Introduction to Aircraft Control System

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Week – 03

Lecture – 13

Closed-Loop Transfer Function Analysis for Aircraft Attitude Control

In this lecture, we'll be studying how the different control inputs are going to be affected by the attitude convergence of the aircraft. We'll find the relationship between the attitude of the aircraft with respect to the reference signal, disturbance, as well as the noise. Then we'll see the combined effect of these control inputs on the attitude convergence of the aircraft. And also we'll be studying how the controller is going to place an important role for mitigating this disturbance, which is basically an unfavourable disturbance to the system. Then we'll conclude the lecture. We'll continue from the last lecture.

In the last lecture, we had the closed loop feedback controls for the aircraft attitude systems. Let me redraw the figure again, then we'll proceed. So we had our control transfer function, $G_c(S)$, and the output from the controller, we had $U(S)$ and we had a summing point. And in the summing point, there's two inputs, one is $M_d(S)$, which is the disturbance and the sum output from this block was going to the plant, which is $G_p(S)$ and the output from the plant was $Y(S)$.

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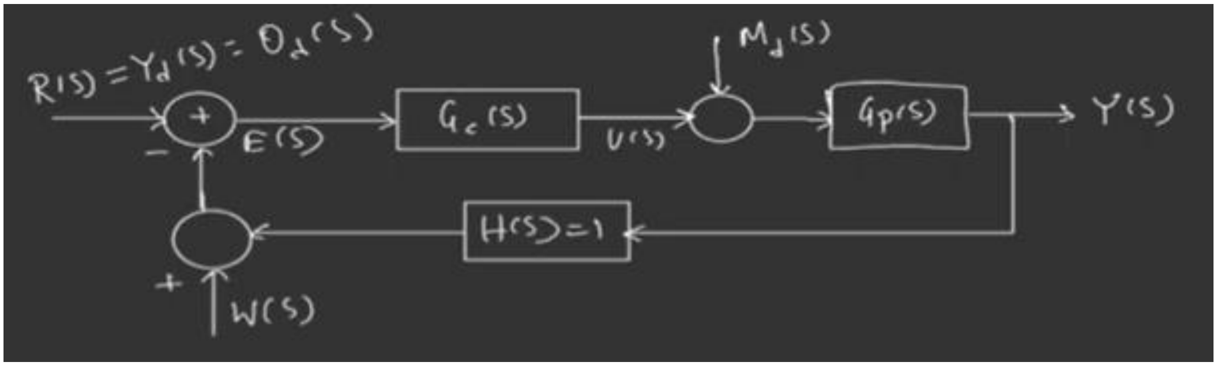

And we had another summing point, we are having the reference input $R(S) = Y_d(S) = \theta_d(S)$ And we had another input for this summing point. So we had noise in the system, $W(S)$ this is negative feedback and the sensor transfer function we assume $H(S) = 1$. This is basically a unit feedback system. And this is the sensor transfer function and the error was going to the controller. This is $E(S)$.

Now, in this lecture, we'll be finding the transfer function $Y(S)$ with respect to disturbance $M_d(S)$, $Y(S)$ with respect to $R(S)$, the reference signal, $Y(S)$ with respect to $W(S)$. So how can we derive a different transfer function? So let's start. So since we are handling the SISO system and transfer function generally we come up with the SISO single input and single output system. So we can't take all the inputs at the same time, we have to take one at a time.

So let $Y_r(S)$ be the response to the reference signal with $M_d(S) = 0$ and $W(S) = 0$. So first we find the transfer function $Y(S)$ with respect to $R(S)$ which is when we are having $Y(S)$ with respect to $R(S)$, I'm denoting $Y(S)$ be $Y_r(S)$. And we are assuming the other input signal to be 0. And in this situation, our closed loop transfer function can be drawn as, this is the summing point where we are having $R(S)$ as a reference signal and here we have a controller block, $G_c(S)$. And since we are not assuming a disturbance, the input is going to the controller.

This is my $U(S)$, this is the $G_p(S)$ and the output for this case, I'm assuming $Y_r(S)$. And also we are assuming there is no noise. This is my negative feedback, this is positive. So this is how in this condition we can run the block diagram. And let me run this figure number one, this is figure two.

