Introduction to Aircraft Control System

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Week – 05

Lecture – 25

Classical Control Method

This is one of the most important lecture in classical control, which is root locus method. So root locus method basically is the graphical technique for designing and analyzing the control system. Here we will see how the locus of the poles or the roots of the system going to change with the varying controller gain. We will have some rules through which we will be designing the root locus plot. And from the plot we will see how the locus going to start from poles of the system and ends at zeros of the transfer function. So it is basically will be varying the controller gain from zero to infinity and we will see the range of the controller gain for which the system will be stable. And also we will be finding the controller gain for which the system will be marginally stable and from that marginal value of the controller gain we will be designing the PID controller using Ziegler's Nichols Rules. So in next few lectures we will be spending on this particular topic and we will see how one can design the closed loop control system using root locus method.

In this lecture we will be discussing one new topic on control system design. From the last lecture we have discussed that how the system stability can be analyzed based on the controller gains. So how the controller gains going to be effective and based on the range of gains we can comment on the stability. Based on the controller gains we can comment on the stability. Basically the range of the gains on which the system will be stable this part we have discussed and in today lecture we will be coming up one new concept which is the concept of root locus. In this concept or technique we will be discussing how the system poles going to change with the change with gains. So how the system poles and zeros going to behave with the change in the system poles and zeros in the system parameter.

If you say the parameter is the gain so how the system poles going to behave on the s-plane. So how this poles going to change on this s-plane with the change with gains and these change in poles is called locus. How the locus of the poles or zeros going to be effective with the change in controller gains. So let us work on this part this is quite interesting and this part we will be dealing next few lectures and the concept this technique will be using for the autopilot design of the translation and rotational motion of the aircraft. So let us begin.

Let us consider the following closed-loop system.

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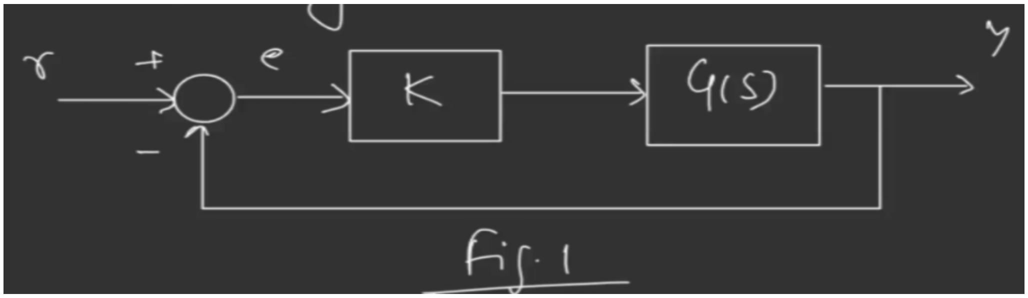

The above closed-loop system this is we are having the reference signal r and we are having the controller gain K and the plant let's assume we are having G(s) and we are having the output from this y and this is error this is the feedback from the output terminal this is negative this is positive and another important concept in this technique is we can take the open loop system and we can check the stability of the closed loop system that is another important takeaway from this concept we will be talking very soon. The closed-loop transfer function from this figure we can write

$$\frac{Y(s)}{R(s)} = \frac{kG(s)}{1 + kG(s)}$$

In this block diagram we will be checking how the system holds and zero is going to be effective with the change with the gain K. We will take the this controller gain as the only parameter going to be changed but later we will extend how we can win the PID controller gains that is another important topic we will be connecting through this technique.

