Introduction to Aircraft Control System

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Week – 03

Lecture – 15

Classical Control Synthesis (Cont.)

In this lecture, we will be discussing the PID control, proportional integral and derivative control. In the last lecture, we discussed proportional and derivative control, and we have assumed that the disturbance torque or moment to be zero. So, in the PID control system, we assumed the disturbing moment or torque to be zero. But in practical problems, in real life problems, we generally can't ignore the disturbance. There is always disturbance. Suppose the aircraft is flying, there can be wind disturbance, for example.

So, let's assume there is some disturbing torque acting in the system, M_d in the system along with PD control. So, we will take you to the motivation why you have to consider integral control. So, here we'll be going with PD control with disturbance.

Let's see how it is happening. So, the system we had for PD control, $I\ddot{y} = K_p(r - y) - K_d\dot{y} + M_d$. So, this we already had, we derived it. And now we are adding something about the response. Suppose we have some system and the system should follow this line.

Let's assume this is my reference line $r(t)$. In PD control, what you have observed is that the system can start with oscillation and with time the oscillation will decay into the reference signal. But at steady state, there can be some errors. Steady state means if you have any kind of response and if this is suppose $r(t)$, that is your signal or the reference signal to be tracked, if my system is going like this, the system goes to some constant value after some time and the system stays there. For all future time, the system stays to that value.

So, from these two goes to the origin to this time, it goes to the steady state, it is called transient behavior of the system. And once it reaches the steady state value, it is called steady state behavior, steady state response.

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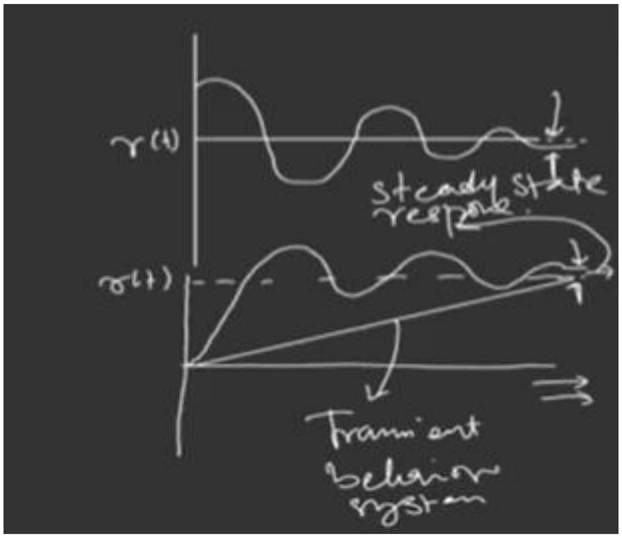

So, even after the oscillations die out in the PD control system, there will be steady state error in the attitude. So, this is the problem that happens in the PD control system if there is a disturbance.

Now, how to tackle these issues? Let's work on it. Let's assume similar to before, the reference attitude $r(t)$ is constant. So, in this situation, if it is constant $r(t)$ at steady state, as I said, it stays to some values. So, $r - y = e$. So, error will be there forever after the system reaches steady state behavior.

So, it means r is constant here. And if e is constant here, after steady state, so y should be also constant, which cannot be varied. So, at steady state, we can say $\dot{y} = \ddot{y} = 0$. That is quite obvious at steady state, because y should be constant at the time. Let me write this equation number one.

We can rewrite at this condition $K_p(r - y) + T_d = 0$ or we can write $K_p e + T_d = 0$ or we can write $e = -\frac{T_d}{K_p}$. So, this problem arises if you have a disturbance in the system in the PD control system. So, which is basically, non-zero. And if you plot graphically, if you see the response, how it grows with time,it will have something like this.

This is my t and this is $e(t)$. So, if you see the magnitude $-\frac{T_d}{K_p}$, it is constant and we can say this is the error area basically. And this area we can write $A = \int_0^t e d\tau$. And let me define this as figure one. So, this will happen at steady state, this is the condition that will arise.

