Introduction to Aircraft Control System

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Week – 03

Lecture – 11

Linearization of Aircraft Kinematic and Moment Equations

In this lecture, we'll be revisiting the nonlinear equation of motion of the aircraft rotational dynamics or attitude dynamics. Then we'll apply the linearization technique to the system based on the assumption of small angular displacement and the inertia matrix is assumed to only have the principal moment of inertia. After applying this assumption, we can come up with the linear form which relates between the angular displacement and the applied torque and moment or moments and disturbance acting on the system. This equation will help us to find the transfer function of the attitude dynamics of the aircraft. In this lecture, we'll be discussing how we can design the autopilot of an aircraft. So we'll be starting with the attitude motion of the aircraft, how we can control the orientation of the body.

So let's assume we have an aircraft system, and we would like to control the orientation of the body. Suppose this is my aircraft and these are the axes, for example, x , y and z. And we would like to maintain the orientation of this body with respect to some frame, maybe inertial frame. And so here, since we are going to discuss orientation control, so mostly we'll be using the moment equation and kinematic equation.

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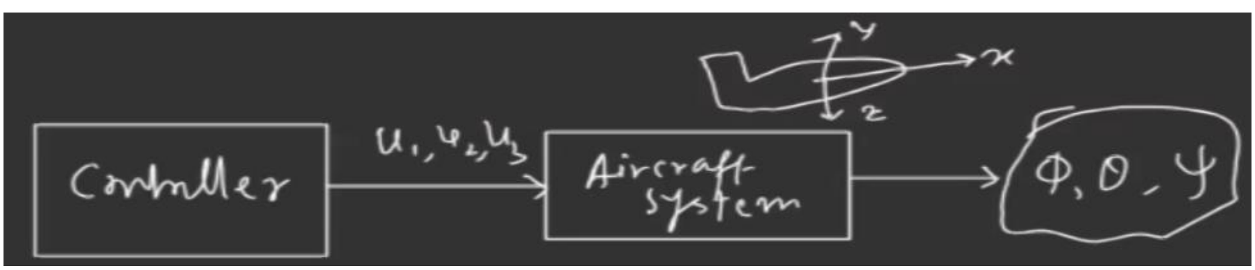

So we'll be having this equation very soon. And we would like to control the orientation of the body, ϕ , θ *and* ψ . So for controlling the orientation, we need to apply some control to the system.

So we'll have some control, u_1, u_2, u_3 . So this control is going to control $\phi, \theta\ \psi$ respectively three angles, so three controls we'll be using for the system.

This is the controller. So controller, we need to design in such a way that we'll get our desired response, $\phi, \theta\ \psi$. So this is the objective of this lecture. And next few lectures will be involved in this part, how we can control the orientation of the aircraft and how we can design autopilot, how we can come up with the logic to maintain the desired accuracy of the system. So with this motivation we'll be moving.

Now, let's write the moment equation. We are not going to derive how the equation came because that is a long part. In this course, mostly we'll be designing the control. So let me write the equation of motion of the aircraft, the moment equation. The moment equation of the aircraft, we can write l is the total moment acting to control the pitch axis of the aircraft.

$$l = \dot{p} I_x - \dot{r}\, I_{xz} + qr\left(I_z - I_y\right) - I_{xz} pq$$

$$m = \dot{q}\, Iy + rp(I_x - I_z) + I_{xz}(p^2 - r^2)$$

$$n = -I_{xz}\dot{p} + I_z\dot{r} + pq(I_y - I_x) + I_{xz} qr$$

l is the total moment acting about the x axis and m is the total moment acting about the y axis. and n is the total moment acting along the z axis. So here p is the roll rate, so let me define here, p q r are rates of x, y, z and I_x, I_y, I_z principal moment of inertia along x, y, z and l ,m ,n total moments acting along x, y, and z. So this total moment can be your torque moment and also disturbance in other part of the system.

