Introduction to Aircraft Control System

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Week – 03

Lecture – 14

Classical Control Synthesis

In this lecture, we'll be discussing the typical controls we use in the classical control synthesis. We'll start with proportional control and we'll see that the response of the aircraft system with the proportional control will be oscillatory around the equilibrium point. To avoid this problem, we'll take the help of derivative control, which will add basically damping to the system and due to which the oscillation in the proportional control can be avoided. Then we'll move to PID control where we'll be adding the integral part to the control. Basically if there is a disturbance acting on the system and due to that disturbance, there is steady state error in the response and through the help of integral control the steady state error can be mitigated over time. Then we'll conclude the lecture.

In this lecture, we'll be discussing what are the typical control algorithms we use in the classical control system domain. Let's redraw the figure we had in the previous lecture, the closed loop diagram. So we had our reference value. This is we can write $r(t)$ and we had a controller $G_c(S)$ and the output from the $G_c(S)$ we had $U(S)$. We can write $R(S)$, the reference signal. And this control, before it goes to the system, we had also the disturbance $M_d(S)$ and the summed output was going to the plant,so $G_p(S)$, this is output, $Y(S)$. In the feedback, we had measurement noise, which is due to the sensor malfunction maybe. And this was measurement noise, $W(S)$ And we had sensor transfer function $H(S)$ and the sensor generally we use to determine or detect the position or attitude of the aircraft. So we assume H is the unity feedback system. So $H(S) = 1$.

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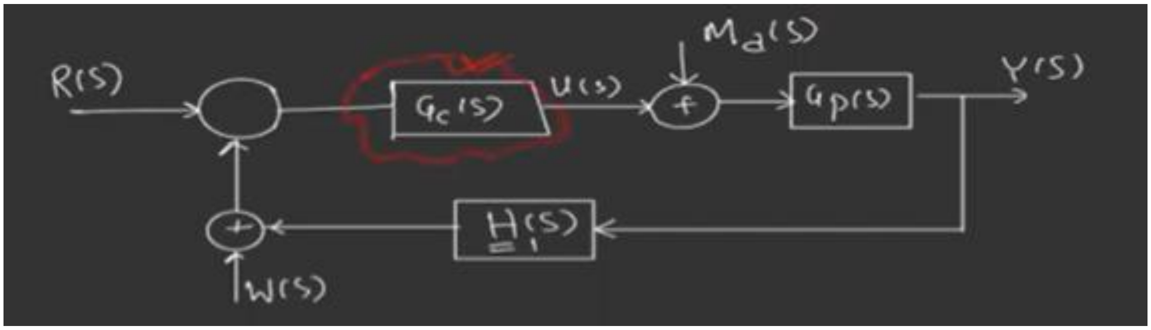

And in this lecture, we'll be discussing how we can come up with different control algorithms in the controller block so that our mission objective is fulfilled.

So we'll be designing different controls in the controller and how we can fulfill our objective. So let me write the heading, typical control laws, we use in classical control techniques. We'll restrict ourselves not to consider the measurement noise for the time being $\omega(S) = 0$. In this assumption, the attitude motion of the aircraft, we can write

$$I\ddot{y}(t) = u(t) + M_d(t)$$

So our motivation is how we can design a control algorithm in place of $u(t)$ so that we can get the desired response $y(t)$. Suppose our desired response is $y_d(t)$ and how we can design control so that I can track $y_d(t) \leftarrow y(t)$. So $r(t) \leftarrow y(t)$ can be your reference signal or it's maybe $\theta_d(t) \leftarrow \theta(t)$. So this is θ actual. So our objective is how we can make $\theta(t)$ to $\theta_d(t)$. So first, let's consider proportional control.

In proportional control, basically, it is just the scaling of the error signal. So it means here we have an error signal, $E(S)$, which is basically negative and positive. And this is my controller.