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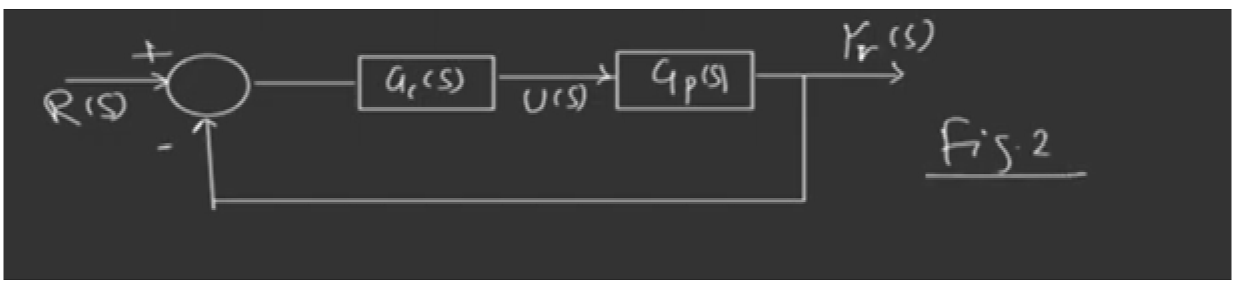

Now let's find the transfer function $Y_r(S)$ with respect to $R(S)$. From figure two, we can write $Y_r(S) = G_p(S)\, G_c(S)\, E(S)$. And also we have $E(S) = R(S) - Y_r(S)$. So that we can write $Y_r(S) = G_p(S)\, G_c(S)(R(S) - Y_r(S) = G_p(S)\, G_c(S)R(S) - G_p(S)\, G_c(S)Y_r(S)$ .And after rearranging the equations, we can write $Y_r(S) + G_p(S)G_c(S)Y_r(S) = G_p(S)\, G_c(S)R(S)$ Further we can write, we can take $Y_r(S)$ common from the left hand side, $Y_r(S)[1 + G_p(S)G_c(S)] = G_p(S)\, G_c(S)R(S)$. So from this we can write

$$\frac{Y_r(S)}{R(S)} = \frac{G_p(S)G_c(S)}{1 + G_p(S)G_c(S)}$$

So this is basically the closed loop transfer function which yields the relation between $Y_r(S)$ and

$R(S)$. This is basically the main transfer function because in this transfer function we are going to track $R(S)$. So let's denote this equation number one. So first transfer function we have found, now we will go to the second transfer function which is with respect to the effect of disturbance on one output.

So now let $Y_d(S)$ be the response with the disturbance $M_d(S)$. So we have to consider one input at a time, so the other input will assume to be zero with $R(S) = 0$ and $W(S) = 0$. And in this situation let's draw the closed loop diagram. This is our summing point and this is $R(S) = 0$ we assumed and controller block would be there. This is $G_c(S)$ and in this case we will assume disturbance which is $M_d(S)$ and this is $U(S)$ and the summed output going to the plant which is $G_p(S)$ and the output we are denoting here $Y_d(S)$ and also we are assuming unity feedback with $W(S) = 0$.

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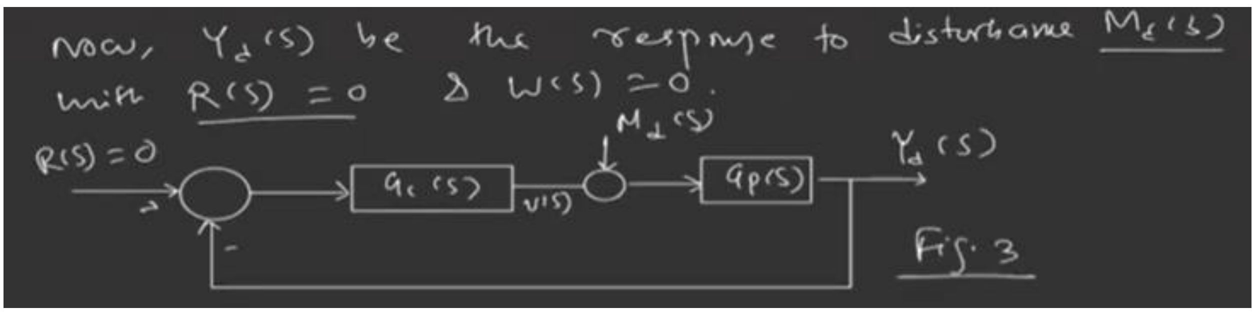

So this is the closed loop transfer function when $M_d(S)$ only acting, other inputs are assumed to be zero. And let's denote this figure number three. From figure three, we can write $Y_d(S) = G_p(S)\,[M_d(S) + G_c(S)\,E(S)]$ and if you multiply the factors $G_p(S)M_d(S) + G_p(S)G_c(S)\,E(S)$. If you notice here, basically $E(S) = -Y_d(S)$ because $R(S) = 0$ here because actually we know $E(S) = R(S) - Y_d(S)$.