So the torque is basically also disturbance but it's the favorable disturbance. This is going to help our system to regulate to our desired orientation. And here I_{xz} , off diagonal term in the moment of inertia matrix. I think we are done with the terms. Now let me define, this is equation number one.

And also we need the kinematic equation because we are going to control the orientation of the body. So also we need to construct the kinematic equation. And the kinematic equation, we can write

$$\dot{\phi} = p + q sin\phi\ tan\,\theta + r\ cos\phi\ tan\theta$$

$$\dot{\theta} = q cos\phi - r\ sin\phi$$

$$\dot{\psi} = q sec\theta\ sin\,\phi + r\ sec\,\theta\ cos\,\psi$$

Let's define this equation number two. So here θ, ϕ, ψ is the angle of the axis.

So now we'll be using this equation, how we can design autopilot for this dynamic system. So here as I said the external moment, let me say something more on this equation. Now this equation is a highly non-linear equation because of the coupling terms. Suppose here p^2 , r^2 , this is the square term or coupling term exists.

And also in the kinematic equation, the $q\ sin\phi\ tan\theta\ r\ cos\ \phi\ tan\ \theta$, these are the coupling terms. And due to which the system becomes non-linear, so the system will not follow the principle of superposition. Now the external moments, (l , m , n) have two components,. As I said it can have the torque as well as the perturbations. L, m , n be the control moment which is given by the controller and L_{dis} , $M_{dis}, and\ N_{dis}$ be the disturbance or perturbations into the system.

Now based on this, we can write L equal to, we can write $l = L + L_{dis}$ and small $m = M + M_{dis}$ and your $n = N + N_{dis}$. And also let's assume the moment of inertia has only the principle moments of inertia. That means we are assuming the off-diagonal terms to be zero, so

$$I = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}$$

The off-diagonal terms assumed to be zero. Now also we'll have another assumption, let's assume the small angle and rates approximation.

So in this case, for small angle we can write $Sin\ \phi \equiv \phi$ and $Cos\ \phi \equiv 1$ and $Sin\ \theta \equiv \theta$ and $Cos\ \theta \equiv 1$, $Sin\ \psi \equiv \psi$ and $Cos\ \psi \equiv 1$. In this condition, based on the above assumption, the equation number two, the kinematic equation gives

$$\dot{\phi} = p + q\phi\ \theta + r\theta$$

$$\dot{\theta} = q - r\phi$$

$$\dot{\psi} = q\phi + r$$

We can also write due to the small angle and rate assumption, the products between angles and angular rates can be neglected.

So $\phi\theta$, $r\theta$, $r\phi$, $q\phi$ These terms we can neglect in equation three. Based on this assumption, equation three can be written as $\dot{\phi} = p$ and $\dot{\theta} = q$ and $\dot{\psi} = r$. So if you notice this equation come up in some nice form but we need more treatment on the other moment equation also. So let's write this as equation number four. Now for small angles and rates approximations, the equation number one can be written as $L + L_{dis} = \dot{p}\ I_x$ because other terms we can ignore in our moment equation.

As I_{xz} assumed to be zero in the off-diagonal terms and these terms we can neglect because of the coupling between the small angle and rates , this is the multiplication of the angular rates terms which are assumed to be zero. So we can neglect these terms. So from the first equation in moment equation we can write this expression and this is how it is coming that is also it's clear because we have assumed that the total moment acting in the axis $L + L_{dis} = \dot{p}\, I_x$ L is the disturbance and L is the torque moment which is given by the controller.And similarly for the y and z direction, we can write $M + M_{dis} = \dot{q}\, I_y$ and $N + N_{dis} = \dot{r}\, I_z$. Let me define this equation number five. If you notice from equation four $\dot{\phi} = P$ right, this expression so $\dot{p} = \ddot{\phi}$ so in place of P we can write ϕ and similarly in place of $\dot{q}$ we can write $\ddot{\theta}$, in place of $\dot{r}$ we can write $\ddot{\psi}$.

