Introduction to Aircraft Control System

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Week – 04

Lecture – 16

Time Domain Specifications

In this lecture, we will be studying the time domain analysis of the system. In this direction, two important concepts will be starting. One is the transient response and is the steady response of the system. For the transient response analysis, we will be considering the PD control system and we will compare the same with the standard transfer function. And we will come up with some relation between the damping ratio and natural frequency of the system. Then we will study how the damping ratio is going to affect the system poles, closed loop system poles. And then we'll study how the response of the closed loop control system is going to affect based on the damping ratio, under which we'll be starting the underdamped response or critically damped response or the overdamped response. We'll be studying those. Then we'll come to the lecture.

This is the lecture number 16. As we mentioned in the last lecture, in this lecture, we'll be talking on the time domain specification of a system. Under time domain specification, as we discussed, time domain specifications, there are two components to study under this. One is the transient specifications and second is steady state specifications. Let me revise again what are these.So suppose we have, this is the current attitude of the aircraft angle. This is my t. And if you want to track the desired attitude, for example, $\theta_d(t)$. And if the aircraft responds like this, for example. So here, till the steady state part is called transient part, transient response.

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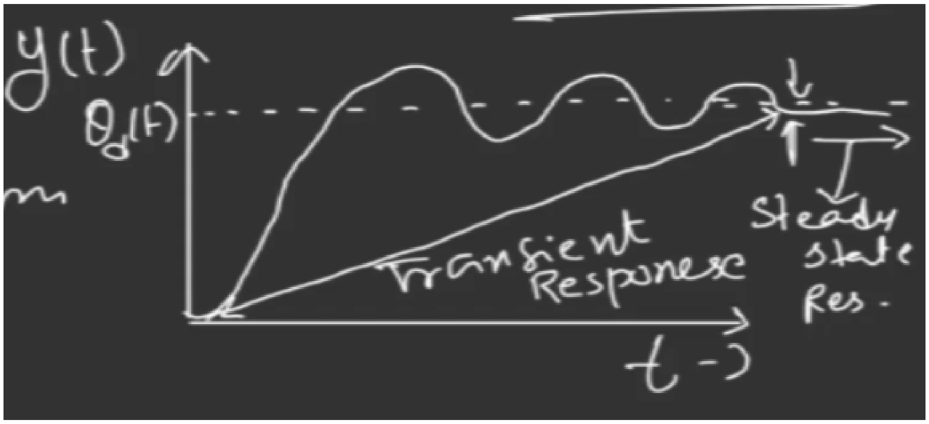

And once the system reaches to the steady state with some error, it's under the steady state response. And we'll start with the transient specification, how we can come up with some kind of specifications which can help us to modify the transient response. Here, how we're going to improve the behavioR(t)he response like the reducing the overshoot, undershoot, how we can reach to the steady state very fast. So we'll have some specification or some kind of terms which will help us to modify this response. Let's start.

So first start transient response. Here, we'll be discussing how the system responds to our step command. So here will be, the specification will be coming up for the theta desired step command. But you can, theta desired can be, for example, maybe 60 degrees. For example, so if you multiply the step command, you can get 60 degrees.

Let me define that so you can understand. So here, the specifications are often described in terms of the response of closed loop systems. Closed loop control system to a unit step function or command. Suppose we have, this is time axis and this is r(t) and we'll be assuming unit step function. This is one and the condition is r(t) equal to one when t is greater than zero, less than equal to greater than zero and zero when t is less than zero.So when this, we apply this kind of command to the closed loop control system and we'll be, based on this, we'll be deriving the specifications.

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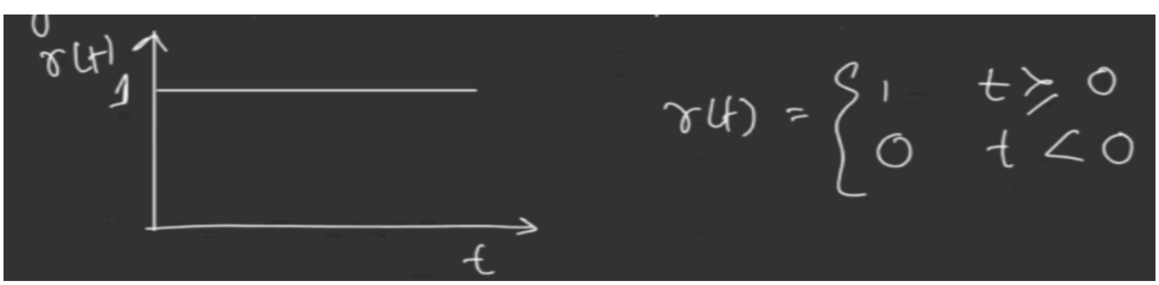