So here $R(S)$ is zero search only $Y_d(S)$. Now if you substitute this expression in this equation we will get $Y_d(S) = G_p(S)\,M_d(S) - G_p(S)\,G_c(S)\,Y_d(S)$. After rearranging the equation we can write $Y_d(S) + G_p(S)\,G_c(S)\,Y_d(S) = G_p(S)\,M_d(S)$ and from this expression we can take common $Y_d(S)\big[1 + G_p(S)\,G_c(S)\big] = G_p(S)\,M_d(S)$ and further we can write

$$\frac{Y_d(S)}{M_d(S)} = \frac{G_p(S)}{1 + G_p(S)\,G_c(S)}$$

So this is the second equation which relates between the disturbance and output signal. So how the disturbance is going to affect the output, output means the attitude of the aircraft.

Now we'll get the final transfer function with the effect of noise $W(s)$. So let $Y_w(S)$ be the response with the measurement noise on $W(S)$. So here we'll assume $R(S) = 0$ and $M_d(S) = 0$. So in this condition let's draw the figure. So this is our summing point, this is the reference signal which is $R(S) = 0$ and this is the error signal we are producing.

So this is your $E(S)$ and the controller is $G_c(S)$ and we can have a summing point here. So here $M_d(S) = 0$ We have already assumed and this response is going to the controller which is $G_p(S)$ and the output from this we are denoting $Y_w(S)$ and here we are assuming there is a summing point where $W(S)$ is acting and this is my feedback loop. This is plus and this is negative feedback.

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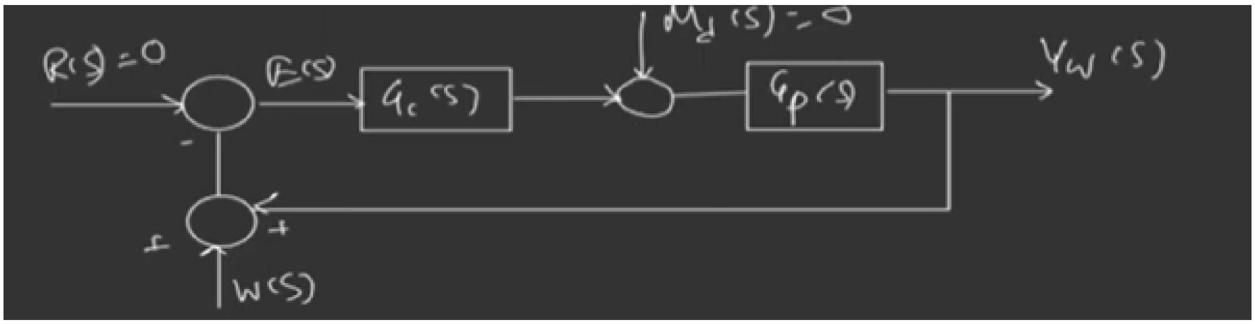

And from this figure four we can write $Y_w(S) = G_p(S)\, G_c(S)\, E(S)$. But here actually since we are assuming measurement noise in the system and we can write $E(S) = R(S) - (Y_w(S) + W(S)$. But here we already assumed that $R(S) = 0$ so we can write the error is basically

$$E(S) = -(Y_w(S) + W(S))$$

Now if we substitute this expression in the previous equation we can get

$$Y_w(S) = -G_p(S)\, G_c(S)\, (Y_w(S) + W(S))$$

$$Y_w(S) = -G_p(S)\, G_c(S) Y_w(S) - G_p(S)\, G_c(S) W(S)$$

If you rearrange the equation we can write

$$Y_w(S) + G_p(S)\, G_c(S) Y_w(S) = -G_p(S)\, G_c(S) W(S)$$

If you take $Y_w(S)$ common from the left hand side we can write

$$Y_w(S)\left[1 + G_p(S)\, G_c(S)\right] = -G_p(S)\, G_c(S) W(S)$$

We can write further

$$\frac{Y_w(S)}{W(S)} = \frac{-G_p(S)\, G_c(S)}{1 + G_p(S)\, G_c(S)}$$

So this is the transfer function which relates between the output and measurement noise in the system.