Now the characteristic equation from this closed-loop transfer function we can write

$$1 + kG(s) = 0 \ldots\ldots Eq(1)$$

so this is basically if you solve this equation we will get the poles of the closed-loop system, roots are the poles of closed-loop system. Now if you assume or if you write G(s) can be one transfer function where we have numerator polynomial and denominator polynomial then equation one can be written as

$$1 + k\frac{N(s)}{D(s)} = 0, \;\; k > 0$$

From this we can further write

$$D(s) + kN(s) = 0 \ldots\ldots Eq(2)$$

Let's assume n and m be the order of D(s) and N(s) respectively. If you notice this equation number two very carefully if k tends to zero we can say D(s) goes to zero. So it means very very important concept which will be used in root locus, poles of G(s) if we assume n poles that because the order of the polynomial is n, n poles and we are assuming there are n branches and if k so n branches means the how the each pole on the root locus going to vary with k and if k tends to infinity in this equation we can write N(s) equal to zero so it means we can write zeros of G(s) where is basically m zeros so if you notice very carefully if you consider k tends to zero or k equal to zero the root locus starting from poles where we can solve this equation basic because from this equation number two if k equal to zero D is equal to zero so we'll get the poles of this equation of this polynomial so as we increase k it is goes to zeros because from this equation we'll get the zeros we'll start from poles and we'll end at zeros so let me write each branches starts at pole of G(s) and approaches to zero of G(s) as k tends to infinity. Now we'll extend this concept for our attitude control system of the aircraft then you can get the better idea how it is happening so let's consider our aircraft PD controlled attitude system attitude system.

So if you remember we had the transfer function

$$\frac{Y(s)}{R(s)} = \frac{k_p/I}{s^2 + \frac{k_d}{I}s + \frac{k_p}{I}}$$

So this is the expression we had and the characteristic polynomial from this transfer function we can write

$$s^2 + \frac{k_d}{I}s + \frac{k_p}{I} = 0 \ldots\ldots Eq(3)$$

Assume $I = k_d = 1$ to one to make our things simplify based on this assumption.

$$s^2 + s + k_p = 0$$

So in this equation we will check with the change of k_p how the poles going to vary so if you find the roots of this equation we can write

$$s_{1,2} = -\frac{1}{2} \pm \frac{\sqrt{1 - 4k_p}}{2}$$

So when $0 \leq k_p \leq \frac{1}{4}$ the closed loop poles are real and lie between -1 and 0. So if you take k_p one by four then the there will be two poles which will vary between 0 and -1 now if you draw these poles in our s plane so there will be two poles one is at zero s equal to zero and another pole is s equal to -1

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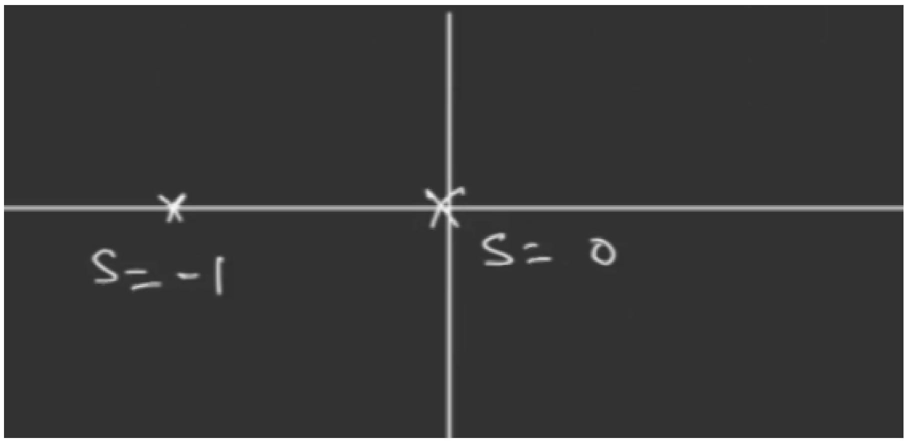

When $k_p = 1/4$ there are two repeated poles at s equal to minus half and in this case if you notice carefully if you plot them at the same point there will be two repeated poles.