And in the controller, basically, we have to assume some kind of constant, which is scale the error signal so that we can get $U(S)$. So let me write the output of the control block, $u(t) = K_p e(t)$. So K_p is the scaling factor of the error signal where $K_p \succ 0$ is the proportional gain. And the associated transfer function for the controller, we can write $U(S) = K_p E(S)$ So output by input equal to transfer function, Laplace transform output divided by Laplace transform of input, basically, it also functions.

So $\frac{U(S)}{E(S)} = K_p = G_c(S)$ and this is the controller transfer function. So now let's assume the reference signal which needs to be tracked, y_d. So here, let's assume the reference signal $r(t)$ or $y_d(t)$ is constant. So if the reference signal is constant, it is kind of a regulator that we already discussed and since you will be finding the transfer function of the entire system and our main motivation is in this block, how we can design the control so that $Y(S)$ can track $R(S)$.

And since we need to find the transfer function between the input and output, this is my reference signal to the block and this is the output from the block. So $R(S)$ can be your input to the entire block here. So we need to find the relation between $R(S)$ and $Y(S)$. So we will not assume the disturbance or noise. So this is the actual main transfer function in the system.

So we'll neglect the disturbance term $M_d(t)$. And based on this assumption, we can write our system attitude aircraft, attitude motion, we can write $I\ddot{y} = u(t) = K_p e(t)$. And this equation further we can write $K_p(r - y(t))$ .So here r is constant, we can just write r. And taking the Laplace transform, we can write $IS^2Y(S) = K_pR(S) - K_pY(S)$. So here we have assumed the initial condition will be zero. So this is the assumption for finding the transfer function that we already discussed. And from this further we can write $(IS^2 + K_p)Y(S) = K_pR(S)$. So from this we can write $\frac{Y(S)}{R(S)} = \frac{K_p}{IS^2+K_p}$. So this is how we can write the closed loop transfer function for this particular system. So this is my reference signal, $R(S)$ and this is the controller, K_p. This is $E(S)$ and this is the plant, $G_p(S)$, which is nothing but $\frac{1}{IS^2}$,and this is the $Y(S)$ and this is basically a feedback loop.

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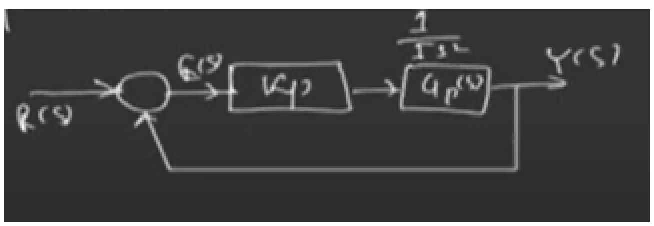

If you want to apply the direct expression, what you have done before, we know $\frac{Y(S)}{R(S)} = \frac{G_p(S)\,G_c(S)}{1+G_p(S)\,G_c(S)}$. This is the closed loop transfer function which relates between the output and input. So if you substitute the term, $G_p(S)$

$$\frac{Y(S)}{R(S)} = \frac{\frac{1}{IS^2}K_p}{1 + \frac{1}{IS^2}\,K_p}$$

So if you solve this expression, you'll get $\frac{K_p}{IS^2+K_p}$. The same expression we are getting. So if you can remember this expression, you can find the transfer function also. So this is our transfer function for this system using PI control. So I can say this is P control, proportional control, closed loop aircraft attitude control system.

So now, as we'll discuss this, the denominator of the closed loop transfer function talks about the number of poles in the system and the numerator talks about the number of zeros in the close loop transfer function. That is very important. So pole and zero, we can come up from any transfer function. This is a closed loop transfer function and this is the poles, this is zeros.

If ,for example, suppose we have an open loop transfer function here,then here also, we can come up with the poles and zeros. The numerator will be poles, the denominator will be 0.

This is the plant transfer function, open loop transfer function we'll discuss later, what is open loop and closed loop. So this is the poles. So poles from this closed loop transfer function, the roots of the characteristic equation of the denominator basically. This is the characteristic equation or the polynomial in the denominator part. The result, if you find from the characteristic equation, maybe I have not discussed it.

