Introduction to Aircraft Control System

Prof. Dipak Kumar Giri

Department of Aerospace Engineering

IIT Kanpur

Week – 05

Lecture – 22

Effect of Disturbance on Steady State Behaviour

In this lecture, we'll be studying how we're going to modify the steady state behavior of the system in the presence of disturbance. Here first we're going to find the relationship between the error due to the disturbance and disturbance acting on the system. Then we'll have some relationship through which if you choose suitably the controller gain or if you add the integral part to the controller, how it is going to mitigate the external disturbance in the system. Then we'll have some example on aircraft control system and how we're going to mitigate the external disturbance if you choose suitably the controller part. Then we'll conclude the lecture.

In this lecture, we'll be discussing the effect of disturbance on the steady state error. If you remember the closed loop system corresponding to W(s)=0, we had

$$Y(s) = \frac{G_p(s)G_c(s)}{1 + G_p(s)G_c(s)}R(s) + \frac{G_p(s)M_d(s)}{1 + G_p(s)G_c(s)}$$

W(s) equal to 0 because we are not considering W(S) for the time being. This is the expression we had in the last lecture and also we had the error expression for the error.

$$E(s) = R(s) - Y(s)$$

$$= \frac{1}{1 + G_p(s)G_c(s)}R(s) - \frac{G_p(s)M_d(s)}{1 + G_p(s)G_c(s)}$$

In the last lecture, we have discussed this part, the effect of reference signal on the steady state error. Based on the type of the system, we have discussed how the steady state error gets affected. In today's lecture, we'll be discussing the second part, the effect of disturbance on the steady state error. For this, we will not consider the contribution of the reference signal. Here we say ignore the contribution of the reference signal R(s). Since we are dealing with a linear system, we can see the effect of individual reference signal or disturbance signal or noise on the system and we can combine them together once we are done for the individual effects. So that's why we are going with one by one. Now the controller, so our purpose here,

$$E(s) = -\frac{G_p(s)M_d(s)}{1 + G_p(s)G_c(s)} \dots \dots Eq(1)$$

Let's define the kind of plant and controller transfer function we'll be considering here. We can write the plant and controller transfer function in the following form.

$$G_p(s) = \frac{k_{pl}\left(T_{a1p}s + 1\right)\left(T_{a2p}s + 1\right) \dots (T_{amp}s + 1)}{s^{Np}\left(T_{b1p}s + 1\right)\left(T_{b2p}s + 1\right) \dots (T_{bnp}s + 1)}$$

Numerator part yields the number of zeros. The denominator gives the number of poles in the system. So from this transfer function, we can further write

$$G_p(s) = \frac{k_{pl}}{s^{Np}}\frac{N_p(s)}{D_p(s)} \dots \dots Eq(2)$$

So here, this is the gain and s^{Np}, this is basically gain in the plant transfer function. And this is the numerator polynomial which yields the number of zeros in the system. And $D_p(s)$ yields the number of poles in the system. So we can write in this structure.

$$G_c(s) = \frac{k_c(T_{a1c}s + 1)(T_{a2c}s + 1) \dots (T_{amc}s + 1)}{s^{Nc}(T_{b1c}s + 1)(T_{b2c}s + 1) \dots (T_{bnc}s + 1)}$$

Here c denotes the controller, here in the above p denotes the plant. And we can write this expression also in the following form, similar to the plant transfer function,

$$G_c(s) = \frac{k_c}{s^{Nc}}\frac{N_c(s)}{D_c(s)} \dots \dots Eq(3)$$

From equations (2) and (3), it can be shown that

$$k_{pl} = \lim_{s\to 0} s^{Np}G_p(s)$$

$$k_c = \lim_{s\to 0} s^{Nc}G_c(s)$$

We will substitute equation number two and three equation one, we can write

$$\frac{G_p(s)}{1 + G_p(s)G_c(s)} = \frac{k_{pl}S^{Nc}N_p(s)D_c(s)}{S^{Np+Nc}D_p(s)D_c(s) + k_{pl}k_cN_p(s)N_c(s)} \dots \dots Eq(4)$$

Why you are writing this sS^{Np}, S^{Nc} structure? Because if you remember in our earlier lecture we have discussed due to the presence of integral term in the PID controller, we can ignore the disturbance effect in the system. This is in this direction we are moving here. How we can come up with the integral term in the system and how it is going to affect our different steady state error due to the disturbance. Now we can write, so if you, we are having this structure and if you substitute this expression in the equation number one, we can come up with the error, the steady state error. Error, let me write, define this equation number four. So substitute equation number four in equation one we can get

$$e_{ss} = \lim_{s \to 0} sE(s)$$

This is we get from the final value theorem.

$$e_{ss} = -\lim_{s \to 0} \frac{k_{pl} S^{Nc} N_p(s) D_c(s) M_d(s)}{S^{Np+Nc} D_p(s) D_c(s) + k_{pl} k_c N_p(s) N_c(s)} \ldots \ldots Eq(5)$$

It is clear that to able to completely reject the constant disturbance ie. $e_{ss} = 0$ we must have $N_c \geq 1$.Why? Because if it is N_c here, if it is greater than one in this case then s will be multiplied to the entire function and if you put s equal to zero then e_{ss} goes to zero. So what is N_c? N_c actually if you look this is the open loop poles in the controller. So that's why to reject completely the disturbance effect on the steady state there must be must be an integrator (1/s) in the controller so to reject the effect of disturbance on the steady state error. This is very important takeaway from this discussion.

