Introduction to Airbreathing Propulsion
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Lecture - 09
Review of Compressible Flows (Contd.,)

So let us continue the discussion on compressible flow. We have started with one dimensional steady compressible flow equations and then from there we moved to the normal shock relation and then towards the end of the previous discussion, we have actually looked at when there is a flow is isentropic, what happens to the stagnation properties and when the flow is actually across a normal shock wave, then what happens to the stagnation properties. So that is what we have already discussed. Now moving ahead, we will talk about now the oblique shock.

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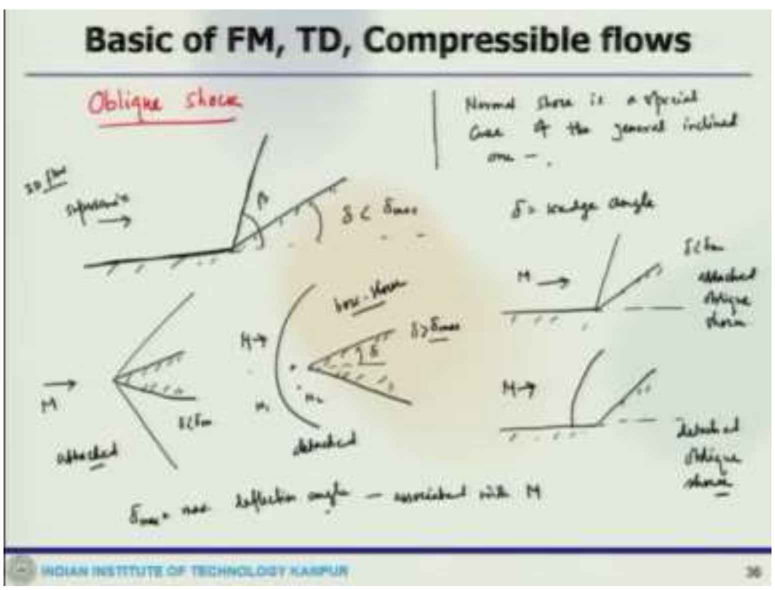

So this is a kind of shock when we looked at the normal shock. Our shock was always perpendicular to the flow field but when you talk about oblique shock. So one can always say that normal shock is a special case of the general inclined one, because when the shock is inclined to an object, let us say in two dimensional flow, then that is what I mean obviously the flow has to be supersonic and for example let us say if you have a kind of base like this and then it goes like that.

So there would be inclined shock here. This is the incline shock. There is an angle to that; this is called beta and there would be some angle which is delta max. So let us say when this is a kind of, this is how the flow comes in. This is in wedge shape kind of geometry and this is in two dimensional flow. So when the flow comes in, a plane attached shockwave may occur here and this is with some angle beta and flow Mach number and the wedge angle together.

So this is the delta is so called the wedge angle. So this kind of shock is called the oblique shock and it is generally inclined one and if you think about when we say that normal shock is a special case, then that is a perpendicular to the flow field, obviously. So when a supersonic flow encounters a concave corner with an angle delta, there are two possibilities of attached or detached shock like we can have. So this flow is also supersonic here.

Now I can have, let us say configuration like this. Let us say, like this and the flow here is inclined there, then we can have a shock like this. This is the one which is called attached. So this is another example of oblique shock, which is attached or we can have another configuration like this, when the shock would be like this. Still, the Mach number incoming flow is like this. This is called the detached.

So that means in the supersonic flow, you have this kind of object sitting in the flow field, when it is supersonic, then either you can have this kind of attached shock or you can have detached shock, but these are not obviously perpendicular to the flow field. So these are all can be clubbed together and called as sort of an oblique shock. Now then, there could be some other possibilities like another possibility could be like this one, the wedge kind of situation where the shock could be like this.

Again the flow is supersonic or I could have a situation like this, when the flow could be like this. So this is again, one can say this is attached oblique shock. This is so called detached oblique shock. So these are the possibilities, which may occur there. Now in the deflection angle, there is a maximum deflection angle which is called the delta max. So delta max is the maximum deflection angle.

So there is always existent maximum deflection angle, which is obviously one can this is associated with Mach number. It is not that it is irrespective of the Mach number, because depending on the Mach number this deflection angle could be different. When the deflection angle, let us say delta max, exceeds then a detached shock forms which is a curved wave front. So this is called the bow like pattern or bow shock.