Characteristic equation basically, the characteristic we write $1 + G_p(S)\, G_c(S) = 0$. So in the closed loop transfer function, this part we call the characteristic equation, $1 + G_p(S) = 0$. So now from this expression, we can get the characteristic equation, $IS^2 + K_p = 0$. And further we can write

$$S^2 = -\frac{K_p}{I} = S_{1,2} = \pm\, i\sqrt{\frac{K_p}{I}}$$

So from these poles, it is clear that the system will be oscillatory, why? Because the solution will be in the terms of sine and cosine. So if you place in our real and imaginary axis, these poles, as I said before, are basically $S = \sigma + j\omega$ So what is this? This is the real part and this is the imaginary part. So in this case, we are having only the imaginary part. And in the S-plan, this is basically the S-plan we call, and this is the real axis and this is the imaginary axis.

I can say imaginary (S), this is the real (S). So since we are having only imaginary poles, we are having two poles, one is here and one is here. Right, I can say this is $\sqrt{\frac{K_p}{I}}$, and this is $-\sqrt{\frac{K_p}{I}}$, in the imaginary axis.

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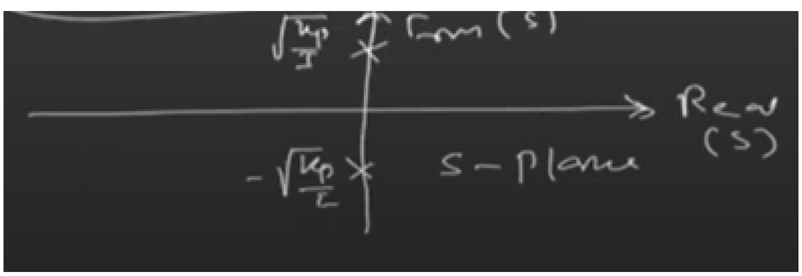

So now if you see the response of the system, the system response will be something, $y(t)$ and t, if you see them in the time domain, the response will be something like this. So it will be oscillatory without peaking in the response.

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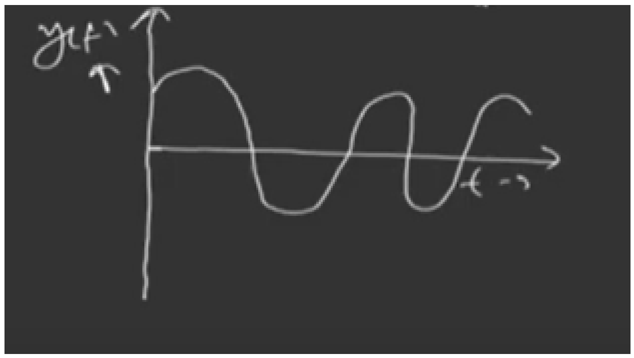

So what is the takeaway from this example? If you use PD, proportional control in our control block, the response will be oscillatory behavior and if you connect to the aircraft system, what exactly is practically happening? Let me draw this figure. It will give you some practical implications of this concept. So this is the line my aircraft would like to follow and this is the aircraft, for example.

So this is the aircraft x-axis, this is my desired signal, r, and so aircraft should follow this line. So here, about this line, the aircraft will oscillate. So suppose this is my spring, and spring constant is K_p , and this is the error e.

