Introduction to Aircraft Control System

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Week – 03

Lecture – 12

Feedback Control Design for Attitude Dynamics

In this lecture, we will be discussing how we can find the transfer function of the aircraft attitude dynamics. In this direction, we will be finding two different transfer functions. One is the angular displacement with respect to total torque or moment acting on the system. And another is the angular displacement with respect to the total disturbance acting on the system. From this transfer function, we will discuss the concept of poles and zeros in the system from the transfer function. Then we'll move to the closed loop control system.

In the closed loop control system, what are the different subsystems connected in the loop and how can we fulfill our mission objective. We'll discuss them briefly. In this lecture, we'll be talking on how we can design a control algorithm for an aircraft and how we can come up with the closed loop control system diagram, where we'll be considering the disturbance and noise in the system and how we can mitigate over time. Let's go back to our previous lecture, the equation we had, the aircraft attitude motion in one axis, we had $I\ddot{\theta} = M + M_{dis}$ or we can write M_d, M disturbance.

Here θ is the angle we would like to control, M is the total control torque given by the controller and M_{dis} is the disturbance coming from the environment. If you apply the Laplace transform, let me write this equation number one, taking Laplace transform, yields

$$I\,S^2\,\theta(S) = M(S) + M_d(S)$$

In terms of the control variables $y(t)$ and $u(t)$, if we write in that form, we can write in terms of $Y(S) = \theta(S)$ and $u(S) = M(S)$. Let's write this equation number two. Here the initial condition is assumed to be zero, so we have assumed the other initial condition is zero.

So now equation four we can write, equation two yields

$$Y(S) = \frac{1}{IS^2}\left(U(S) + M_d(S)\right)$$

And as you know the transfer function basically relates between the input and output and also the transfer function is generally defined for the SISO (single input and single output) system. But if you notice this equation number three, there are two inputs $U(S)$ and $M_d(S)$ but one output $Y(S)$. But if you look they are actually acting separately, we can say $U(S)$ coming from the controller and $M_d(S)$ coming from the disturbance, external perturbations maybe. Now if you write the ratio that

$$G(S) = \left.\frac{Y(S)}{U(S)}\right|_{M_d(S)=0} = \left.\frac{Y(S)}{M_d(S)}\right|_{U(S)=0} = \frac{1}{IS^2}$$

basically the transfer function if we denote and $Y(S)$, how $Y(S)$ going to reflect it with respect to $U(S)$ when $M_d(S) = 0$. So this is basically the relation between the output and input when one other input is zero. So in both the cases if you notice the transfer function remains the same.

So $G(S) = \frac{1}{IS^2}$. So for both the cases it remains the same. So now we can write since there are two inputs. We actually have two plant transfer functions but they are actually the same . One from the control input to the output and the other from the disturbance to output. So we can write we have two inputs, one from controller to output, another one is the disturbance to output.

So you can see that equation I mean four equation, two and three are the same for both the cases. Hence we can write $Y(S) = G(S)(U(S) + M_d\ (S))$ So here this is the procedure transfer function for this particular system but if you have any general system for example any other LTI system and we are having one input and one output for general. So this is what I'm going to do is the concept of pole and zero. For the LTI system we can write the transfer function. We can write

$$G(S) = \frac{a_0 S^m + a_1 S^{m-1} + \ldots + a_m}{S^n + b_1 S^{n-1} + \ldots + b_n}$$

One more thing,here actually if the system is proper , let's remember $m \leq n$. It means the polynomial degree for the denominator should be higher or equal than the polynomial degree in the numerator. Let me write this equation number four. Here a_i and b_i are the real coefficients of $G(S)$ and the equation four can be factored into we can write

$$G(S) = K \frac{\prod_{i=1}^{m}(S - Z_i)}{\prod_{i=1}^{n}(S - P_i)}$$

So here we can write this polynomial numerator, polynomial we can factor in this form and the denominator polynomial we can factor in this form.