This is called bow shock or bow-like shock, which is formed. So in this case, that delta is greater than delta max; that means the deflection angle which is here this is the delta. The deflection angle is greater than delta max. So that time you can have this kind of pattern or when it is delta less than delta max, they will remain attached. So behind this bow shock or curved shock, so we can find all shock solutions.

So this is the upstream of it and this point would be the downstream of it. So we can find all possible shock solution associated with the initial Mach number M_1. So if this side is M_1 and we can find M_2 and all sort of things here. Now at the center a normal shock also exists, when there is a bow shock, there is a center could be a normal shock with subsonic flow result. As the wave front curves around the shock angle actually decreases continually with a resultant decrease in the shock strength.

So with this the strength also decreases. Eventually one reaches a point where supersonic flow exists after the shock run. Now this oblique shock, there are different kind of oblique shocks which are preferred in these all engineering applications compared to normal shock. One of that we can associate it with these oblique shock relations as we go on discussing about that, but there is a fact that combination of oblique shocks is more favorable for poor shock conditions.

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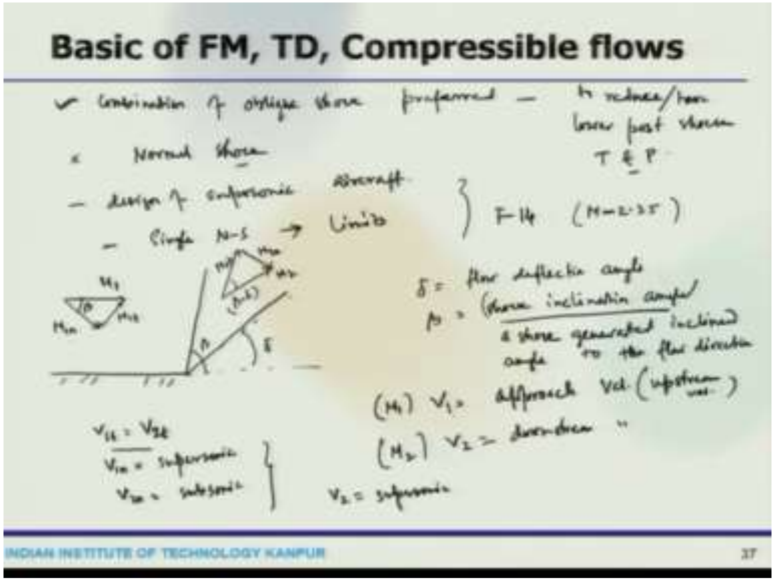

So combination of oblique shock, which is preferred because the post shock temperature to reduce or have lower post shock temperature and pressure. So these are important. Now this combination is more preferred compared to the normal shock and the reason being that across a normal shock flow will become always subsonic. So the change in properties are quite drastic, but as we go in the discussion we can see that behind the oblique shock, the flow still could be supersonic or subsonic depending on the situation.

So this combination one can think about. This is a good combination for design of supersonic aircraft, especially the intake portion, which are well set to compress the air into the combustion chamber while minimizing the thermodynamical losses. But early days this kind of supersonic jet engines which are designed using compression form of a single normal shock, but when you use a single normal shock, this has some limits of operation because behind the shock the flow is going to be immediately subsonic and which has drastic change in properties.

Now typical Mach number, so one can see these things, wedge kind of patterns are used in like F14 fighter aircraft which has maximum Mach number of roughly 2.35 or something like that. Now for analyzing oblique shock, let us take this wedge kind of pattern and let us say then this would be the shock sitting there; this is beta; this is delta. So the flow comes in here. This is the velocity triangle for that. So this is Mach number 1. This is β. This is M_{1n} M_{1t}

That is the tangential component of the Mach number. One is normal component; one is tangential component and then here we will have the velocity triangle like this. So this is M_2, this angle is $\beta - \delta$. This is M_{2t}. This is M_{2n}. So we take this figure where the delta is the flow deflection angle and beta is the shock inclination angle or other one can say the shock has the generated shock inclined to an angle beta to the flow direction.

One can say that or can be said that a shock generated inclined angle to the flow direction. So that is what one can say. Now the flow approaches with the velocity V_1. So this is the approach velocity or one can say upstream velocity ahead of the shock and it turned through an angle delta. It passes through the shock leaving with a velocity V_2. So this is the downstream velocity, downstream of the shock. So obviously the V_1 is associated with Mach number 1.

This is Mach number 2 and there will be an angle $\beta - \delta$, which is respect to the shock. The inlet and exit velocities can be decomposed in the normal component and the tangential component; where similarly we can have the velocity. So like the Mach number we can have the similarly velocity components, the tangential velocity components upstream and downstream are equal. So that means V_{1t} would be V_{2t}, which means V_{1n} is supersonic and V_{2n} is subsonic. But even that V_2 is supersonic at the downstream. So we can write this tangential component to be equal and using this Mach number relationship.