Also, the standard PD control, what we have done before, the standard PD control is

$$u(t) = k_p e(t) + k_d \dot{e}$$

$$= k_p(r(t) - y(t)) + k_d(\dot{r} - \dot{y})$$

$$= k_p e - k_d \dot{y}$$

And the modified closed loop control system, the block diagram of the modified closed loop control system, we can have, here we are having R(s) and let me draw the figure that I will explain. This is Kp and we are having summing point $M_d(s)$ and the summed output goes to the plant which is $1/Is^2$ and this is output from this, we can denote Y(s). If you notice carefully, this is the term, this term we can write, this is k_d, we can write k_d is the derivative of y, we can write as input to the summing block minus, this is plus, this is plus. So based on this function, based on this function, we can write this loop and we have another outer, this is basically inner loop and the outer loop we can draw, outer loop is basically minus and plus, so this is error function. So based on this k_d control, we can come up the figure like the closed loop diagram like this.

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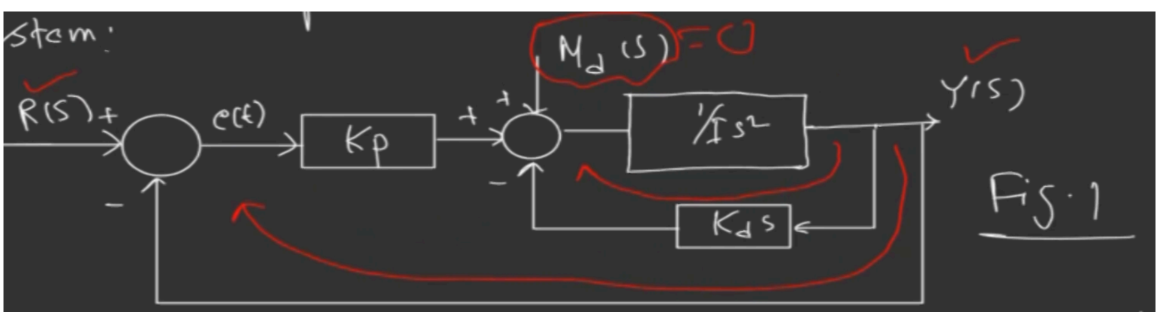

So here is two loops, one is inner loop like this, another is outer loop like this. So for the inner loop, we can have a transfer function and before that we have another assumption that since we are going to define the specification for the step command and with the application of step command how the output response is going to respond. So in that case, we are going to have the relation between the output and input, reference signal and Y(s), so this part we can assume to be zero. So we are going to have transfer function between R(s) and Y(s). So here, so let me write the response, response to a step command is the attitude angle of the aircraft is relevant at this stage.

So we said the disturbance $M_d(s) = 0$ and W(s)=0. This is the assumption we can take. And from figure, this is, let me draw this figure one. So from figure one, we can come up with the transfer function for the inner loop. So from the inner loop transfer function figure two, the transfer function, we can write $G_p(s) = \frac{1}{Is^2+k_d(s)}$.This is very simple, we can use the same logic what you have done before. And so this is, okay, let me write the another part then I'll explain. And the effective control transfer function, we can write $G_p(s) = k_p$. So here, this is the $G_c(s)$, okay, only k_p is there. Now, the modified block diagram of figure one can be drawn as, this is the summing point where I'm applying R(s) and we are having p(s) which is going to the controller k_p. And from this, we can write no need to have summing point here because we already have the transfer function for the inner loop. And also we have assumed the disturbance to be zero. So we can write the inner transfer function here, which is $\frac{1}{Is^2+k_d(s)}$. This is output Y(s) and this is the feedback. So we can come up the transfer function like this for the PD control system.

