Introduction to Aircraft Control System

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Week – 05

Lecture – 24

Stability of Closed Loop Control System

This is one of the most important lecture in this course. Here we are going to provide the stability of the closed loop control system without solving the characteristic equation of the closed loop transfer function. Here we are going to form a table by inspecting the first column of the table we can comment on the stability of the system and also we can comment how many poles are there in the left hand side or right hand side. Here also we are going to find the range of controller gain for which the system will be stable and also we will find the ultimate gain for which the system will be marginally stable and then we will conclude the lecture with some examples.

In this lecture we will be discussing the stability criterion. Here we will be discussing how we can come up with some conclusion on the range of gains should be considered to maintain the stability of the system.

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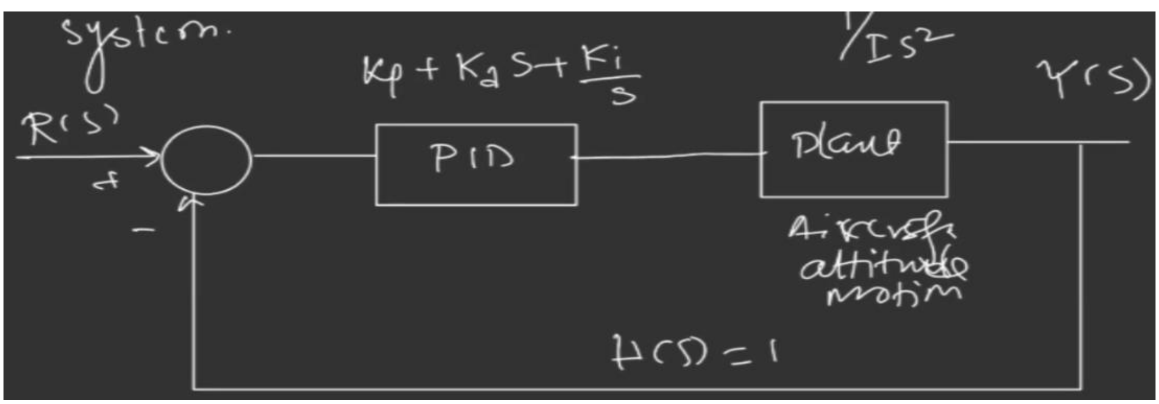

If you consider PID controlled aircraft attitude control system, suppose we have a step command R(s) and here in the control system in the control block let's say we are having PID controller and we have plant here which is aircraft attitude motion dynamics and we are having output. This is the feedback and in plant we had $\frac{1}{Is^2}$ the plant transfer function and in the transfer function for PID control we know $k_p + k_d s + \frac{k_i}{s}$. Now if you want to find the closed loop transfer function CLTF we are going to have

$$\frac{Y(s)}{R(s)} = \frac{G_p(s)}{1 + G_p(s)G_c(s)}$$

$$= \frac{k_d s^2 + k_p s + k_i}{Is^3 + k_d s^2 + k_p s + k_i} \ldots \ldots Eq(1)$$

Where $Is^3 + k_d s^2 + k_p s + k_i$ is the characteristic equation and if you notice here we are having three unknowns namely k_p, k_d, k_i but the previous methods what you have done it is difficult if you have if you are going to apply that method here to find these gains. So in this lecture we'll come up with some conclusion how we can come up with some range on these gains and based on that we can come up with the stability criteria. System will be stable if you have this kind these values for these gains. So let's consider most of the linear system, first let's go with the concept linear system. The closed loop whose transfer function has the following form

$$\frac{Y(s)}{R(s)} = \frac{a_0 s^m + a_1 s^{m-1} + \cdots + a_n}{b_0 s^n + b_1 s^{n-1} + \cdots + b_n} \ldots \ldots Eq(2)$$

Here generally we define the stability of the system based on the characteristic equation or the poles of the closed loop transfer function.