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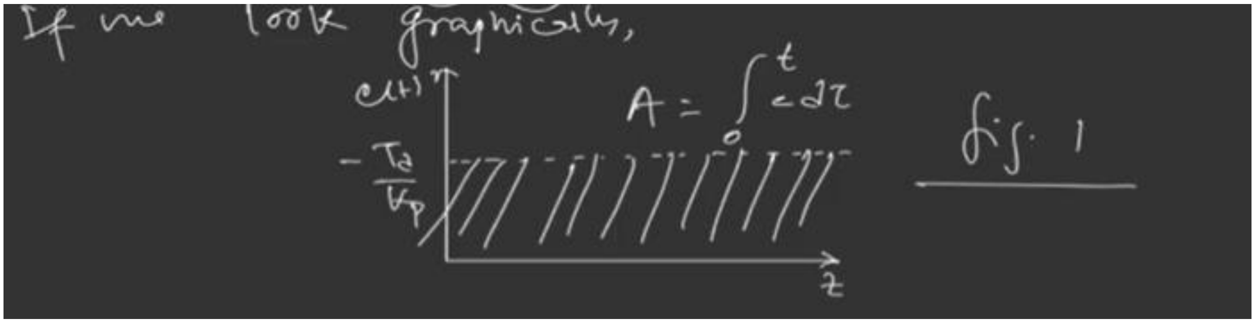

Why integral term should come into picture. So, PD control, proportional derivative control, does not have a memory. And to provide the controller with capability of driving the error to zero in the presence of constant disturbance, we need to add a integral term in the control law. So, let me write, in the presence of external disturbance in the aircraft attitude motionless equation, an integral term is added to the controller.

So, we have to add an integral term like $K_i \int_0^t e(\tau)\, d\tau$ where $Ki > 0$ and this is basically what we call integral constant. So, hence the PID control, so because of this our integral term is coming into the PD control system. Hence, the PID control law is given by $u(t) = K_p e(t) + K_d e(t) + K_i \int_0^t e(\tau) d\tau$. And the controller transfer function, we can write, if we take the Laplace transform, we will have $U(S) = K_p E(S) + K_d S\, E(S) + \frac{K_i}{S} E(S)$ And from here, we can write

$$\frac{U(S)}{E(S)} = K_p + S\, K_d + \frac{K_i}{S} = G_c S$$

And if you see the closed loop control structure, we have a reference signal $r(s)$, we have error $E(S)$ from this summing point, we have a controller here, $K_p + SK_d + \frac{K_i}{S}$. And the output from this block, we can write $U(S)$. And if also you are having the disturbance, the system, $M_d(S)$.

And we can write our attitude motion equation $\frac{1}{IS^2}$. This is my $G_p(S)$ equal to $Y(S)$. This is the EOD feedback system. In this summing point, this is the closed loop control system with PID control.

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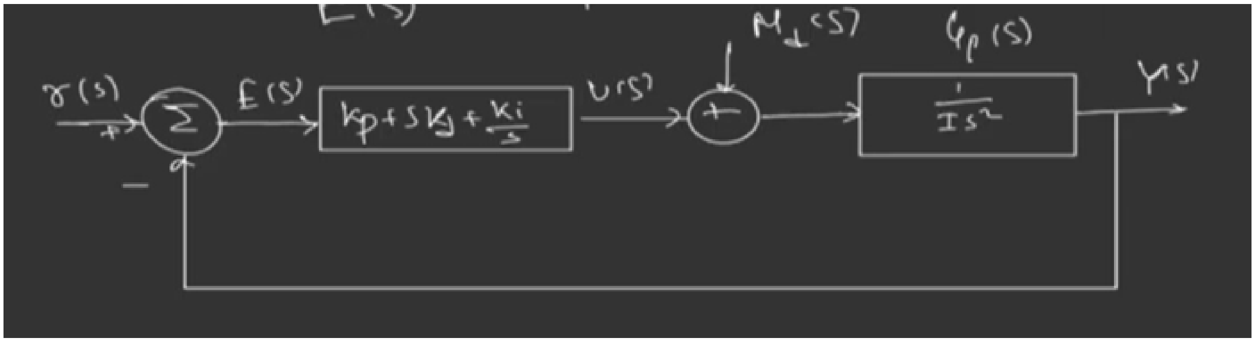

Now, let me write a note here, how this integral term is going to dissipate this term. It's a very important note. The integral term has memory that is proportional to the area under the error curve. A, this is my error curve area . So now, if the error is non-zero, the integral term keeps on increasing to a larger value to provide the correcting torque to the system, until the disturbance torque has been compensated.