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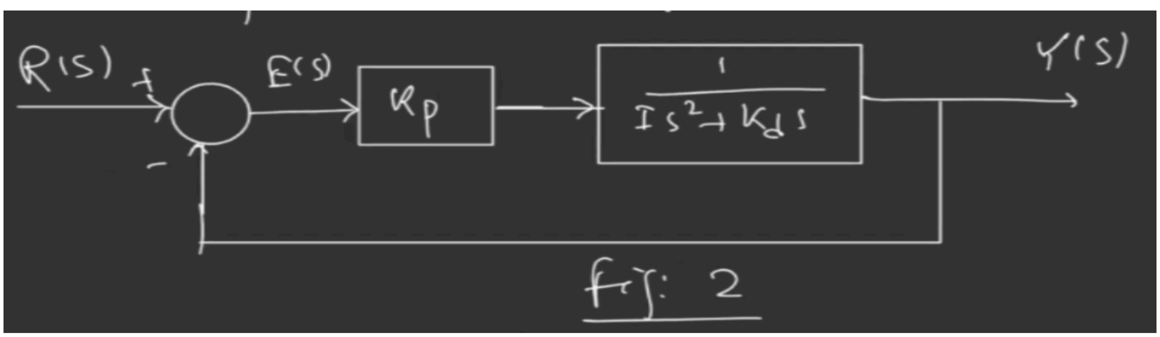

So this is closed loop diagram. And the transfer function from this figure two, we can write, let me write T(s), the transfer function. We can write also G(s), also. So T(s), I'm denoting here T is the transfer function which relates between the R(s) and Y(s).

$$T(s) = \frac{G_p(s)G_c(s)}{1+G_p(s)G_c(s)} = \frac{k_p\left(\frac{1}{Is^2+k_d(s)}\right)}{1+k_p\left(\frac{1}{Is^2+k_d(s)}\right)} = \frac{\frac{k_p}{I}}{s^2+\left(\frac{k_d}{I}\right)s+\frac{k_p}{I}} \quad .... Eq(1)$$

So let's define $\omega_n^2 = \frac{k_p}{I}$ and $2\zeta\omega_n = \frac{k_d}{I}$. So here, let me write here, ω_n is the undamped natural frequency and ζ is damping ratio. We can write and denote it and if you notice carefully, if you can control natural frequency or if you can come up with some gains k_p and k_d, we can change the natural frequency and damping ratio because we can write from this also, you can write $k_p = I\omega_n^2$ and $k_d = I2\zeta\omega_n$. So if you can change the value, some optimal values from controls techniques or we'll be talking later. So if you choose very suitably this k_p and k_d, we can change the damping ratio and natural frequency. So we can change the speed of the system also, we can change the overshoot, undershoot of the system because natural frequency talks about the how speed my response will be and damping talks about how fast I can go to the desired value with less overshoot, how I can damp out the overshoot, undershoot in the response.

So we have to suitably choose k_p and k_d from the control designs or control, I mean whatever the concept will be, will be having, using that we can come up with some kind of relation through which we can change the response properties. So let's go step by step. Based on ω_n^2 and $2\zeta\omega_n$, equation one can be expressed as expressed as

$$T(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} \ldots Eq(2)$$

So this is, if you notice carefully, this is the second order system. So why it is second order? Because the highest power in the denominator polynomial is two and generally we define the ordeR(t)he system based on the highest power in the denominator polynomial.So this is second order system and also we sometimes call this is standard second order system. If you refer any control system book, generally we use this kind of expression for defining the second order system. Now let me define this equation number. This is equation number two and from equation number two, the characteristic equation we can write

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

So based on these poles we will come up with three different conditions.
Based on the polarity of ζ we will have different kind of location of the poles in the system. So there are three cases, there are three different cases for the different values of values of damping ratio. So let's go step by step. Case one, when ζ value lies between

zero and one the system is called underdamped . So this is the condition for underdamped system when zeta lies between zero and one.

So in this case if it is ζ is in this range then if you notice here this term will be imaginary. If ζ is less than one then we can write $\sqrt{\zeta^2 - 1}$ is imaginary and there will be two complex conjugate poles. So what are these? So in this case the pole should be

$$s ==- \zeta\omega_n \pm j\omega_n\sqrt{\zeta^2 - 1}$$

So these are the poles for this underdamped system and if you locate this pole in s-plane there's complex conjugate so negative is here $-\zeta\ \omega_n$ and we have two complex part also so this is the real of s this is the imaginary of s and we have complex conjugate poles can be located here and here and this is actually the value of $\omega_n\sqrt{1 - \zeta^2}$ and this value we can write $-\ \omega_n\sqrt{1 - \zeta^2}$. So it is clear that if the system is underdamped the response will be something oscillatory behavioral response so we'll have for this kind of response.

Note: These poles lead to decaying oscillatory behaviour at damped natural frequency ω_d, where

$$\omega_d = \omega_n\sqrt{1 - \zeta^2}$$

$$s =- \zeta\omega_n \pm j\omega_d$$

This is the damped natural frequency why because damping factor in the right hand side so that's why it is called damped natural frequency where the response will be decaying.