So let me define this equation number three. If you notice very carefully in this three transfer function the denominator part in all equations are the same. If you see the first equation this is our denominator part, the second equation is the denominator, the third equation also is the denominator. So the denominator part remains the same. Now this is a very very important observation how we can see the system with the defined inputs disturbance noise reference signal.

Now since the system is linear we can combine the effects. So let us write the combined response to $R(S)$, $M_d(S)$, and $W(S)$ is the sum. So let us assume $Y(S)$ is the total response. We can write $Y(S) = Y_r(S) + Y_d(S) + Y_w(S)$ so basically we can write $Y_r(S)$ equal to this whole transfer function into $R(S)$. Similarly $Y_d(S)$ we can write the whole transfer function with $M_d(S)$. Similarly the $Y_w(S)$ equal to whole transfer function into $W(S)$.

So let me write

$$\frac{G_p(S)\,G_c(S)}{1 + G_p(S)\,G_c(S)}\,R(S) + \frac{G_p(S)}{1 + G_p(S)G_c(S)}\,M_d(S) - \frac{G_p(S)G_c(S)}{1 + G_p(S)G_c(S)}\,W(S)$$

and let's write this equation number four. So in this equation many things we are going to observe, these are very very important equations. Our main motivation is if our control objective $Y_r(S)$ matches with $R(S)$ or we can say $\theta(S)$ matches with $\theta_d(S)$ or $Y(t)$ should match with $Y_d(t)$. This is our main motivation because R is basically the desired signal or reference signal for the system.

So this is the main motivation in the control system: how we can come up with our desired signal with the application of control. But if you look in equation four $G_p(S)$ is the plant. In the plant we can't do anything. Plant is coming from the natural motion of the system, natural dynamics.

So we can't play anything on this transfer function. This is basically the main plant transfer function so we can't do anything here. And the other transfer function $G_c(S)$ is the controller. So a controller, basically it can be an artificial device or some human being. Some kind of mathematical formulation we are doing here which basically the control engineers do.

So here we can say this is up to the control engineer. So we can modify it. We can change the different control algorithms. It is up to us. So now if we can design some control algorithm where $|G_c(S)| \to \infty$, is very large because in control basically there are a lot of gains. We'll be talking later but the control algorithm we designed is very big. Let me write that the control is large.

So in this case if the magnitude of $\left|G_c(S)\,G_p(S)\right| \gg 1$ Then what do you get? Then we can say the first term in equation four, this is going to be one because this term is bigger than one, this term then both will be cancelled out one you can ignore here. So this term and this term will cancel out, so we can say in this condition we get $Y_r(S) \rightarrow Y(S)$ but that's a problem. Let's not go to the problem again now. So now if it is very large again, let's do the second term, if it is very large $G_c(S)$ compared to $G_p(S)$ so this term we can say goes to zero. So it means even the disturbance is there in the system $Y(S)$ can track $R(S)$ because with this assumption the disturbance acting on the system can be neglected in this condition.

This is very important takeaway from this discussion. So first if you can design our control algorithm in such a way that this condition will fulfill and then we can get from this discussion $\frac{G_p(S)\, G_c(S)}{1+G_p(S)\, G_c(S)} \rightarrow 1$ and $\frac{G_p(S)}{1+G_p(S)\, G_c(S)} \rightarrow 0$. So this is a very good thing if we design control in such a way that both these conditions can be satisfied, so we can get perfect tracking. But in this condition there's a problem on the noise, so with the above condition $Y_w(S) = -W(S)$ because if you look at this equation it's going to be one. What we have done here if you notice here this is what we have done, so with this condition this is the case, so if this arises we can't get perfect tracking because $W(S)$ also in this equation. So this is the compromise we face as a control system engineer.

So this problem can be tackled if you come up with some frequency analysis like Nyquist or Bode plot, those things will be talked about later because noise is basically high frequency signals . So this thing we can tackle while we'll be designing the control algorithm based on in the frequency domain so that using that concept we can tackle this part as well but this is the main motivation if you can do at least we can get our tracking. But this can be done with the help of some navigation or navigation algorithms or some frequency analyst design based control, we can tackle this problem as well and this is the main motivation of designing control algorithms so that we can get our tracking algorithm which means we can get our desired attitude angles with the help of control algorithm which should be designed very carefully so that we can get this kind of objectives.

Let's stop it here. Next lecture we'll be discussing how we can come up with some typical control algorithms and how we can get our desired response in the closed loop control system and we'll be designing all this algorithm for the aircraft autopilot design. How we can control the orientation of the body of the aircraft. We'll continue from the next lecture. Thank you very much.