Introduction to Aircraft Control System

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Week – 05

Lecture – 23

Time Domain Specification with Examples

Hello everyone, in this lecture we will be solving some problems, practical problems for aircraft. We will be starting with a servo system which is very essential tool for an aircraft to achieve and maintain a desired output typically with the purpose of controlling the position, speed and other mechanical systems. A servo system is a control system that uses feedback to achieve and maintain a desired output typically with the purpose of controlling the position, speed and other mechanical system. There are some common uses of servo system, flight controlled surfaces. Servo systems are employed to control the movements of aileron, elevators, radars. These control surfaces are crucial for the aircraft roll, pitch and yaw motions and servo actuators help in precisely adjusting their positions based on pilot input and flight conditions. And autopilot systems, so under this heading hold and attitude hold. A servo actuators are integrated into autopilot systems to maintain a specific heading or altitude. The autopilot receives inputs from the various sensors and adjust the control surfaces through servo mechanisms to keep the aircraft on a speed defined course or at a specific altitude.

Few other uses of servo system are throttle control, flaps and slats, landing gear, stability augmentation system, brake system. So these are the areas we use very often in the servo system, very useful mechanical device which gives the desired controlling output. Now we will be solving one problem on the servo system. First let me design the control system block, how servo control system is generally employed for aircraft applications.

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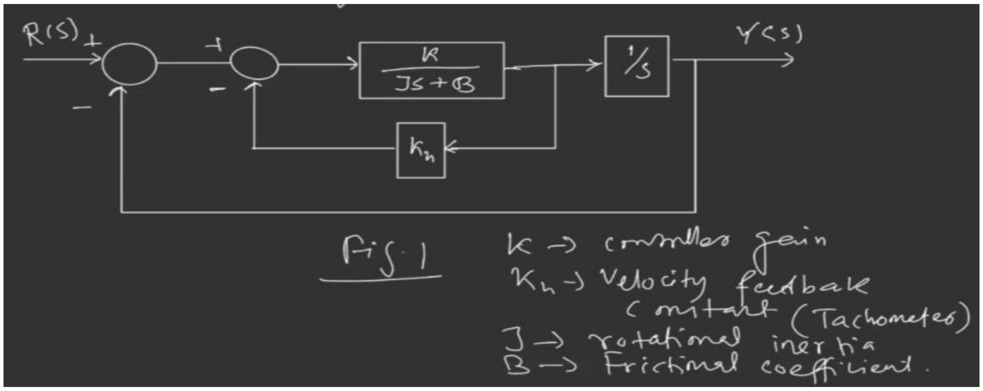

Suppose we have step command R(s) to the servo system and we have another summing point here. Let me draw the figure first then I will explain. The output is the position of the shaft and here we are having negative feedback. This is how the servo system are generally used for application purpose. Here is the controller gain, k_h is the velocity feedback constant. This is basically tachometer which is used for this purpose for measuring the speed of the shaft and J is rotational inertia and B is frictional coefficient. Now from the figure 1, let's go the final transfer function step by step. First we will find the transfer function for this block, for this loop. The inner loop transfer function we can have

$$\frac{G_p(s)G_c(s)}{1+G_p(s)G_c(s)H(s)} = \frac{k}{Js+B+kh}$$

And if you now redraw the block diagram again we can come up summing point.

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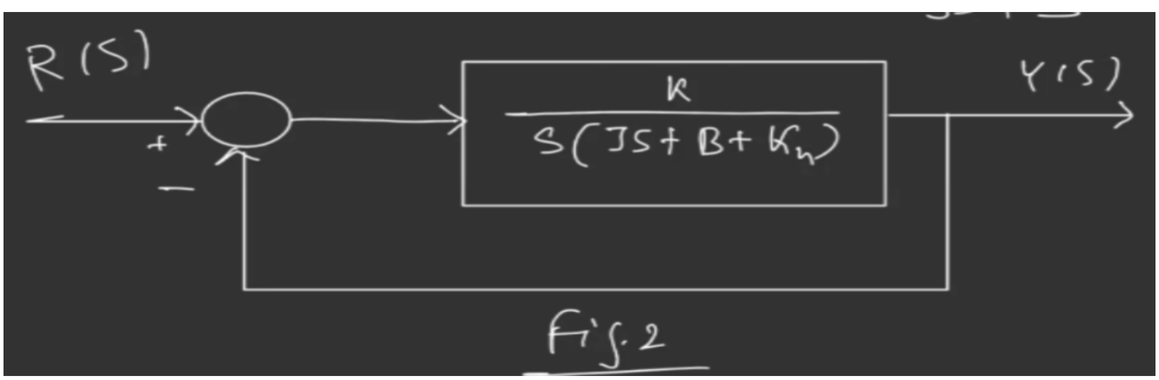

Now we can find the overall closed loop transfer function for this block

$$\frac{Y(s)}{R(s)} = \frac{\frac{k}{s(Js + B + kk_h)}}{1 + \frac{k}{s(Js + B + kk_h)}}$$

$$= \frac{\frac{k}{J}}{s^2 + \left(\frac{B + kk_h}{J}\right) s + \frac{k}{J}} \ldots \ldots Eq(1)$$

So this is the second order closed loop transfer function. Now if you compare this equation, with the standard second order transfer function which is $\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$ we can find the natural frequency and damping ratio with the following relations.

$$\xi = \frac{N + kk_h}{2\sqrt{kJ}}$$

$$\omega_n = \sqrt{\frac{k}{J}}$$

Now the question is determine the values of k and k_h so that the maximum overshoot, in the unit step response is 0.2 and peak time, t_p is one second. With these values, we obtain the rise time, time t_r and settling time t_s. So here you can assume J equal to 1 kg meter squared inertia and B is one Newton meter radian per second. So these are the values given, so we need to find first k and k_h, then we have to find rise time and settling time.