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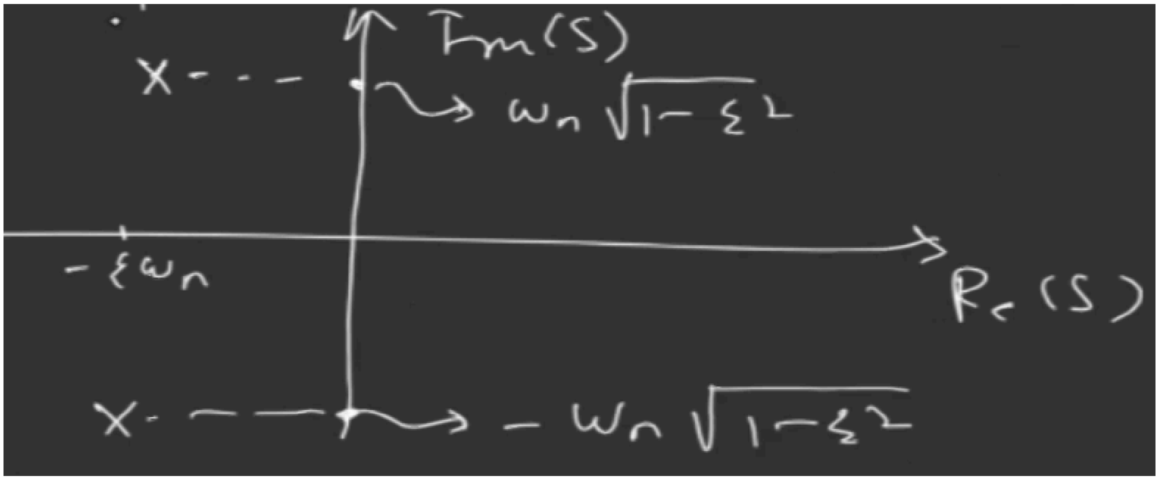

Let's look at case two, when $\zeta = 1$ the system is called critically damped system, let's look how the response will look like or how the pole location will be in the s plane so if ζ equal to one we can see that this part is going to zero I mean this part is going to zero so we can write there will be two poles at the same point, so we have two repeated poles in this case the poles are we can say

$$s = -\omega_n, -\omega_n$$

The location of these poles in the s planes we can write there are two repeated poles at $-\omega_n$ and so this is the critically damped system.

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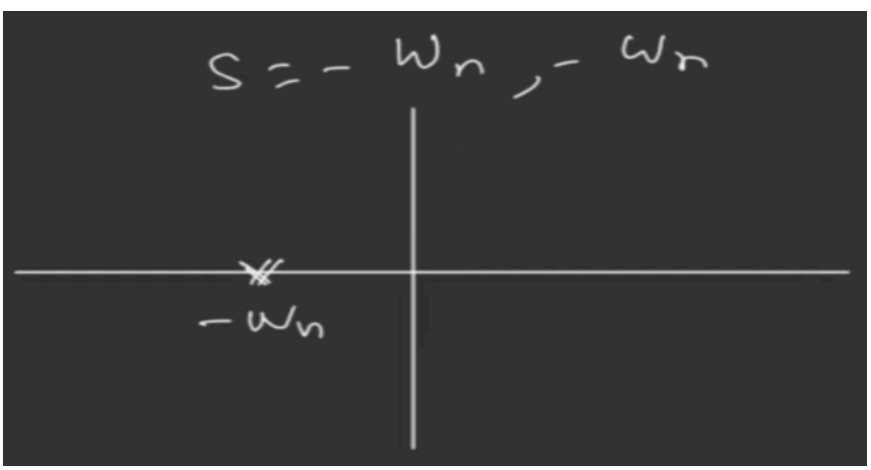

Lets look at case three when $\zeta > 1$ This condition is called over damp because damping is quite high it is more than one so this system is called overdamped in this case $\sqrt{\zeta^2 - 1}$ is real because ζ is greater than one and hence there are there are two distinct negative real poles

$$s = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

So these are the poles which are negative real poles and if you see these poles in s plan this is my s plane and these poles this is the imaginary of s is the real of s and one pole will be here another will be here so this pole will be $= -\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1}$ and this pole will be $= -\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}$.