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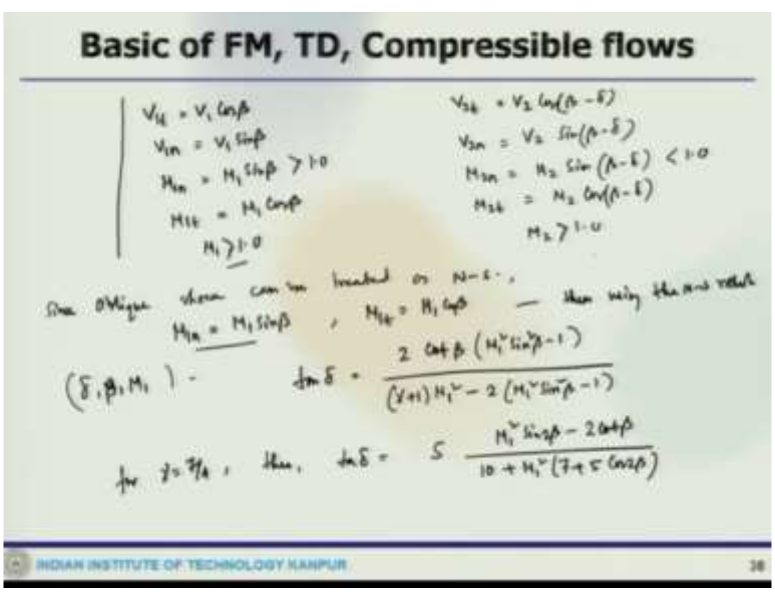

Now what one can write, now that

$$V_{1t} = V_1 \cos\beta$$

$$V_{1n} = V_1 \sin\beta$$

and then

$$M_{1n} = M_1 \sin\beta$$

which would be greater than 1 and you have one tangential component, which is

$$M_{1t} = M_1 \cos\beta$$

and M_1 is supersonic, so which is greater than 1. I mean using the velocity triangle one can easily write these. Similarly, at the downstream location

$$V_{2t} = V_2 \cos\beta - \delta$$

V2 normal component would be

$$V_{2n} = V_2 \sin\beta - \delta$$

So normal component of the Mach number would be

$$M_{2n} = M_2 \sin\beta - \delta$$

which is less than 1 and

$$M_{2t} = M_2 \cos\beta - \delta$$

and M_2 is also supersonic. Now since the oblique shock can be treated as a normal shock, where we have an upstream Mach number like now since oblique shock can be treated as normal shock, So then if we treat like that like these components here, we treat them as in normal shock and then we can use all, then using the normal shock relation what we can write; that we can write some relationship between δ, β, M_1 like that which gives us that

$$\tan\delta = \frac{2\cot\beta\,(M_1^2 sin^2\beta - 1)}{(\gamma + 1)M_1^2 - 2(M_1^2 sin^2\beta - 1)}$$

Let us say, if you consider a value of gamma which is 7/4, then

$$\tan\delta = \frac{M_1^2 sin^2\beta - 2\cot\beta}{10 + M_1^2(7 + 5cos2\beta)}$$

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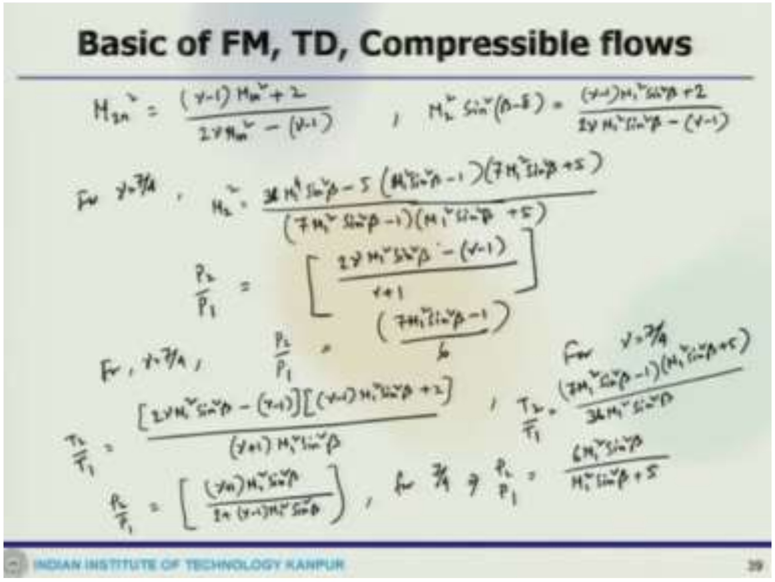

