# Introduction to Aircraft Control System


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**Week – 04**

**Lecture – 19**

## Transient Response Specifications (Cont.)

Hello, everyone. This is lecture number 19. In this lecture, we'll be discussing on some example of control system, how we can design a control system for our aircraft attitude system, and also we'll be looking how the system changes with the location of the poles in this plane. Before we continue, I'd like to refer some specifications of what you have done in the earlier lecture. So the specifications we derived from the second-order system are

$$t_r = \frac{\pi - \beta}{\omega_d}$$

$$\beta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

$$t_s \approx \frac{4.4}{\xi\omega_n}$$

$$t_p = \frac{\pi}{\omega_d}$$

$$M_p = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} * 100\%$$

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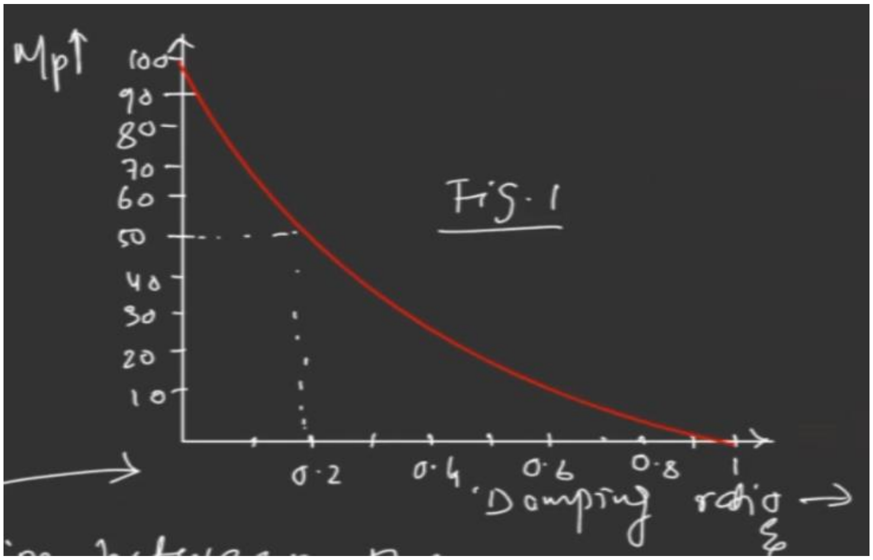


So, these are the specifications we obtained from the analysis what you have done in the last lecture, and if you want to design a control system, we're going to use this specification, how we can use the control and how we can modify this specification, and accordingly we can come up with the closed-loop control system, because these specifications are being derived from the standard closed-loop second-order system. But sometimes some difficulty arises while designing the control system, so there are simple ways we can come up with the relation between the maximum overshoot and damping ratio.

So, this plot indicates the relation between the damping ratio and maximum overshoot, and this relation basically for the underdamped system, because if you see the x-axis, we are having the damping ratio $\xi$, and the y-axis indicates the maximum overshoot, and damping ratio varies from 0.1 to 0.1, and maximum overshoot varies from 10 percent to 100 percent. Now, if you are given, suppose, maximum overshoot in the control system, we would like to have maximum overshoot, for example, 50 percent. So, at 50 percent, we can come up with the relation that roughly you can say the damping ratio will be 0.2.

So, if one of the parameters is given to us, we can get the other parameter. So, sometimes it is very easy to come up with the conclusion of the control system instead of going through the mathematical derivations. Now, look at an example how we can design a control system practically.

Example: Design a PD controller for aircraft attitude/orientation control. Assume the inertia is $I = 1\ Kg - m^2$. Now design the control system (closed loop control system) which satisfy the following time domain specifications:

1. Rise time constraint: $t_r \leq 30sec$
2. Maximum overshoot constraint: $M_p \leq 30\%$

3. Settling time constraint: : $t_s \leq 100sec$

So, these are the specifications given to us and we should design an autopilot which will satisfy these objectives. If you are going to use this formula, we can get the value of $\xi$ and $\omega_n$. But sometimes it is a tedious process to solve this expression. So, we'll use this figure one, and based on it, we can get some idea on the damping ratio because we are given the maximum overshoot in the problem and which should be less than 30 percent.