Now based on b_0, b_1, b_n the values of these will define the stability of the system, so the poles obtained would be specific to the coefficients b_i. Now based on this let us come up two important points we'll come up with some kind of table and based on that table or we can say array and that array will tell us the stability of the system and the way we follow the process we follow that array actually called Routh stability criterion methods from that array we can come up with some kind of comments that system is stable or not so the way we'll be solving in this method in this approach is called Routh stability criterion. So in this part there are two important points, one is the Routh stability criterion tells us whether or not there are unstable roots of characteristic equation exists in the positive real axis without solving for them. So from this stability criterion we can come up with the conclusion whether the system's poles are in the right hand side or not so as you know if any poles comes in the right half of S plane then we generally call the system is unstable and if they are in left half of S plane the system is stable.

So from the Routh stability criteria we can say any poles exist on the right hand side or not and also we can come up with the gains in the system if based on the gains we can comment for these gains the system will be stable and unstable we can come up with some conclusion. So let's go to the another point ie. the range of range of $b_i's$ also can be guaranteed for stability. What does it mean because if you compare both equation number one and equation number two, so here b's these are what, this is you can say the controller gain may exist so based on these gains also we can comment on the stability for the range of this coefficient. So let's go with the condition the coefficients $b_1, b_2 \ldots b_n$ must be positive if any of them are negative or zero a pole exists either on the imaginary axis or has a positive real part for the stability all the values of coefficient should be positive. If any of these values are negative or zero then it is quite confident that system is going to the unstable. Now another condition is if $b_i >$we can form a routh array. So let's look how the Routh it looks like

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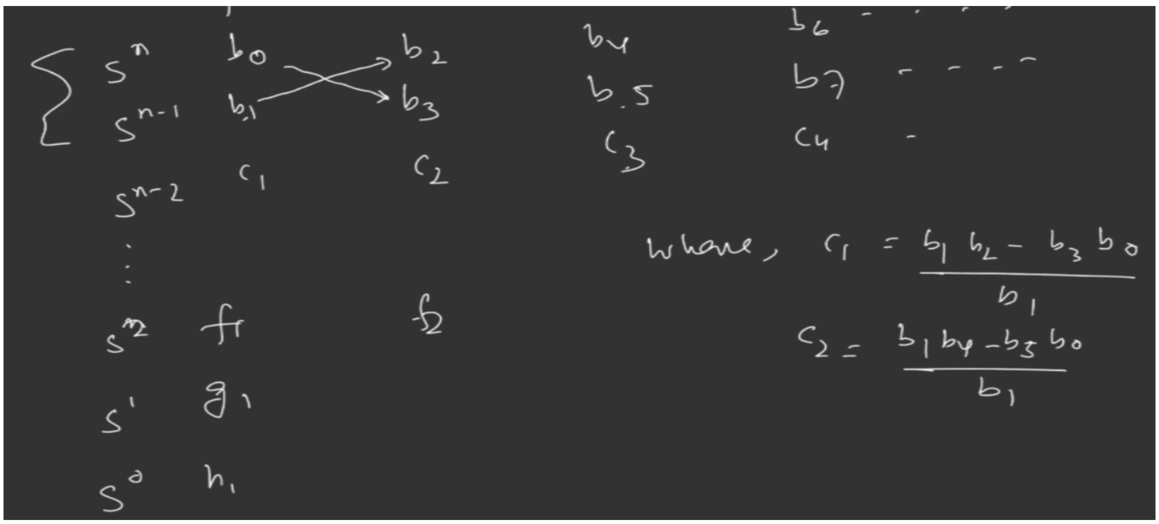

$$c_3 = \frac{b_1 b_n - b_7 b_0}{b_1}$$

So we'll start with if it is n is odd we'll take the coefficient of odd s's if n is even if the for example in this case if n is even then we'll take the even value of s's coefficient. We will come will come up the example then it will be clear to you and based on these two rows we can form the rest of the rows.

This process we will continue until the nth row has been completed, once the Routh array has been constructed asymptotic stability can be determined by first column of the coefficient. So what does it mean so after forming this Routh array we'll check the first column the terms in the first column and if all the terms are positive then we can say the system is asymptotically stable if any of the terms comes out to be negative then it will be ensured that the system will be unstable that there are poles existing on the right hand side.