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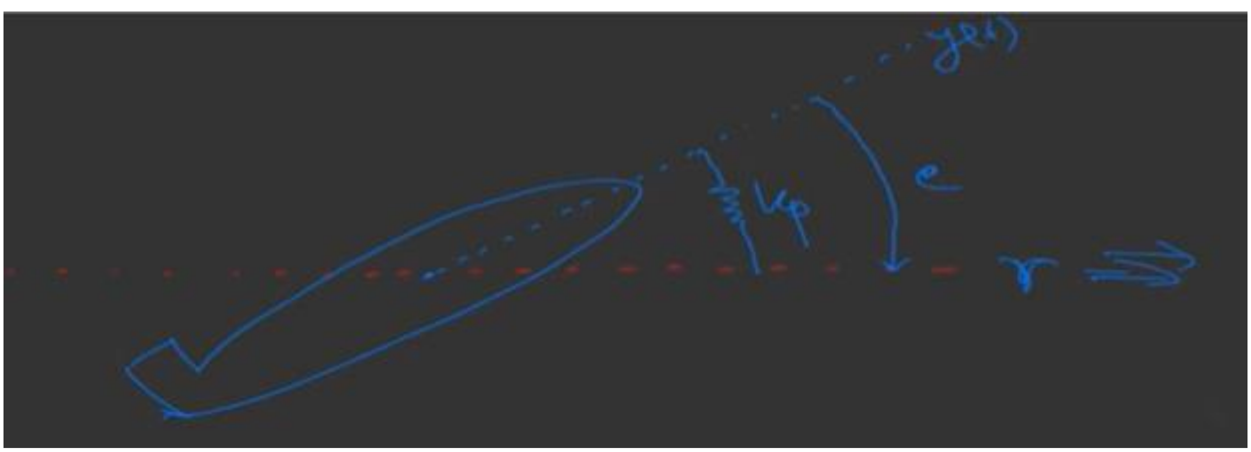

So this is the actual value of $y(t)$. So it means aircraft will oscillate about the reference line r, and with the oscillation, $\omega \sqrt{\frac{K_p}{I}}$.

So this is practically happening. This aircraft will not come to the reference line forever because of the nature of the system response, the aircraft will oscillate about the reference line. This is actually what will happen. So now, if you draw the closed loop diagram in this case, so we are having, this is my r, this is the error e, this is the control block K_p , and this is output $u(t)$, this is the plant, $\frac{I}{IS^2}$, this is $y(t)$, but this is the negative feedback.

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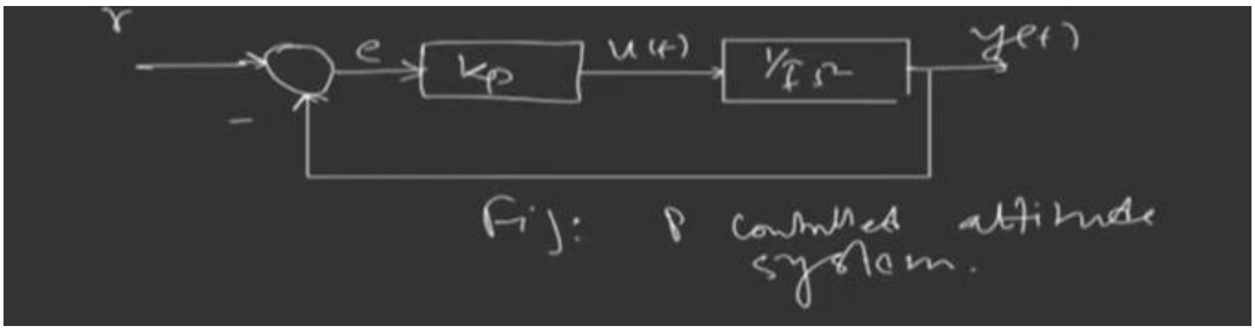

So this is the structure of the P controlled attitude system. And this is the problem. So from this kind of response, we can say P controlled system is just a mass spring system. So if you connect to this kind of concept to the mass spring system, it is the same kind of response you'll be getting because of the mass spring damper system, what is that expression? We know $m\ddot{x} + b\dot{x} + Kx = 0$.

So $b\dot{x}$ does not exist here. So here, where I'm having this kind of response. If you solve this equation, we'll get exactly the same kind of response. So it is similar to the mass spring system. So it means from the behavior, from the response, undamped oscillatory attitude motion of the aircraft will be there. So this is the problem if you were spring for this particular attitude control system of the aircraft.

