Introduction to Aircraft Control System

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Week – 04

Lecture – 17

Time Domain Specifications (Cont.)

In this lecture, we will be starting with the transient response specification. Before we proceed to the specification part, we need to highlight why we have to consider the underdamped system practically for implementing the control system design. Then we'll give the expression of the step response of the PD control system from which we'll be deriving the specification part. Then we'll conclude the lecture with the definition of the different specifications which we'll be using to improve the transient response of the control system. Then we'll conclude the lecture.

Hello everyone, this is the lecture number 17. We'll continue from the last lecture on the transient response specifications. In the last lecture, we came up with three different conditions on damping ratio. When the damping ratio lies between 0 and 1, this is basically underdamped response. When it is equal to 1, critically damped response and when the damping ratio is greater than 1, overdamped response. But for aircraft applications, generally we consider the damping ratio lies between 0 and 1. This is the condition generally we use for aircraft control system. Now I'd like to highlight why we have to consider the damping ratio in this range. Let me highlight a few important points.

If you consider this condition, first let me highlight first point, faster response. Under damped systems, more quickly the change in input or disturbances compared to overdamped. Or critically damped system. This faster response can be considered in situations where rapid adjustment are necessary. For stability or performance. This is the one reason why you have to consider the underdamped system. Second, improve handling characteristic. So, under this under damped system, response can contribute to better handling quality characteristic. Making the aircraft more responsive to pilot inputs. More responsive to pilot inputs. This is particularly important for agile and maneuverable aircrafts. This is also very important point why you have to assume the damping ratio in that range. Third point, reduced settling time. We'll discuss what is the settling time, we'll discuss very soon. Under this, under damped systems tend to settle with the final value more quickly than overdamped systems.

This characteristic is also desirable in applications where minimizing settling time is a priority. Fourth point, enhanced sensitivity to control under damped systems exhibit oscillatory behavior. This is already we've discussed how the oscillatory behavior arises for the under damped system behavior in their response. These oscillations can enhance the sensitivity of the control system to small inputs. It's a very, very important point.

So, if you use this kind of system, it will be the sensitivity of this control system to small input will be increased, enhanced. Allowing final or finer control and shift point we can write stability margins. The under damped system's response can provide greater stability margin in certain situations. Making the system less prone to instability. So, these are the points why you have to consider the damping ratio should be in this range. But it depends also in some situations it may change, but most of the cases we generally consider the damping ratio in this range. Also, there another point can be the control authority because if you want to consider damping ratio higher, so there will be control authority will be here, requirement will be increased because to provide that kind of damping to the system, you should have that kind of control power in the system. That is another reason maybe. Let's go back to the dynamic system what you have considered before.

So, we had the transfer function. Now, you will consider the damping ratio in between 0 and 1 under damped system. So, based on the above points we are going to consider the damping ratio should be in this range. Now, the transfer function we had

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

So, as we discussed that will be considered the step command in the system.
So, we will assume r(t) equal to 1 which is step command and we will see how the response going to be for this step command and if you take the Laplace terms from $R(s)$ =1/s r and for this R(s) is the step response. Why it is step because the command is 1/s step command the step response is given by

$$Y(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}\frac{1}{s}$$

Now, if you use the partial fraction approach we can Y(s) in a partial fraction

$$Y(s) = \frac{1}{s} - \frac{s+2\xi\omega_n}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$= \frac{1}{s} - \frac{s+\xi\omega_n}{s^2+2\xi\omega_n s+\omega_n^2} - \frac{s-\xi\omega_n}{s^2+2\xi\omega_n s+\omega_n^2} \text{......}Eq(1)$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = \left(s + \xi\omega_n\right)^2 + \omega_d^2$$

$$\omega_d^2 = \omega_n^2\left(1 - \xi^2\right)$$

Substituting the above expression the above in equation 1 else we can write

$$Y(s) = \frac{1}{s} - \frac{s+\xi\omega_n}{\left(s+\xi\omega_n\right)^2+\omega_d^2} - \frac{\xi\omega_n}{\omega_d}\frac{\omega_d}{\left(s+\xi\omega_n\right)^2+\omega_d^2}$$

So, we have multiplied ω_d here in both side in the denominator and numerator to make the things in more simplified form and taking inverse Laplace we get

$$y(t) = 1 - e^{-\xi\omega_n t}\left[\cos cos\ \omega_d t + \frac{\xi\omega_n}{\omega_d}\sin sin\ \omega_d t\ \right]\text{......}Eq(2)$$

$$e^{-at}\sin sin\ \omega\ t = \frac{\omega}{(s+a)^2+\omega^2}$$

$$e^{-at}\cos cos\ \omega\ t = \frac{s+a}{(s+a)^2+\omega^2}$$

So if you take the inverse Laplace transform you'll get this expression so this is these are the terms we have used in the equation number two and equation number one now we have our step response of the second order system and on this system we'll be defining the specification what are the specifications will come up.