So let me write the closed loop transfer function is asymptotically stable if and only if all the coefficient in the first column are positive. Let's take an example then this problem will be clear. Let's assume we have a characteristic equation

$$s^4 + 2s^3 + 3s^2 + 4s + 5 = 0$$

This is the characteristic equation we are getting closed loop transfer function and we can form the Routh array so there is highest power of s is four so we'll start with the even then we'll start with odd

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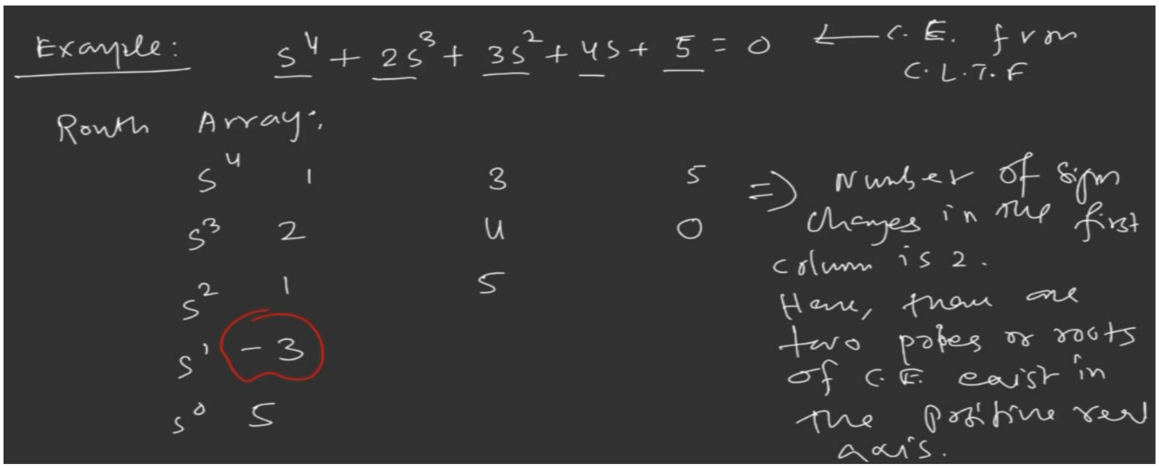

If you notice there are two sign change in the first column we can say number of poles exist on the right hand side number of sign changes in the first column is two hence there are two poles or roots of characteristic equation exist in the positive real axis so this is how we can comment on the stability so it means the system is unstable two roots are there in the positive half. Let's take another example where we'll find the range of gain in the system so that system will be stable. Let's consider the following example

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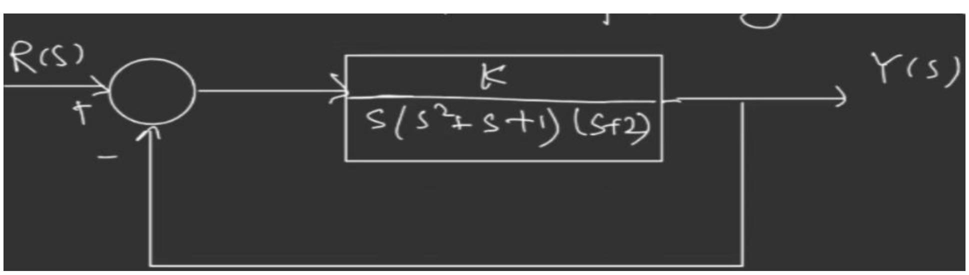

So here the question is determine the range of k for which the system stability is ensured so it means first we have to find the first Routh array and we will take the first column and based on which we will find the value of k for which the first column will be always positive all the terms in the first column, so the closed loop transfer function we can come up as