So basically from this concept will be coming up the concept of pole and zero. So here Z_i are the roots of $a_0 S^m + a_1 S^{m-1} + \ldots + a_m = 0$ and actually these roots are called the zeros of $G(S)$ and

P_i are the roots of $S_n + b_1 S^{n-1} + ... + b_n = 0$ and these roots are called poles of $G(S)$. Now let's design the feedback control system for the attitude dynamics , attitude motion. Why feedback? Because if you consider ,we have controller let's assume $G_c(S)$ with a controller which will be giving the input to the plant $U(t)$ of the transfer function for plant $G_p(S)$ and this is my output $y(t)$ which is nothing but $\theta(t)$ but if you want to track your $y(t)$ to some desired $y_d(t) = \theta_d(t)$ some desired angles, so how will make $y(t) = \theta(t) = y_d(t)$ So for that we need to come up with some kind of form of error which is $e(t) = y_d - y \rightarrow 0$, so it means $e(t) \rightarrow y_d \rightarrow y_t$.

So this is how we can design the control system but in the open loop structure we can't come up with the error function because the output variable should be available at the controller so that i can find the error, but in this open loop function it's not possible, so we need to consider the feedback control system where we'll have sensor on board and sensor will measure the current value of the attitude and this current value will go to some kind of summing point where we'll find the error and that error will go into the controller and control provide the necessary control to the system.So that's why we need to design the feedback control system where our plant is $Y(S) = G_p(S)(U(S) + N_d(S))$ so this is our system and for this system we'll be designing controller. So our mission objective is $y(t)$ follow a reference or a desired signal, let us denote $r(t)$ which is desired attitude that we would like to maintain for the aircraft $\theta_d(t)$ and the laplace transform of $r(t)$ we can write $R(S) = \theta_d(S)$. This is what we would like aircraft should follow this attitude angle with the application of control so our control input must correct error $e(t) = \theta_d(t) - \theta(t) = r(t) - y(t)$ because $\theta(t)$ we denoted as $y(t)$ and $\theta_d(t)$ we denoted as $r(t)$. So objective is $e(t) \rightarrow 0$ as $t \rightarrow \infty$ since we need to come up with some motivation.

Let's look at the closed loop control system which will provide the desired angle with the application of control. So now let's have a control plug here we will provide the desired torque to the system $G_c(S)$ the controller transfer function and this is the output from this controller, let's denote $U(S)$ and there is disturbance also in the system $N_d(S)$ and the sound is going to the plant $G_p(S)$ which is basically $\frac{1}{IS^2}$ the plant and this is plus this is plus and the output from this is $Y(S)$.

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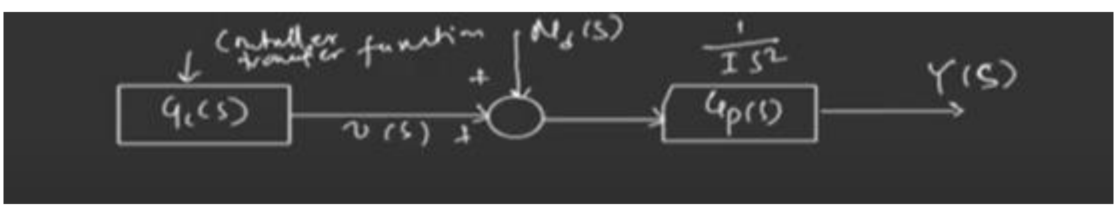

Let's assume we have an onboard sensor on the aircraft with sensing the current orientation of the body. This is my $H(S)$, this is we can say sensor transfer function and we are having desired output $y_d(t)$ or we can say $\theta_d(S)$ or we can also write $Y_d(S)$ in laplace domain. This is the forward loop and we are having the feedback from this sensor going to the summing point , this is negative and this is positive and from this block diagram this is my error $e(t)$ which is basically the difference between $Y(S)$ and $Y_d(S)$ or i can say $E(S)$ in laplace domain. but in practically due to the sensor measurement noise,sometimes the signal we are getting through this sensor is not perfect, so there can be noise in the system in the major noise measurement ,so we can have another summing point here which is basically entering the measurement noise to the system.