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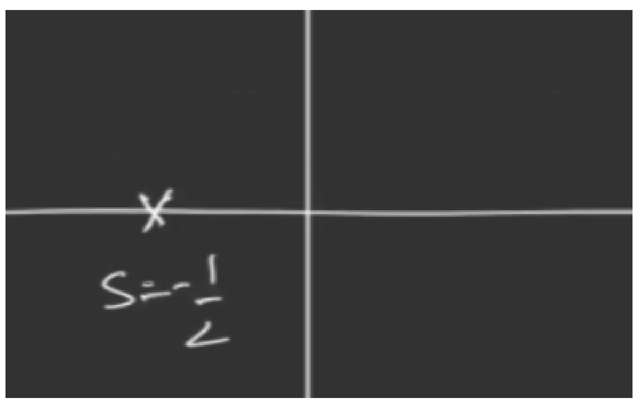

So let's assume this is minus one so we can write here there will be two repeated poles s equal to minus half and so if you notice carefully with the change of k_p the poles are shifting on s plan so now if you assume k_p is greater than one by four the roots are imaginary so how so k_p is greater than one by four we have two poles which are

$$s_{1,2} = -\frac{1}{2} \pm j\sqrt{4k_p - 1/2}$$

And in this case a real part is minus half so let me draw the in s plane how these poles are evolves

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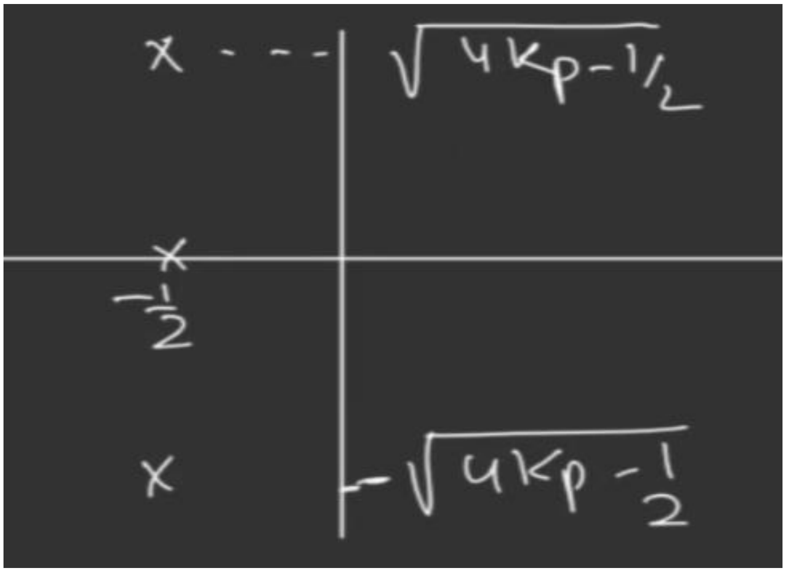

If you combine all of the above locus we can draw in s plane

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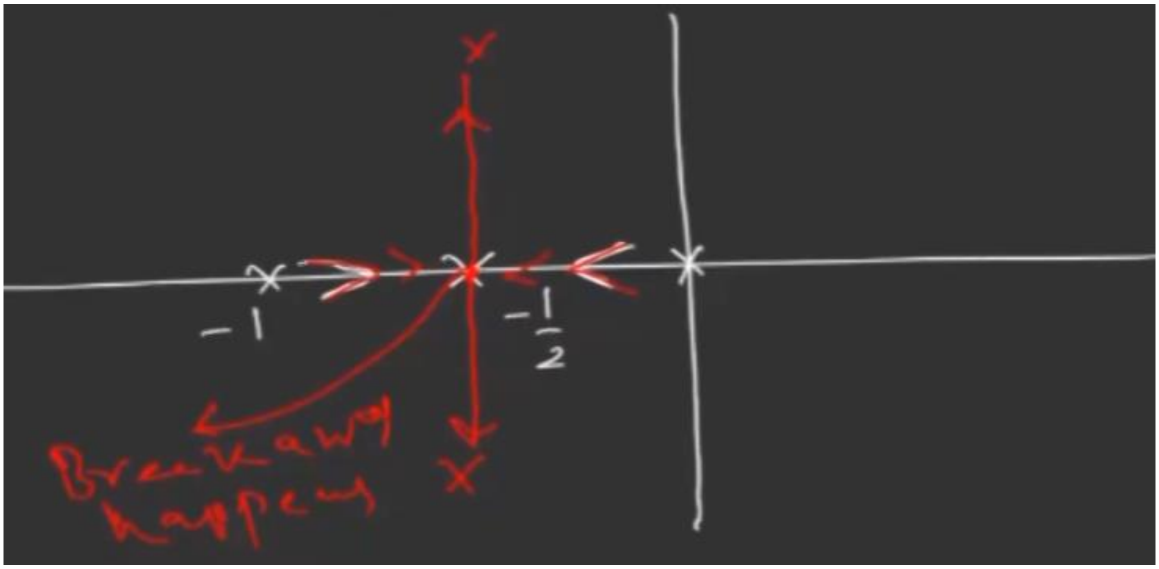

