Introduction to Aircraft Control System

Prof. Dipak Kumar Giri

Department of Aerospace Engineering

IIT Kanpur

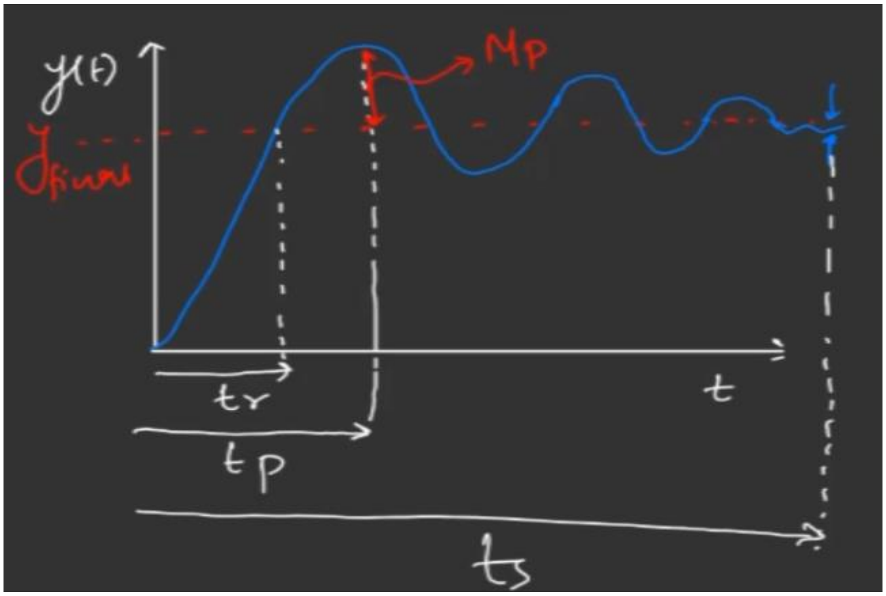
Week – 04

Lecture – 18

Transient Response Specifications

In this lecture, we'll be studying the transient response of PD-controlled system. In this direction, we'll be finding the expression of rise time, settling time, peak time, and maximum overshoot. And these parameters can be used to modify the transient response of the system. If you'd like to reach the desired values very fast with less magnitude in overshoot and undershoot, how you can use this parameter to modify the response? Then we'll conclude the lecture. In the last lecture, we discussed about the specification and how can we define this specification in the response plot. In this lecture, we'll be finding the expression of this specification in time domain. We'll start with finding the rise time. As for the definition, the rise time is the time taken to first reach the final value.

(Refer Slide Time 00:46)



So from the step response, let me define this the figure 1, from the step response in figure 1, we see that at $t = t_r$, the output is y(t) equal to 1. Hence, if you remember the equation we had,

$$y(t) = 1 - e^{-\xi\omega_n t}\left[\cos\omega_d t + \frac{\xi\omega_n}{\omega_d}\sin\omega_d t\right] \ldots \ldots Eq(1)$$

So this is the expression we found and if you substitute y(t) equal to 1, let me write the equation number 1, in equation number 1 we can write

$$\cos\omega_d t_r + \frac{\xi\omega_n}{\omega_d}\sin\omega_d t_r = 0$$

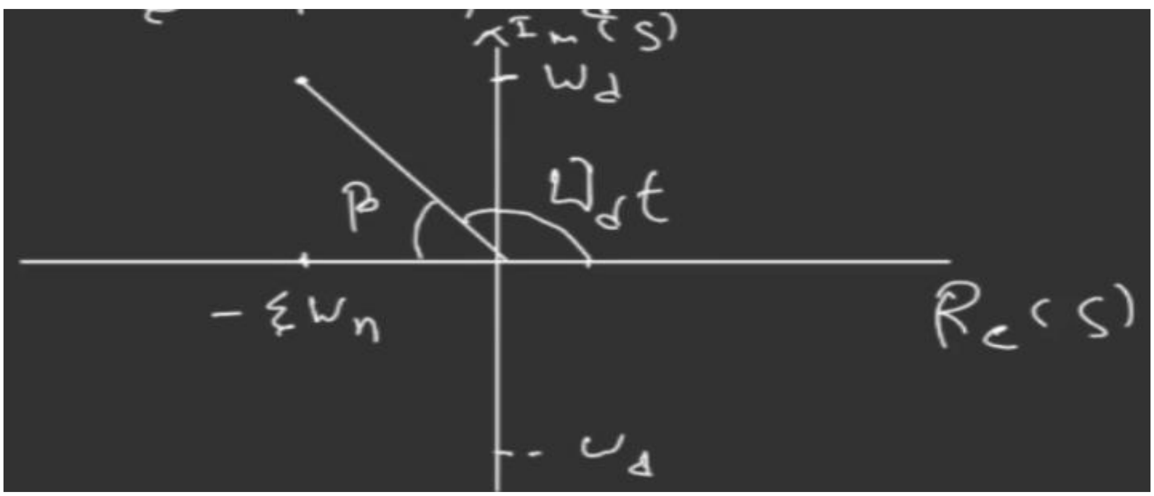
After rearranging, we can write

$$\tan\omega_d t_r = \frac{\omega_n\sqrt{1-\xi^2}}{-\omega_n\xi}$$

For the underdamped systems, as we have discussed before, for the underdamped system, the pole locations are as following.

$$s_{1,2} = -\xi\omega_n \pm j\omega_d$$

(Refer Slide Time 05:56)