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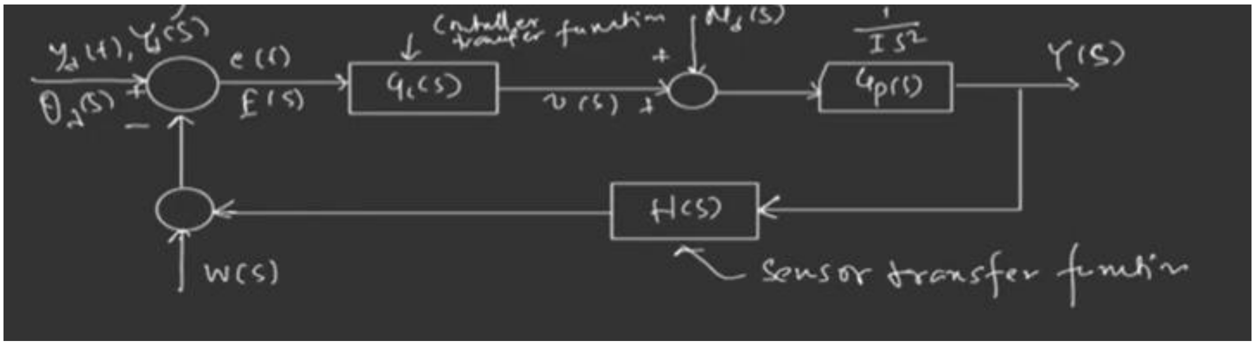

Let's denote $W(S)$ So let me write here what this actually is. For any real sensor, the measurement is not perfect and this is corrupted by the noise $W(t)$ (laplace of $W(t), W(S)$) . So hence the sensor does not provide the actual output $y(t)$,instead it provides $y(t) + \omega(t)$, so this is actually the feedback signal going to this summing point, so if i say this is my $Y(S)$ so this is plus and plus, so at this terminal the input to the summing block is basically $Y(S) + W(S)$ .Hence, the error signal goes to the controller part we can say $e(t) = r(t) - y(t) + \omega(t)$. This is the error signal actually goes to the system the controller block not the actual $y(t)$ or $Y(S)$. This is the most critical situation we can handle. In practical also there are error in the system than in the measurements.

So I can say this is not the true error which is basically $y(t) = y_d(t) - y(t) = r(t) - y(t)$ But instead of this, we are actually practically giving this error signal to the plant or to the controller. So now here one more assumption we have to consider , in figure one the sensor transfer function $H(S) = 1$, for the time being we are assuming the sensor transfer function is one. so once the $H(S) = 1$,we call this kind of system unity feedback system because if it is $H(S) = 1$, then it is just a straight line, there is no transfer function in the loop. So we can say this is just feedback and also if you noticed from figure one that there are three external inputs to the system and one is reference input which is nothing but $R(S)$.

So if you see this block diagram there are three different external inputs to the system, one is reference input, another one is measurement noise which is $W(S)$ and another one is the disturbance which is $M_d(S)$. But as per the transfer function concept this basically relates between the single input and single output, but we are having three inputs, so we need to calculate individually the effect of inputs on the output.So we need to find three different types of function from the system, one is $\frac{Y(S)}{R(S)}$, $\frac{Y(S)}{\omega(S)}$, $\frac{Y(S)}{M_d(S)}$ and we will see how the response changes with respect to the individual inputs and since the system is linear ,the combined effect on the output is just the sum of the individuals effects.

I hope this is clear to you. So we'll find three different transfer function with respect to $R(S)$ or $W(S)$ and $M_d(S)$ and we'll just sum the effect and we'll see how the system is going to behave. Let's stop it here, we'll continue from the next lecture on how we'll find the transfer function with respect to different inputs on the output and we'll see the combined effect of how it is changing the system and how the controller is going to be affected on the system. Also noise and disturbance we'll look at separately.