So we get, now what happens to the normal component. So the normal components would be

$$M_{2n}^2 = \frac{(\gamma - 1)M_{1n}^2 + 2}{2\gamma M_{1n}^2 - (\gamma - 1)}$$

Similarly, what we can write

$$M_2^2 sin^2(\beta - \delta) = \frac{(\gamma - 1)M_1^2 sin^2\beta + 2}{2\gamma M_1^2 sin^2\beta - (\gamma - 1)}$$

Now if we say gamma is 7/4, what we get

$$M_2^2 = \frac{36M_1^4 sin^2\beta - 5(M_1^2 sin^2\beta - 1)(7M_1^2 sin^2\beta + 5)}{(7M_1^2 sin^2\beta - 1)(M_1^2 sin^2\beta + 5)}$$

We get

$$\frac{p_2}{p_1} = \left[\frac{2\gamma M_1^2 sin^2\beta - (\gamma - 1)}{(\gamma + 1)}\right]$$

So when we use, for this case also for gamma 7/4 this becomes

$$\frac{p_2}{p_1} = \left[\frac{7M_1^2 sin^2\beta - 1}{6}\right]$$

Similarly,

$$\frac{T_2}{T_1} = \frac{[2\gamma M_1^2 sin^2\beta - (\gamma - 1)][(\gamma - 1)M_1^2 sin^2\beta + 2]}{(\gamma + 1)M_1^2 sin^2\beta}$$

Similarly, for gamma = 7/4, this becomes

$$\frac{T_2}{T_1} = \frac{[7M_1^2 sin^2\beta - 1][M_1^2 sin^2\beta + 5]}{36M_1^2 sin^2\beta}$$

Now the other quantity which would be also retained like the density ratio

$$\frac{\rho_2}{\rho_1} = \left[\frac{(\gamma + 1)M_1^2 sin^2\beta}{2 + (\gamma - 1)M_1^2 sin^2\beta}\right]$$

For gamma=7/4, This becomes

$$\frac{\rho_2}{\rho_1} = \left[\frac{6M_1^2 sin^2\beta}{M_1^2 sin^2\beta + 5}\right]$$

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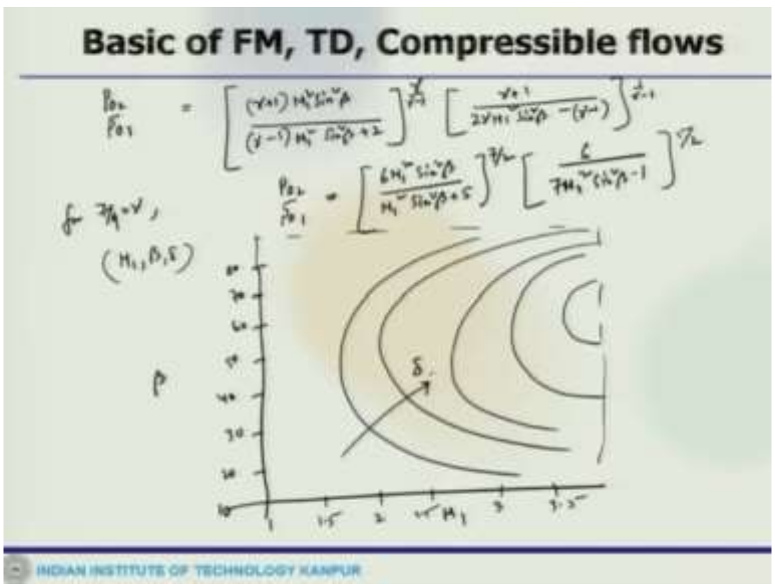

So we can also write down the relationship for the stagnation condition right

$$\frac{p_{02}}{p_{01}} = \left[\frac{(\gamma + 1)M_1^2 sin^2\beta}{(\gamma - 1)M_1^2 sin^2\beta + 2}\right]^{\frac{\gamma}{\gamma - 1}} \left[\frac{(\gamma + 1)}{2\gamma M_1^2 sin^2\beta - (\gamma - 1)}\right]^{\frac{1}{\gamma - 1}}$$