So, at 30 percent, we can get some idea of damping ratio. So, let's work on the problem. From the relation in figure one, we can get for 30 percent overshoot, the damping ratio yields to be 0.4. Now, we are having damping ratio 0.4 and we can find the natural frequency if you use the relation for settling time. So, we know the settling time, the relation between $t_s$ and damping ratio and natural frequency, we can write

$$t_s \approx \frac{4.4}{\xi \omega_n}$$

$$\omega_n = 0.11\, rad/s$$

And now, we can go with the PD gains, since we are asked to design PD controller here. So, as you know, the relation between the damping ratio and natural frequency and PD controller gains,

$$k_p = \omega_n^2 I = 0.0121$$

So, for these conditions, we are getting the proportional gain as 0.0121. Now, we will find the derivative gain, the relation between the damping ratio and natural frequency as you have done for analysis.

$$k_d = 2\xi \omega_n I = 0.08$$

So, now we are having the controller gains which satisfied the above specifications. Now, the controller for the above system, we can write

$$u(t) = k_p e - k_d \dot{y}(t)$$

$$= 0.0121e(t) - 0.088\dot{y}(t)$$

If we take Laplace transform,

$$U(s) = 0.0121E(s) - 0.088sY(s)$$

And if you draw the closed loop attitude control system of the aircraft, we are giving the step command, which is R(s)=1/s, basically r(t)=1. So, here we have $k_p$, which is nothing but $G_c(s)$, and we are having a summing point here and this is our plant, plant is $\frac{1}{Is^2}$. This is our current attitude or y(t), or we can say Y(s) in Laplace transform.

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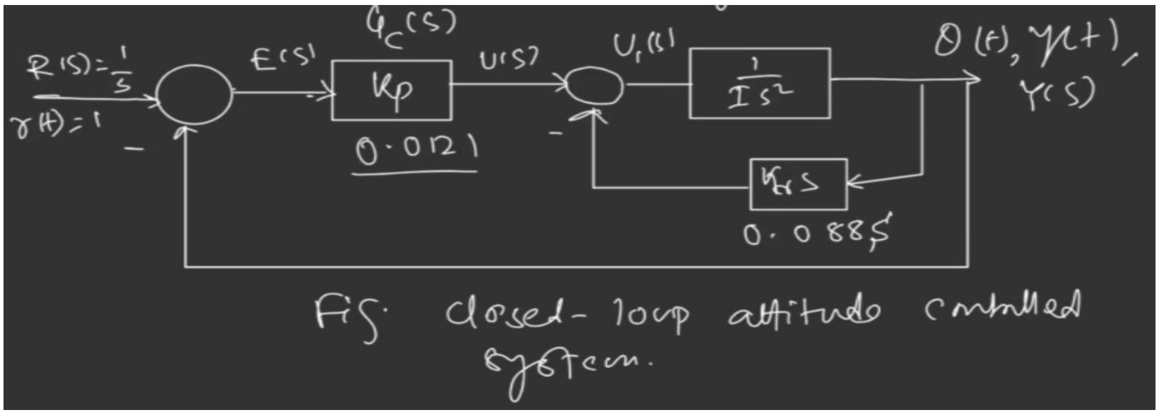


And in the inner loop, we are having, in the feedback, we are having $k_d s$, basically $k_d s$ is negative 0.088 s, and our $k_p$ gain is 0.0121, and this is E(s), and this is the outer loop of the control system, negative. So, if you look this expression, it is quite well validated in this block diagram. So, 0.121 into E(s), this is basically our U(s), we can write, this is another controller, may be $E_1(s)$, which is the sum value of the two different inputs. So, this is how we can design the closed loop control system. Now, let's extend this concept for any other system, any other dynamical system, because here we know the attitude dynamics, and also from the attitude dynamics, how we can come up with the closed loop control system, and how we can maintain the desired attitude, this is we have discussed so far, and based on the standard system, standard second-order system, how we can come up with the control gains, which can help us to fulfill our mission objective. Now, let's extend this concept for any other dynamical system. Suppose, sometimes what happens, sometimes system linear model is given and you are asked to design the control system.

So, to tackle this kind of problem, how we will handle, let's discuss on this. Suppose, we are given system

$$\ddot{y}(t) + y(t) = u(t)$$

You are asked to design U(t) such a way that this will help you to track y(t) to our desired step command, which is r(t)=1. So, this kind of problem we can easily tackle if you have full understanding on standard second-order system. So, standard second-order system we know

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Suppose, you are given some ideal response, ideal response means you are given some the values of $t_p, t_r, t_s, M_p$, for example. So, based on these given parameters, your ideal step response, you can come up. Suppose, this is my ideal step response, something like this,

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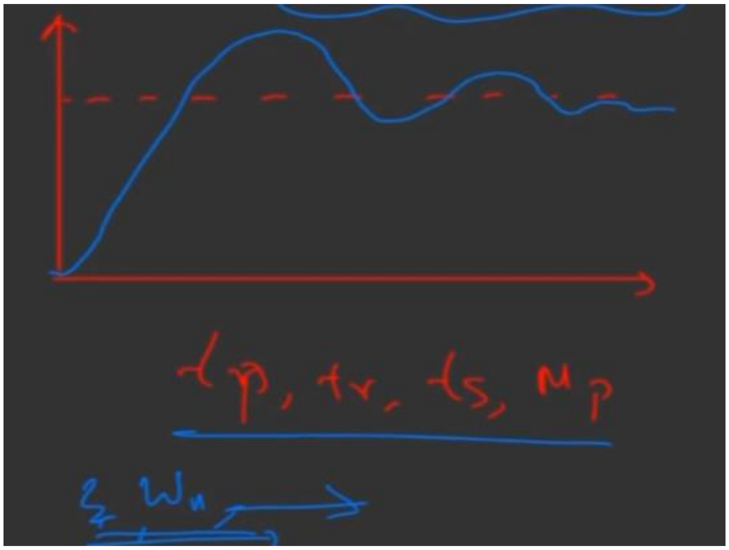