From this plot, we can write

$$\omega_d t_r = \pi - \beta$$

$$t_r = \frac{\pi - \beta}{\omega_d}$$

$$\tan(\pi - \beta) = -\frac{\sqrt{1-\xi^2}}{\xi}$$

$$\beta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

So this beta can be used to find the t_r expression because this is the t_r expression. So here ω_d is known and β is unknown. So if you can find β from this expression, we can find t_r. So if you notice carefully here, this β is a function of ξ and this ξ also depends on the control parameters what you are defining. We have defined some expression with ξ, some expression k_p, k_d and ω_n. So using these control variables, we can modify the ξ and if we can modify ξ also β be modified and based on β, we can modify t_r. So this is how we can improve the rise time. So now let's go to our second specification, peak time. In peak time, the definition was the peak time occurs at the first time when $\frac{dy}{dt} = 0$. So of course this is from the plot at t_p here the value is constant.

So we can write at t equal to t_p we can write $\frac{dy}{dt} = 0$. So for this we will take the time derivative of the equation number one.

$$\frac{dy}{dt} = e^{-\xi\omega_n t}\sin\omega_d t\left[\frac{\xi^2\omega_n^2}{\omega_d} + \omega_d\right] \dots\dots Eq(2)$$

So we can write the peak time $t = t_p$ we must have $\sin\omega_d t_p = 0$. Hence $\omega_d t_p = 0, \pi, 2\pi, \dots$

From this clearly the first peak correspond to $\omega_d t_p = \pi$

$$t_p = \frac{\pi}{\omega_d}$$

So this is the expression for peak time so here again you can see that $\omega_d = \omega_n\sqrt{1-\xi^2}$ So here ω_n and ξ depends on control parameters.

So we can modify the ω_d and also you can modify the peak time now let's go find the maximum overshoot. The next specification maximum overshoot which is denoted by M_p ,the maximum overshoot occurs at the peak time $t = t_p$ that is obvious it is already we have done the maximum overshoot occurs $t = t_p$ so based on this definition we can say that the maximum overshoot occurs at the peak time $t = t_p$ with the maximum response.

$$y_p = 1 - e^{-\xi\omega_n t_p}\left[\cos\omega_d t_p + \frac{\xi\omega_n}{\omega_d}\sin\omega_d t_p\right]$$

We know that $\omega_d t_p = \pi$ such that $\cos\omega_d t_p = -1$ and $\sin\omega_d t_p = 0$

Therefore the maximum response

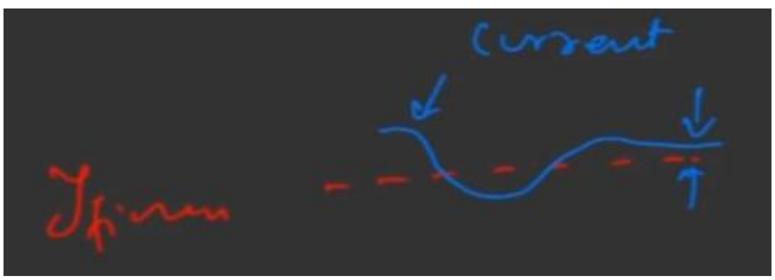
$$y_p = 1 + e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}$$

The maximum percentage overshoot of the final value is

$$M_p = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} * 100\%$$

So this is how we define the maximum overshoot of the response similarly the previous specification here also we can modify the maximum overshoot using ξ and ξ is also dependent control parameters this is how we do the modification. Now we'll go to our last specification settling time t_s. As you know that it is basically the difference between the current and final value and we had response something maybe something like this

(Refer Slide Time 18:03)



So now the settling time we can write here the difference between the difference between current and final value of y(t) so we can

$$y(t) - 1 = -\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}[\cos\omega_d t \sin\beta + \cos\beta \sin\omega_d t] \ldots\ldots Eq(2)$$

$$\because \tan\beta = \frac{\sqrt{1-\xi^2}}{\xi}$$

$$\cos\beta = \xi, \sin\beta = \sqrt{1-\xi^2}$$

Since we know that the magnitude of $|\sin\omega_d t + \beta| \leq 1$ If you use this condition in equation number two we can write

$$|y(t) - 1| \leq \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}$$

The percentage deviation from the final value we can write

$$\frac{|y(t)-1|}{1}*100\% \le 2\%$$

So the two percent we have assumed while we are defining the response all this stuff so two percent we can tolerate in the response suppose this is the difference between two percent between the current and final value so we can write this expression or you can write but in some books they are having different values so if it is different we can calculate accordingly for t_s

$$t_s = \frac{ln\left(0.02\sqrt{1-\xi^2}\right)}{-\xi\omega_n} \dots\dots Eq(3)$$

So this is the expression for settling time so if you notice here again the settling time also depend on the natural frequency and damping ratio so using the control we can modify the settling time how fast if we want to reach if your aircraft needs to be reached to the final value very quickly then by using the control we can modify the settling time but this is how we do the I mean analyze the system using the controls let me write one note here this maybe we will be using in some places

Note: for damping ratio between 0.1 and 0.9, the quantity $-ln\left(0.02\sqrt{1-\xi^2}\right)$ varies between 3.9 and 4.8.We may approximate the expression for t_s by

$$t_s \approx \frac{4.4}{\xi\omega_n}$$

So sometime we use this value for the underdamped system and yeah this is also the designing control we use this expression also so let's stop it here we will continue from the next lecture. I will have some examples how we can design a controller for aircraft attitude systems. Thank you.