And the attitude error of the aircraft system, which is $(r-y)$ is to zero. Also, we can write here the integral term in the PID control learns what the disturbance torque acting in the system is. So, this is a very important note, why we need to consider the integral term in PID control. In this motivation, we'll be moving in the sports that control design and these three types of control will be considering for designing the autopilot for the different systems or different equations of motions. Now, let's look at how we can modify the system response.

As I mentioned, for any kind of response, we can have two components, one is transient response and other is the steady state response. So, suppose this is my $y(t)$ and this is t- axis. So now, if our desired value somewhere is here, and if the aircraft attitude response, for example, is something like this and after some time it gets settled here. Now, the question is how we can reduce this steady state error, this overshoot or undershoot. So, we'll define some kind of parameters which will help us to modify the entire response.

So, here we can say before the system goes to steady state, this is the part of the transient response and once it reaches the steady state, and after that time, we can say this is the steady state response.

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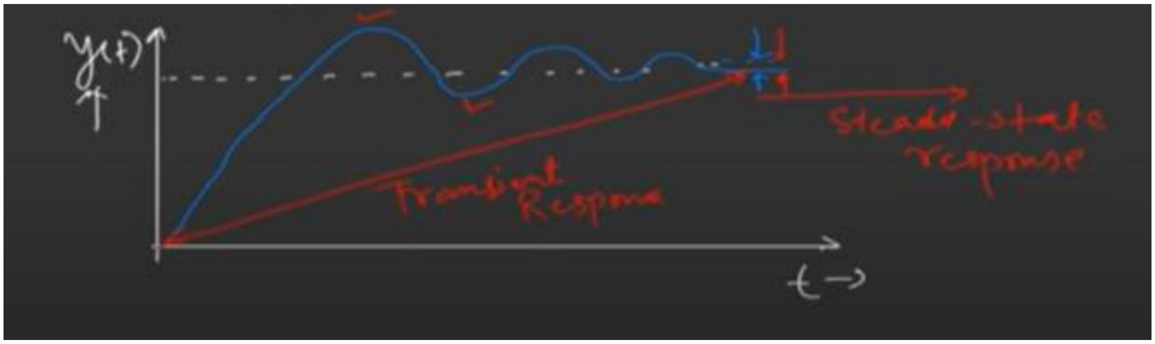

Now, we have to design the controller in such a way that we can improve transient response as well as the steady state response. Now, how will we think about this? How will we be designing the controller precisely so that our mission objective is met? So, here the first and foremost part is, the controller must provide asymptotic stability. So, the objective is we need to design the controller in such a way that the controller must provide asymptotic to the proposal of the system.

If you refer to the equation or the transfer function which relates between the input and output, reference signal and actual output, we had $Y(S) = \frac{G_p(S)\ G_c(S)}{1+G_p(S)\ G_c(S)}\ R(S)$ So, $R(S)$ is the reference signal and $Y(S)$ is the actual output from the system. So, the asymptotic stability means,, if you take the Laplace transform of this equation, $y(t) = \mathcal{L}^{-1}\left\{\frac{G_p(S)\ G_c(S)}{1+G_p(S)\ G_c(S)}\right\}$ inverse Laplace transform of this equation. So, if you notice in this equation, the system has been transformed into a time domain from the frequency domain. Now, we need to come up with some kind of specification which will help the system to follow asymptotically stability.

So, here asymptotically means, this expression, let me write this equation number one, for example, must asymptotically go to zero. So, this means the pole of the system must lie in the negative real axis. So, I should say this means the poles of the characteristic equation $1 + G_p(S)\ G_c(S) = 0$ must have negative real parts. So, this pole should lie in this region, in the negative side. It may have a complex conjugate part, but it should have a negative real part, that is very important.

Now, we have to come up with some kind of specification which will help us to modify our response. So, as I said, how I can reduce the statistic error here, or how I can reduce the overshoot, undershoot. So, we need to come up with some kind of specification which will help us to do this. And this is called the time domain specification.

What does it mean? So, we are working till now on the Laplace transform, right? I mean, the transfer functions we have arrived at in terms of Laplace domain. Now, we need to convert from the Laplace domain to time domain by taking the inverse Laplace transform. And in the time domain equation, we'll specify some terms which will help us to do our job. And this part is very, very important, how to modify the transient response and steady-state response of the system. And let's continue on this part from the next lecture.