so initially our with the condition there are two poles which are zero and minus one so there are two poles one is zero one is minus one and if you take the another condition where the pole locus come to here which is minus two so it means the locus is coming like this and at minus half it become imaginary so we can say and here it become complex so we can say at this point the breakaway happens and the poles goes like this

this is how we can draw the locus of s this is very important concept from this so with the change of gain the locus is changing it's going another direction and this point actually breakaway happens.

So we'll be discussing in detail how we can find this breakaway points and where these poles are going so this is how the locus being created with the change of controller gain so from this discussion let's have following points

1. At $s = -1/2$ the root locus or loci on the real axis meet and then split the complex split to the complex plane that what I have noticed here in this in the above this plane and this the point at which the split happens is called breakaway point
2. The root locus shows how the closed poles change in the s plane with the changing parameter, here parameter is k
3. From the change in parameter, we can come up with the controller design concept

This part is very very important so this technique will be using very frequently for the autopilot design of the different motions of the aircraft.

Now we'll be coming up some rules how we can draw the root locus if you follow that those steps we can easily come up with this root locus this is very very important concept in control system so let me write down these rules for constructing the locus

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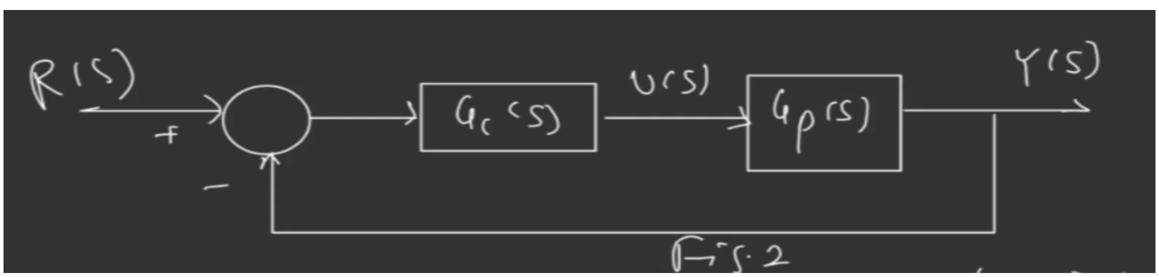

First let's go with the closed loop system and let me write the open loop transfer function from the closed loop system we have summing point here this figure is very very important we are frequently using this figure because this is the closed loop control system and how the autopilot thing developed based on this closed loop control system concept

$$C.L.T.F: T(s) = \frac{Y(s)}{R(s)} = \frac{G_p(s)G_c(s)}{1 + G_p(s)G_c(s)}$$

$$C.E.: 1 + G_p(s)G_c(s) = 0$$

$$G_0(s) = G_p(s)G_c(s)$$

$$= k\,\frac{(s - z_1)(s - z_2)\ldots(s - z_n)}{(s - p_1)(s - p_2)\ldots(s - p_n)}$$

So if you notice here we are going to consider this open loop transfer function and we are going to study the stability of the closed loop system that is the beautiful thing in root locus we will be considering the open loop transfer function and we will see the behavior of the system in the closed loop sense. The list of rules will be discussed from the next lecture and we'll also coming up some example how we can come up with the root locus plot based on those rules. Thank you.