Now to solve this problem, we need to consider another term, which is called damper in the system. So in the PD control system, we'll put a derivative term to compensate for this kind of sustained oscillation behavior in the system. Now let's look how the derivative control looks like. So the problem of sustained oscillation in PD, in P control system, P means proportional control system, is not desirable. For this, we're gonna add proportional control, plus derivative for PD control, to consider.

So here, the PD control is given by law $u(t) = K_p(t) + K_d\dot{e}(t)$ .This is the part of proportional control. So K_p is scaling factor and derivative control is $K_d\dot{e}$. So here we are adding the derivative of error to the proportional control.

And here, $K_d \succ 0$ is the derivative gain. Now, we'll find the transfer function for the controller law. So the controller transfer function, we can write, if you take the Laplace transform of this equation, $U(S) = K_pE(S) + K_dS\, E(S)$. And from this, we can write $\frac{U(S)}{E(S)} = K_p + K_dS$. So how the sustained oscillation can be damped out using the PD controller.

So this is my reference attitude the aircraft should follow. Let's assume this is $r(t)$ or $\theta_d(t)$ or $y_d(t)$, same thing, the notation just changed. And this is the aircraft system. And this is the actual value output $y(t)$.

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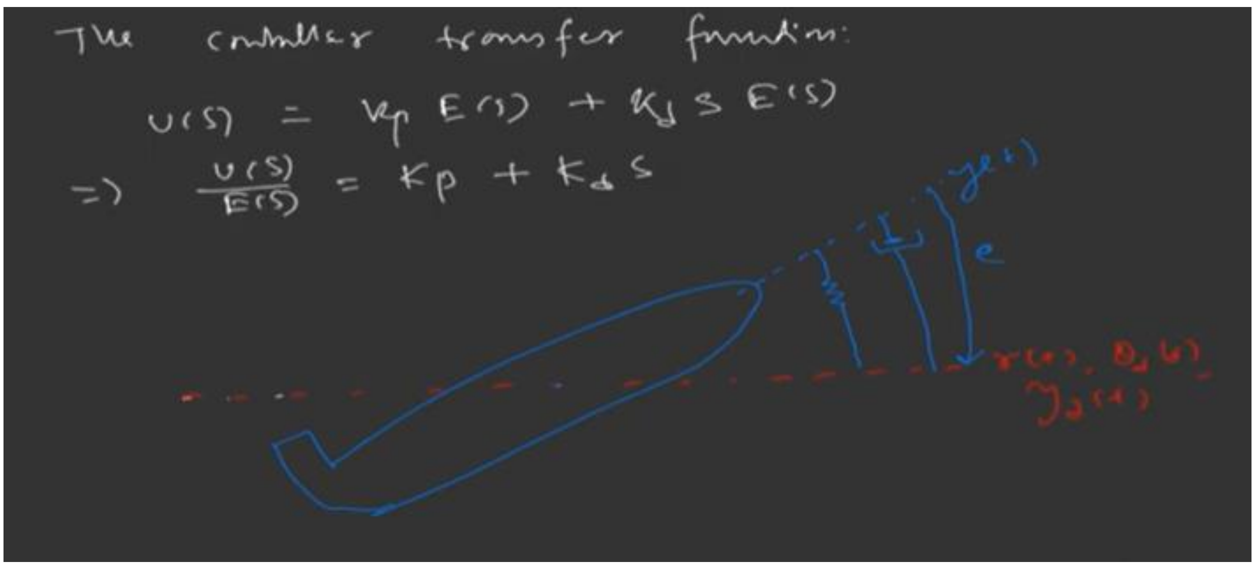

And in proportional control, we have spring here. And derivative control we are damping, we are adding a damping term in this expression. So due to the damping, the oscillation which is due to the spring can die out. So this is the error e. So initially, it may have some oscillation, but after some time, and due to this damping term, it will go out.