And from this given specification also you can come up with the damping ratio and natural frequency. So, based on these values, you can come up with this transfer function, what should be the transfer function? This is the ideal case, you can come up with the values. We'll look just for a moment how you can come up.

So, now this is, you are asked to design the control here. So, here in this example I'm taking for the motivation if you want to design control system for any other dynamic systems. How will you tackle? I'll proceed. So, now from this given system, this is my plant

$$\ddot{y}(t) + y(t) = u(t)$$

And from this plant we can come up with the plant transfer function. If you take the Laplace transform of this equation, we get

$$\frac{Y(s)}{U(s)} = \frac{1}{s^2 + 1} \rightarrow G_p(s)$$

And you are asked to design PD control. So, this is basically my $G_p(s)$ plant transfer function, which is fixed, which you can't change. The plant transfer function is difficult to modify, because this is coming naturally. And you are asked to design PD controller

$$u(t) = k_p e(t) + k_d \dot{e}$$

And from this, if you apply the Laplace transform for this equation, you can get

$$\frac{U(s)}{E(s)} = k_p + k_d s$$

And if you want to come up with the closed loop transfer function for this particular plant,

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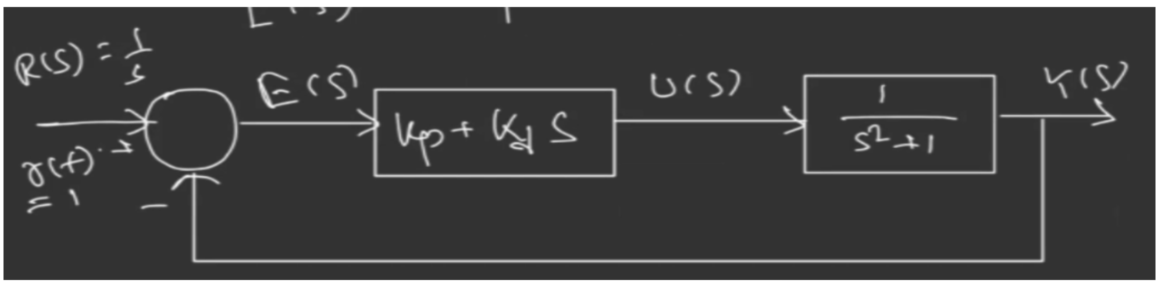


And from this closed loop control system, we can find the transfer function. If you notice carefully, here $k_p$ and $k_d$ are unknown. So we need to find them, this actually we are doing the same procedure what you have done in the previous example. So the closed loop transfer function, we can write

$$\frac{Y(s)}{R(s)} = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)}$$

$$= \frac{k_p + k_d s}{s^2 + k_d s + (1 + k_p)}$$

So this is our actual system, actual closed loop control system where the unknowns are $k_p$ and $k_d$. Now let's go back to here. So this is the standard second order system. Please don't forget this is very very important. Basically the peak time, rise time, settling time, maximum overshoot are defined on the standard second order system.

And for this given values, some numerical values, let's say this is the response we are getting. And for this given specification we can come up with $\xi$ and $\omega_n$. Let's say $\xi$ is 0.5 and $\omega_n$ also 0.5. First let's say, so from these values we can come up with

$$\frac{Y(s)}{R(s)} = \frac{0.25}{s^2 + 0.5s + 0.25}$$

So this is the second order system, some ideal response we'll have and this is the response we are having here. This is the response and our system should follow this response because this is our step response. So this system also should follow Y(s)/R(s), should follow this response. So what we'll do is we'll compare both the above equations,

$$k_d = 0.5$$

$$1 + k_p = 0.25$$

So this is how we can find the value of $k_p$ and $k_d$ which will help us to come up with the relation with the control system for this particular plant. And if you consider this $k_p$, $k_d$ in this controller you can get a desired response which will track this step function. So this is how we can design a control algorithm for any arbitrary systems and this is the procedure we follow. So we should have some standard system where you know all the system specifications and you can compare both the system and you can come up with the control parameters.

So this is one way we can design the controller. In this course we'll be talking different approaches how we can design the controller, this is one of them. So let's stop it here, we'll continue from the next lecture with the new topic there we'll be talking on how the system behavior will differ or change with the addition of poles and zeros to the system. Thank you.