This one can find or look at the detailed of these calculations in any of the compressible flow book. So I am just giving it or going through these things quickly because these are all required, but not the part of detailed discussion of this course. So same thing for 7 by 4 gamma, you get

$$\frac{p_{02}}{p_{01}} = \left[\frac{6M_1^2 sin^2\beta}{M_1^2 sin^2\beta + 5}\right]^{\frac{7}{2}} \left[\frac{6}{7M_1^2 sin^2\beta - 1}\right]^{\frac{5}{2}}$$

So these are the different relationship between, I mean, temperature ratio, pressure ratio, density ratio and stagnation condition. So this actually one can have these relations or there could be a nice diagram of what can happen between M_1 beta delta using that where you can get a diagram of that kind. For example, if I draw it qualitatively, so this is M_1 and this side is beta which starts from 10, 20, 30, 40, 50, 60, 70, 80 and so on and this side 1, 1.5,2, 2.5, 3, 3.5 like that.

So you have kind of again as I said, so you get this kind of curve like that. So these are for different increasing delta and this is beta and this is M_1. So this is called the $M_1, (\delta - \beta - M)$ relationship plot, which is quite important because one can use this graph to find out the conditions for a particular Mach number of what could be the deflection angle, what could be the other angle.

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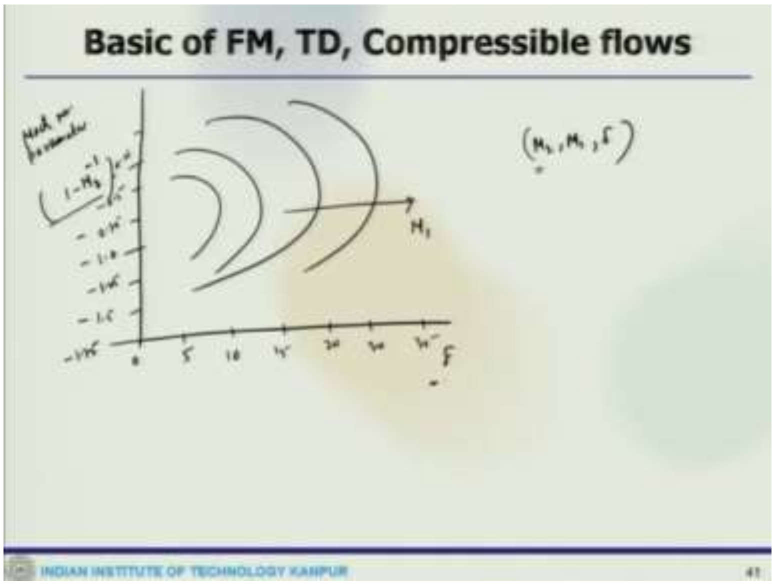

Similarly, you can have the another plot, which can also give you the idea about the downstream condition. So if I plot that which goes from 0, 5, 10, 15, 20, 30, 35 and so on; this is the delta and this side it would be $(1 - \frac{1}{M})$ So this is called Mach number parameter. So you get, this goes from -1.75, 1.5, 1.25, 1,75, 0.5, 0.25 like that. So qualitatively, you get curves like this. So these are again this direction is increasing M_1

So instead of using those complicated equations, one can use these two curve where when you have the upstream Mach number from there you see for a given value of delta or wedge angle you can find out the beta or rather for beta given beta and M_1 you can find out delta, either of them with that when you come to the downstream of the system, where you need to find out the Mach number at the back of these things like the downstream of the shock.

Then this is the Mach number parameter, which is not directly M_2 which is written as $1/M_2$ and then the qualitative plot looks like this. I mean I would suggest you guys should look at the book properly to see this diagram with proper numbers, because these are again, I mean, repeatedly I have been mentioning this. These are qualitative plot which I am trying to draw just to give you an idea, how they look like, what are these parameters all about and how one can find out the details.

Coming back to that, once you identify, so this is the upstream situation which you can get and once you find out either beta or delta depending on Mach number, using that here is a relationship between delta M_1 and M_2 So this is M_2, M_1 and delta relationship curve. So you can use that and actually find out the Mach number downstream of the shock. So this is how the oblique shock relations can be used, not necessarily one has to sort of mug up those or keep remembering those equations like this complicated equations.

This is not expected, I mean these are the handbooks or tables which are available for obliques of relations or this kind of plot or the graph which are very, very handy for designers to use this relationship to find out upstream and downstream properties and once you find out the Mach number, then rest of the things can be followed very easily and so that is what will pretty much want to just touch upon the discussion on oblique shock. We will stop here and continue on the other portion in the next class.