So let's look at how it is happening. Similar to the previous control here, let's assume $r(t)$ is a constant reference attitude to be tracked. And similar to neglecting the disturbance in the aircraft equation of attitude motion, yields to be

$$I\ddot{y} = K_pe + K_d\dot{e} = K_p(r - y(t)) + K_d(\dot{r} - y(t))$$

So here r is constant, we are assuming. So we can write $K_p(r - y(t) - K_dy(t)$. So this is the expression we are getting. And further we can write $I\ddot{y} + K_d\dot{y} + K_p y = K_p r$. And if you take Laplace Transform, we have $IS^2Y(S) + K_d S\,Y(S) + K_p Y(S) = K_p R(S)$. And further we can write

$$\frac{Y(S)}{R(S)} = \frac{K_p}{IS^2 + K_d S + K_p} = \frac{\frac{K_p}{I}}{S^2 + \frac{K_d}{I}S + \frac{K_p}{I}}$$

So this is the closed loop transfer function of the PD controlled attitude motion of the aircraft. So now let's look at how this oscillation is going to be damped out.

And as we said, this is the characteristic polynomial which gives us the number of poles in the closed loop transfer function. So let's write the characteristic equation. We can write $S^2 + \frac{K_d}{I}S + \frac{K_p}{I} = 0$. From this we can write, so this is the second row system, we have two poles. We can write

$$S_{1,2} = \frac{-\frac{K_d}{I} \pm \sqrt{\left(\frac{K_d}{I}\right) - 4\,\frac{K_p}{I}}}{2}$$

So this is the number of poles in the system. So now if you see these poles located in S domain, this is maybe somewhere here. So there are two poles somewhere here. So based on the values, even if the value, this part comes out to be imaginary somewhere, maybe here or here, but the real part is negative. So due to this, if you solve this equation, we'll have an exponential term in the solution. So the system may start maybe like oscillation, but after sometime it will damp out.

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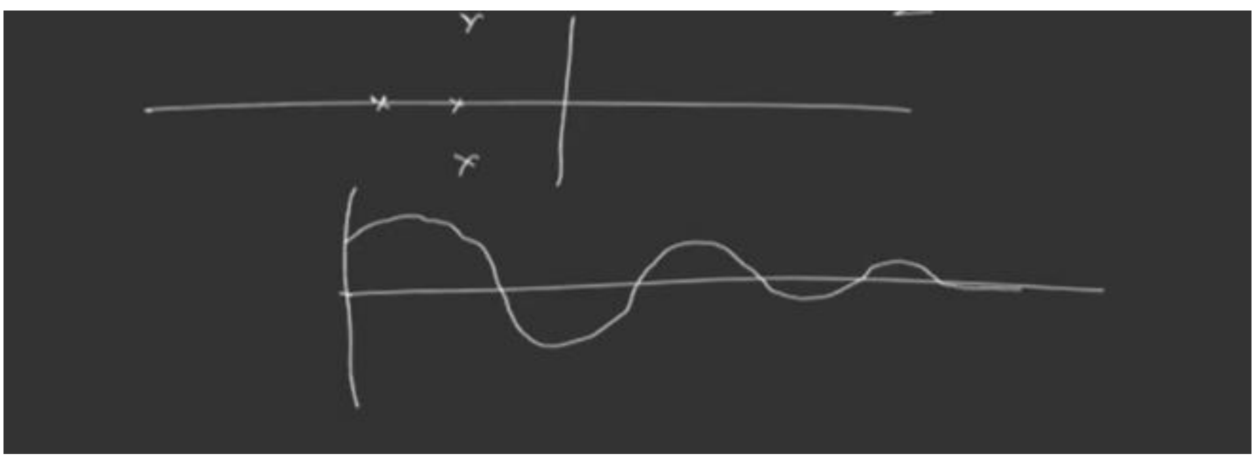

So this is why it is damping out? Because of the damp part in the system and because of the negative term in the poles. This is how we can modify the response, what you had in the proportional control. But let us stop it here. In the next lecture, we'll be discussing how we can design PID control and how the disturbance in the system can be considered and how due to the disturbance we can modify the control response. Thank you very much.