm\big(180° + 360°(l - 1)\big) + \phi$$
When $l = 1,2, \ldots$

ϕ is the net angle contribution at the pole of $G_0(s)$ of interest due to all other poles and zeros of $G_0(s)$. The angle of a departure from a complex pole $= 180° -$ (sum of the angle from zeros to complex poles from other poles) + (sum of the angle of vectors to a complex pole from other zeros)

Angle of arrival at a complex zero $= 180° -$ (sum of the angle of vector to a complex zero from other zeros) + (sum of the angle of vectors to a complex zero from other poles)

This is what we have done before because generally the locus will start from the pole and end at zero if there is no zero then it will go to infinity but if there are complex pole it will start from the complex pole with the angle of departure angle some angle which is called angle of departure and with some angle it will arrive to the zero which is called angle of arrival sum of the angle of the vectors to a complex zero from other zeros plus sum of the angles of vectors to a complex zero from poles these poles can be complex or not complex but generally arrival and departure we measure at the complex pole and zero but the total angle we measure from the poles and zero those can be real or complex

7. We said the asymptote will go like this and it will cross the imaginary line and it'll go to the unstable region now how to find this point if it is angled through θ and at what point it will cross the $j\omega$ axis so this is what we'll talk in rule seven. Find the points where the root locus may cross the imaginary axis and this point on the imaginary axis can be found by two ways, first is by using a routh stability criteria, we can find this value because in routh stability criteria we form an array and we check the first column of this array whether all the points are positive or not and also we can find some range on gains and through these gains we can comment when the system will be unstable when the system will cross the imaginary axis and at that point the poles which crosses the imaginary axis at this point we can find the value of the gain through routh stability criteria we can find that value and another way is substituting $s = j\omega$ in the characteristic equation and equating both the real and imaginary part to zero and solving for ω and a the gain the values of ω gives the frequency at which the root locus across the $j\omega$ axis and the value of k gives value at that frequency at ω

These all rules we will be using through examples can validate these rules. Let's stop it here, in the next lecture we'll have few examples and how we can use these rules and we can construct the root locus plot and also you can comment on the stability of the system Thank you.