The maximum overshoot is given by,

$$M_p = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} = 0.2$$

$$\xi = 0.456$$

So this is under damped system, because the damping ratio if it is varies from 0 to 1 then it is under damped system. Now we'll find the ω_n, the peak time is given, the peak time t_p is one second. So we can write

$$t_p = \frac{\pi}{\omega_d}$$

$$\omega_d = 3.14$$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\xi^2}} = 3.59$$

Now we'll find natural frequency as we have done. This is the natural frequency we have. Now we can find the value other parameters easily.

$$\omega_n = \sqrt{\frac{k}{J}}$$

$$k = 12.5\ N - m$$

$$k_h = \frac{2\sqrt{kJ}\xi - \beta}{k} = 0.178\ s$$

$$\beta = \tan^{-1}\frac{\omega_n\sqrt{1-\xi^2}}{\xi\omega_n} = 1.10$$

$$t_r = \frac{\pi - \beta}{\omega_d} = 0.65\ s$$

$$t_s = \frac{4.4}{\xi\omega_n} = 2.68\ s$$

If you have servo system we can solve the different parameters how the system evolves and if you see graphically from below figure.

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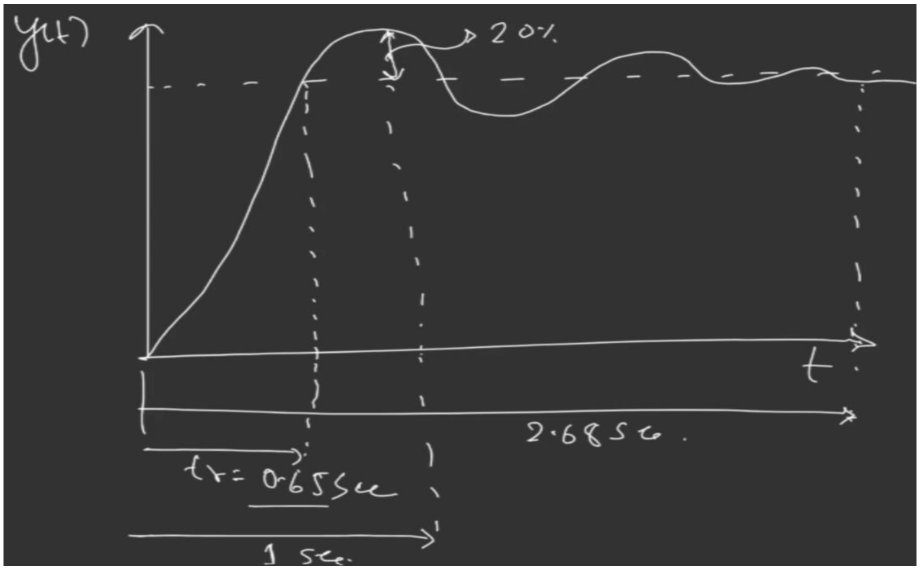

Now let's have another example. Consider the following block diagram for electric motor position control problem

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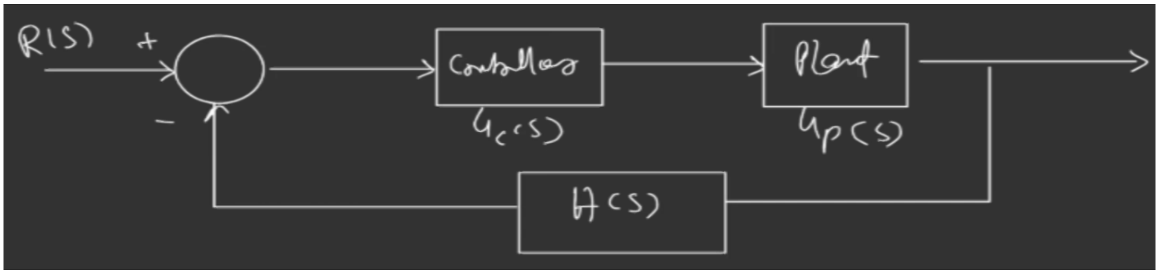

Where $G_p(s) = \frac{1}{s(s\tau+1)}$

$$G_c(s) = k_p$$

$$H(s) = 1 + k_t s$$

Determine the system type and relevant error constant with respect to reference input step and ramp. So in this question we'll be finding the system type and relevant error constants the steady state error basically. Let's start with the problem the open loop transfer function we can write

$$G_0(s) = \frac{k_p(1 + k_t s)}{s(s\tau + 1)}$$

This is basically type one system because in the open loop transfer function we have one pole at origin. Now we'll find the steady state error

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{s}{1 + G_0(s)} \qquad \left[\because E(s) = \frac{R(s)}{1 + G_0(s)}\right]$$

For a step input, R(s)=1/s

$$e_{ss} = \frac{1}{1 + \lim_{s\to 0} \frac{k_p(1 + k_t s)}{s(s\tau + 1)}} = 0$$

For a ramp input, $R(s) = 1/s^2$

$$e_{ss} = \frac{1}{\lim_{s\to 0} \frac{k_p(1 + k_t s)}{(s\tau + 1)}} = \frac{1}{k_p}$$

So this is the non-zero steady state error and this is how we can solve the problems if you are having given system how we can design control algorithms so that we can come up with our desired time domain specification and also the steady state specification. So let's conclude here in this lecture for this time domain specifications. From next lecture we will be going to term stability analysis technique and how we can find the gains or controller gains through some other technique. Thank you very much.