So now using equations four and five we can write

$$I_x\, \ddot{\phi} = L + L_{dis}$$

$$I_y\, \ddot{\theta} = M + M_{dis}$$

$$I_z\, \ddot{\psi} = N + N_{dis}$$

We can write equation number six. Now if you notice the conclusion from equation six we can write that equations are decoupled and have the same form. Therefore if you can take one equation and if you can design the control for that particular equation then it will be the same technique also we can apply for all other equations. So let's take one equation because the concept will be the same for the other equation as well, the same concept we can apply for the rest of the systems $\ddot{\phi}$ and $\ddot{\psi}$. So let's consider only one axis at a time to be controlled and $\theta\ (t)$ t to be controlled. So in this direction let's assume $I_y\, \ddot{\theta} = M + M_{dis}$. This is the equation we are going to consider for designing control for it. For this particular equation we'll be spending around five to six lectures on how we can design a controller to control θ.

For this, instead of I_y let's write I only ,for simplification $I\ddot{\theta} = M + M_{dis}$. Now this is our main equation to be regulated with design control. The quantity here that we are interested to control is the attitude orientation angle θ. θ is the only angle we want to design the control which is generally called the plant output and in that case we can write the plant output of $y(t) = \theta(t)$. So for easy access of the systems instead of writing $\theta(t)$ we would like to control θ so that's why we are writing this $y(t)$ is the variable to be controlled and the quantity that can be used to effect the attitude is the control torque which is generally the control input or plant input and which can be labeled as $u(t) = M$, the same thing we are trying to connect to the control perspective $y(t) = \theta(t)$.

So if you write in block diagram form, so this is our plant which is basically $I\, \ddot{\theta} = M + M_{dis}$ so here this is the total moment coming to the system $M + M_{dis}$, so here basically I have a summing point which is M_{dis} coming from the external perturbations from the outside, it may be disturbance and it is coming from the controller which is a $u = M$ and the output from this system we can write $y(t) = \theta(t)$, so this is how we can design in block diagram form.

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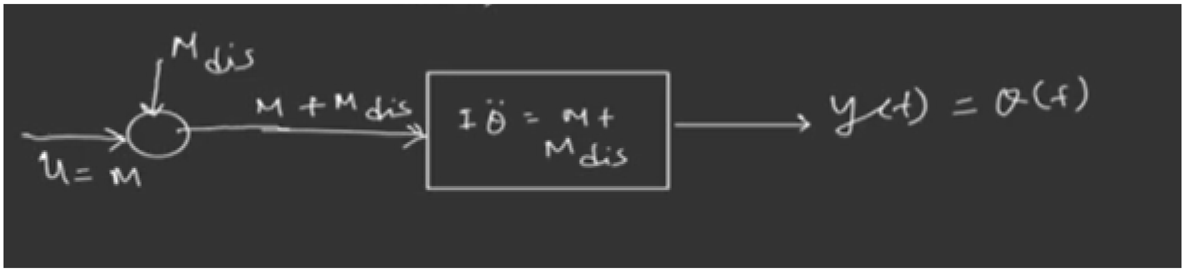

Now we have to design a controller u which will regulate the system and which will help us to maintain our desired attitude of the system. So now let me write a note here why for this particular system we have to assume the initial condition zero. So assumption on initial conditions ,we are not interested in the response of the output to non-zero initial conditions, this is the first assumption and that is a valid also that is we already discussed why we should not consider and second is even if you consider initial condition non-zero ,this non-zero initial condition will disappear to zero provided the control system is designed to be asymptotically stabilizing. Third, in this particular contour synthesis, we are interested in how the system responds to an input which will affect how the control system is designed, that is our main purpose. So that's why we will set the initial condition. Therefore we assume the initial conditions to be zero. The other reason also we have discussed in the previous lecture is why you should not consider the initial conditions to be zero since we'll be dealing with the causal system which is if the time is less than zero, then the initial condition will assume to be zero. So this thing you have already discussed extensively, let's stop it here. We'll continue from the next lecture on how we'll come up with the transfer function for this particular system and we'll try to connect with the controls. Thank you.