Let me define what are these specifications then we'll find those the expression of those specifications based on them let me write based on them so this is the unit step response why is unit step because we have we have taken r(t) equal to one unit step this is the expression we have taken that is our unit step instead of response a number of different specifications may be given as the first specification we are going to derive is rise time which is denoted by t_r so let me define what is this the time taken for the step response step response to first and each of final value in response plot we'll so it will be showing how it will look like how we'll find this rise time just after some time we'll be showing this another specification pick time at t_p the time taken to first achieve the peak value for the peak response and this because when we have maximum we have maximum over note we did it by M_p the maximum percentage overshoot from the final pattern. Fourth specification will have settling time so the time taken for the output to get within two percent of the final value we'll assume two percent maybe some cases there are different system different percentage used but we'll be using two percent final value and stay there.

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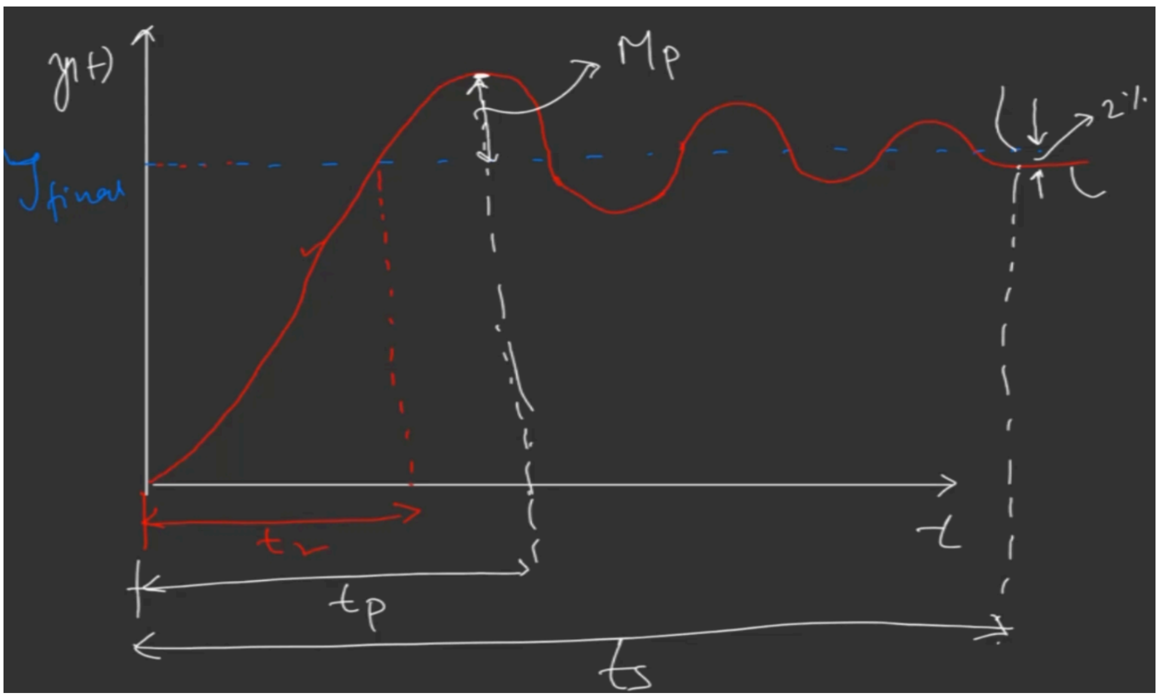

Now we'll see we'll see how this specification can be seen in the response plot let me go to new figure let's assume we have a response this time axis and the step response y(t) and since we are assuming step command so we are having the final value my aircraft should maintain this final value for example let me write y_{final} and the response I'm having starting from here this goes like this and it is something that is now as for the definition first let's go the right style the time taken for the response to first reach the final value so here this is our final value so this is the response this is the response plot and it first reaches to the final value here.

So this is the we can denote so this is the we can denote this time taken from this to this this we can denote by t_r and the second is peak time the time taken to first achieve the peak response so this is our peak response here the maximum peak so time takes two to this peak we can denote as t_p peak time and now let's move to the maximum overshoot the maximum percentage overshoot from the final value so in this case this is our maximum overshoot from the final value so we can denote this is M_p now the settling time, the time taken for the output to get within two percent of the final value so we can say let if you assume this is the value let's assume this is two percent from the final value this is the response and this is the final value and between these two percent error is there so once the system reaches to this value we can say so this is the value we get at this time the time takes to reach this value we can denote this is t_s so this is how we can define the specification for response now that as a control engineer the control system will modify this specification so that you can fulfill your mission objective. If you want to fulfill your mission objective very quickly then we have to modify the settling time or if you want to or if you want to have less overshoot less oscillation in the system, so we have to work on the maximum overshoot how we can modify it or if you want to raise the final value very fast so that we have to modify it here so how the controls algorithms should be designed so that you can modify this response and you can get the better response better objective, so this is how we'll be designing the control system let's stop it here we'll continue from the next lecture and how we will be coming up the expression of this specification using these definitions. Thank you.