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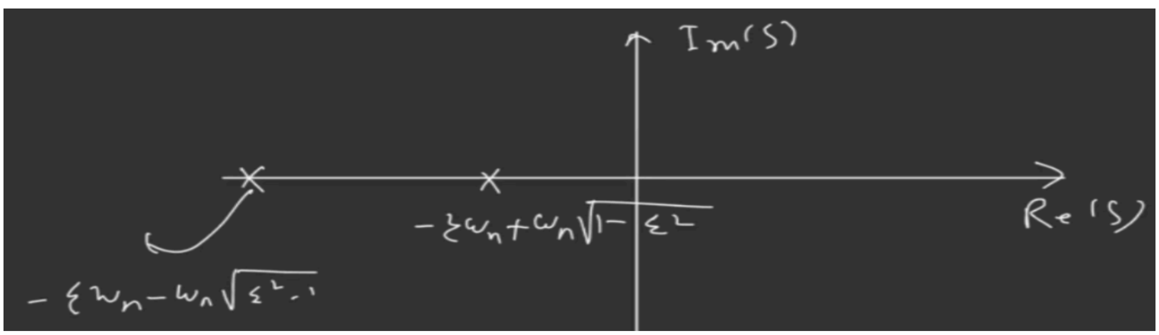

So this is quite stable system and now let's have a let's look the response for the different values of damping ratio how the response will be look like so this is my the response of the system this is time axis this is for example y of t the response for example and since we are considering the step command this is the step command so here R(s) equal to one upon s or R(t) this is R(t) this is the desired response my system should follow and we have this kind of response and so in this case this is the value of ζ when it is in between zero and one so this response called we can say under damp response if we little bit increase the damping ratio let's assume 0, then system will have something like this very minimal something like this and this is the case when damping ratio is 0.7 so this is also oscillatory but the magnitude of the oscillation is very less reduced and let's have the case when damping ratio is one only so in this case it will scroll something like this and very small oscillation then it will settle down.

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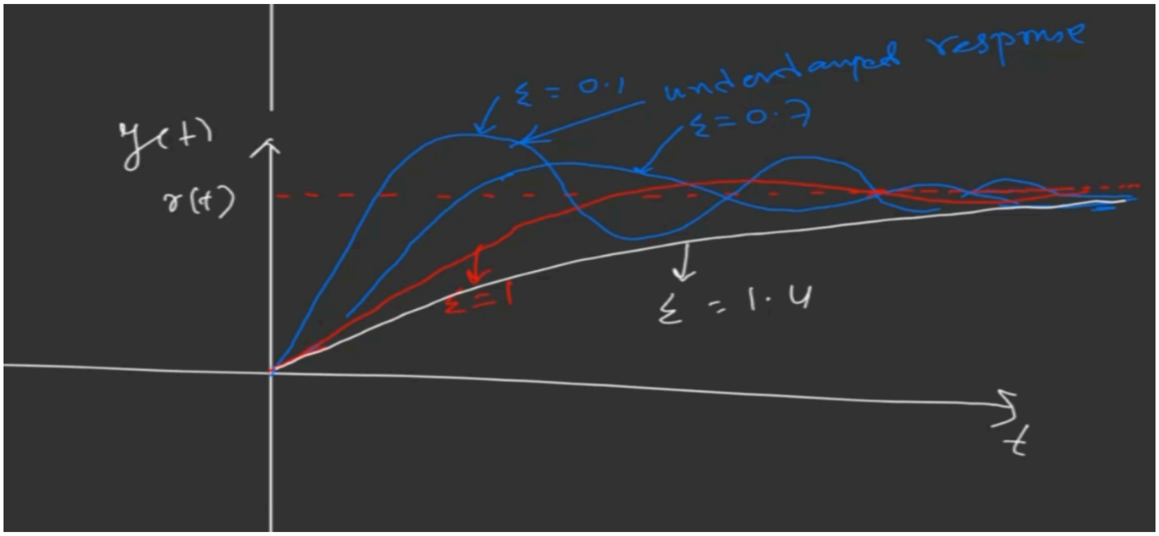

So this is the case when damping ratio is one and let's have another case when damping this is greater than one in this case the response will go straight away something like this so this is the case when damping ratio is for example 1.4 so as we increase the damping ratio the oscillation is reduced, there are other effect also we'll be talking later in this lecture what you have learned actually if you increase the damping ratio the oscillation will be reduced but there will be other problem like settling time will increase if you decrease the damping ratio oscillation will more but steady state it will be reduced that we'll be discussing in the coming lecture so this is how we analyze the system and we connect how the damping ratio and natural frequency plays a major role in the response of the system because this is how we'll be designing the autopilot how we can modify the response of the aircraft if it is position or attitude how we can come up with very precise observations with the help of this kind of techniques so let's stop it here I will continue from the next lecture and we'll define the specification how we can relate the system response to the time domain specification, thank you.