Let us consider if there is no integrator in the controller for example, for example pd controller, the structure is

$$u(t) = k_p e + k_d \dot{e}$$

$$U(s) = k_p E(s) + k_d s E(s)$$

And for PID

$$PID = k_p e + k_d \dot{e} + k_i \int_0^t e(\tau) d\tau$$

Let's assume there is no integrator in the controller it means N_c equal to zero. if N_c equal to zero then

$$N_p(0) = N_c(0) = D_p(0) = D_c(0) = 1$$

In this situation, if you assume there is no open loop poles on the plant transfer function then we can write

$$e_{ss} = \frac{-k_{pl}M_d}{1 + k_{pl}k_c}$$

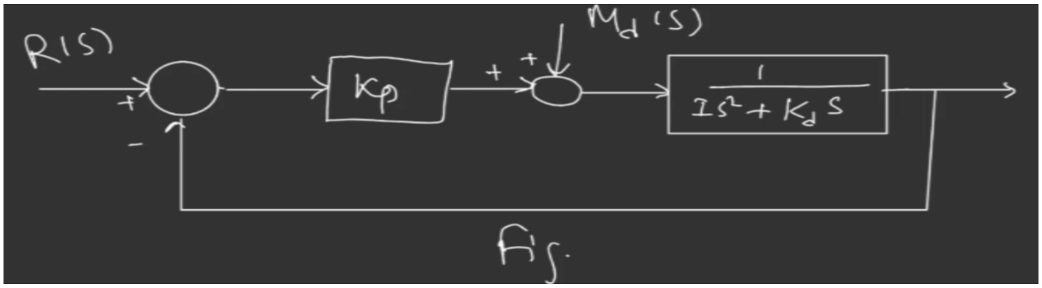
This is the case when $N_c = 0$ and $N_p = 0$. Again if there is open loop pole in the plant transfer function one which is $N_p \geq 1$ then we can write

$$e_{ss} = -\frac{M_d}{k_c}$$

This is how we can say how the integrator plays an important role in control system. If you add integrator in the controller we can reject the disturbance effect in the system so we have two both two cases in the control transfer function if there is no integrator exists in the system then how the steady state error would be. So this is the case if you notice very carefully in this equation so here we have assumed there is no controller, there is no open loop poles in the control system controller block but due to the presence of the open loop poles in the plant transfer function still we can manage the steady state error how if you increase the k_c gain here if M_d is a constant disturbance in the system. If we increase k_c gain considerably high we can manage steady state error this is also possible so let me write here one note, if there is no integrator in the controller which is $N_c = 0$ then we must have to increase the controller gain to reduce the steady state error. This is very important takeaway from this part now.

We'll take an example how our system going to behave. So we'll take the aircraft control system what you have done before. Effect of disturbance on PD (means proportional derivative) aircraft attitude or orientation control system let's have the following block diagram

(Refer Slide Time 27:09)



We are having summing point here and we have reference input R(s) and we'll look the modified pd control attitude system so it is k_p is here and we are having disturbance on the system $M_d(s)$ which is we can say constant. We'll assume the constant disturbance acting on the system and we have the modified plant transfer function

$$\frac{1}{Is^2 + k_d s}$$

So this is a closed loop control system and from this let me define this figure one and we have to find the effect of disturbance $M_d(s)$ on the steady state error in the system.

From figure one we can write the plant transfer function

$$G_p(s) = \frac{1}{k_d s\left(\frac{1}{k_d}s + 1\right)}$$

and the controller transfer function we can write

$$G_c(s) = k_p$$

It is clear that there is no integrator in the controller because here $N_c = 0$. However in $G_p(s)$ we have one integrator $N_p = 1$. Therefore the steady state error can be obtained and obtained as

$$e_{ss} = -\frac{M_d}{k_c}$$

This is very very important so this is what exactly we have done in the previous example. There will be steady state error

So now as a control engineer if you are very curious to know how much k_c should be increased so that steady state error can be minimized and for example if you have some value and if you want to maintain some steady state in the system for example if you are given a specification on the steady state error such that $|e_{ss}| \leq e_{max}$. As a autopilot designer you want to maintain this steady state error and you don't want to have more than this maximum value. So from this condition the proportional gain k_p must be chosen such that

$$k_p \geq \frac{M_d}{e_{max,ss}}$$

So this is how we can come up with the condition how the k_p or the controller gain should be chosen so that it can fulfill the objective on steady state error This is very very important how we can achieve your mission objective if you want to maintain a desired attitude of the aircraft or position whatever so and if you want to maintain some steady state error in the final stage how what are the condition we have to fulfill what are the condition we have to follow so this is how we can improve the performance of the controller let's stop it here we'll come to the next lecture we'll have some examples so far whatever you have done on this whole topic we'll have some example and we'll see how we can design the controller. Thank you.