$$\frac{Y(s)}{R(s)} = \frac{k}{s^4 + 3s^3 + 3s^2 + 2s + k}$$

The characteristics equation is $s^4 + 3s^3 + 3s^2 + 2s + k = 0$

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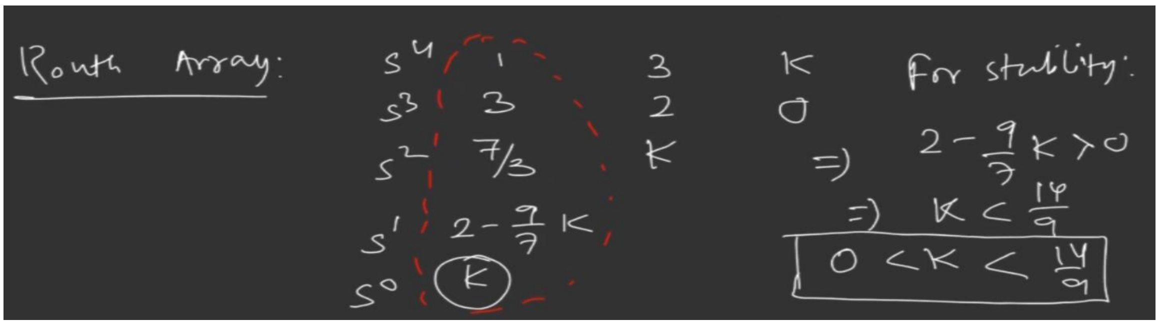

For stability as you know the first column should be all the terms in the first column should be positive. So this is the range of k for which system will be stable so this is kind of the gain tuning how we can come up with some gain value for who is the overall closed loop system will be stable so this is very I mean simple way to find the gain range for which the system will be stable. If you choose the controller gain in this range the system will be stable. So let's go with another example this is a little complicated example let's work on it consider the following characteristic equation of a closed loop system.

$$s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0$$

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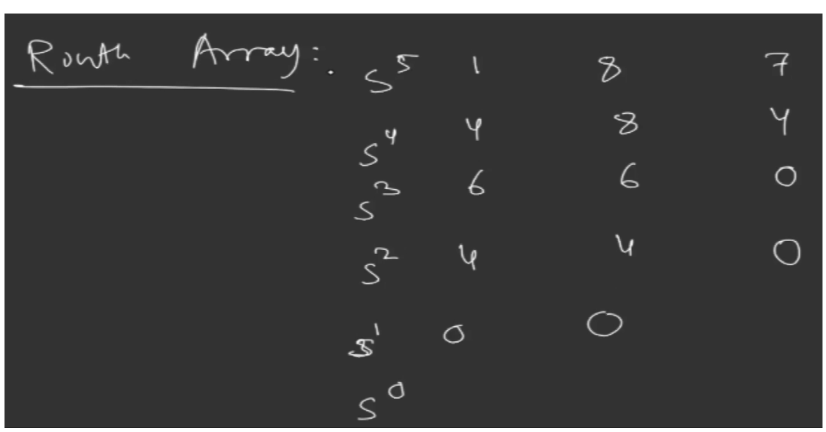

so this is one of the critical case if 0 0 comes into the in any of the rows so what will happen. so let's work on this let me write it because of row of zeros appears prematurely we form a auxiliary equation using coefficient of s^2 row. This is very very important if any of the row comes up to be zero then we will choose the previous row and we'll form a auxiliary equation from the previous row so here in this row we are having zero so we'll choose this row as a auxiliary equation so let's write so auxiliary equation is we can write

$$A(s) = 4s^2 + 4 \dots \dots Eq(3)$$

The derivative of A(s) with respect to s yields

$$\frac{dA(s)}{ds} = 8s \dots \dots Eq(4)$$

So now from which we can come up with the coefficient basically 8 and 0. What we'll do is this equation coefficient will fill with s^1 row. So what we'll do is 8 and 0 will replace zeros in s^1 row of the original array.

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Solving Eq(3) and Eq(4) we get two roots which are basically at $s = j$ and $s = -j$ s which are also two of the roots of characteristic equation which is very very important, so if you solve this equation we are having two roots which are complex conjugate and these root also the root of this our main characteristic equation. So this we can write hence the characteristic equation has two roots on the $j\omega$ axis and we can solve the marginal value of system parameter for system stability.

So it means if there is some unknown a coefficient exist in the system and we can come up with the condition of that unknown coefficient the system will be marginally stable or fully stable or unstable so this is the way we can also find the gain of the system because in the previous example if you notice if for example k if for example k equal to 14 by 9 if you choose this case then there will be there will be zero in the row will come and due to that the system will be marginally stable so that's why we can find the range of k on which the system will be stable so this is very very important method to find the values of controller gain for stability what are the range of the controller gains for that range we can comment that system will be stable outside the range the system are unstable so this is how we can check the stability of the system of a closed loop system transfer function so let's stop it here we'll continue from the next lecture on new topic. Thank you.