Introduction to Airbreathing Propulsion
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Lecture – 07
Review of Compressible Flows (Contd.,)

Okay so let us continue the discussion on compressible flow. So we are looking at the relationship between Mach number and the velocity and this is what we stopped at.

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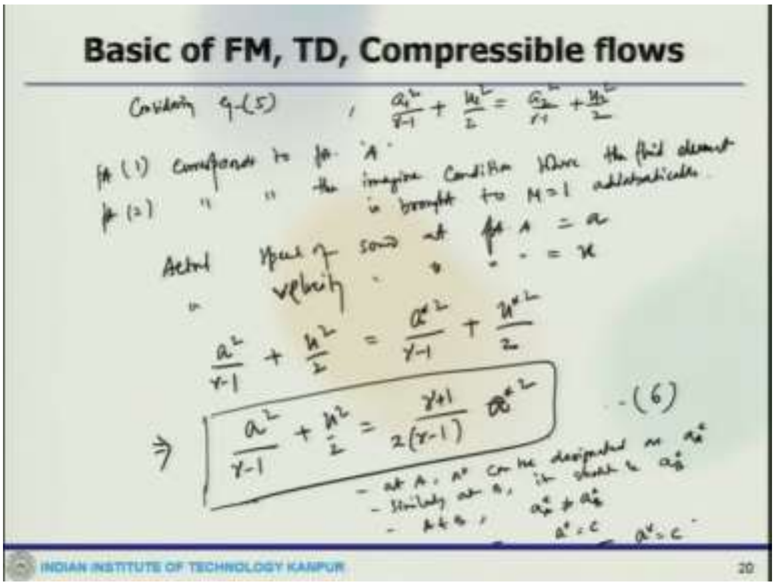

In the last session where you get the relationship between Mach number in the local velocity now just we derived that equation 6 but some points here to be noted which are important is that at A, a^* can be designated as a_A^* similarly at B it should be a_B^*. So, if the flow is not adiabatic between A and B then a star would not be same or if the flow is adiabatic everywhere then $a_A^* = C$ for isentropic flow also $a_B^* = C$.

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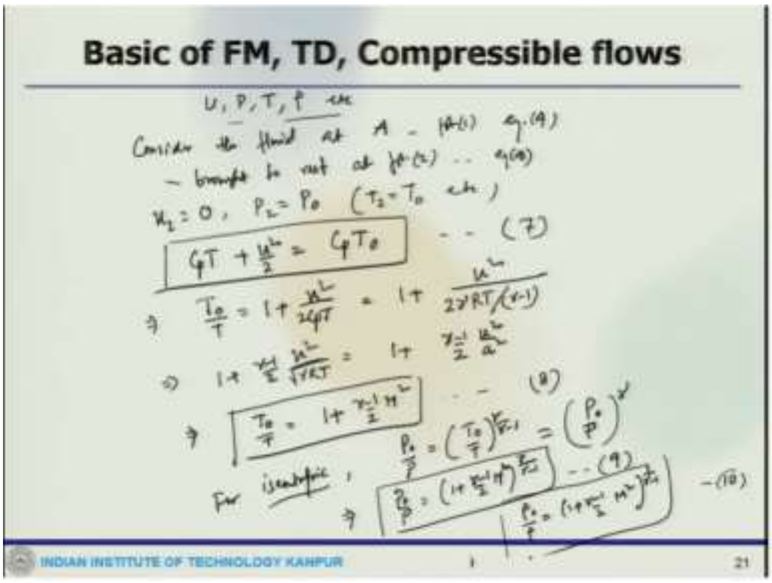

Now if we return to the stagnation conditions like for U, P, T density like that now let us say consider the fluid let us consider the fluid at A which is point (1) equation (4) and this fluid is brought to rest isentropically at so brought to rest at point (2) of equation. So, what will happen then $u_2 = 0, p_2 = p_0,\ T_2 = T_0$ or something like that. So, from equation (4) which is true for both adiabatic and hence isentropic flow we can write

$$C_p T + \frac{u^2}{2} = C_p T_0$$

So that is what you can write and you can further simplify

$$\frac{T_0}{T_1} = 1 + \frac{u^2}{2C_p T} = 1 + \frac{u^2}{2\gamma RT/(\gamma - 1)}$$

if we simplify bit further what we get

$$1 + \frac{(\gamma - 1)}{2}\frac{u^2}{\sqrt{\gamma RT}} = 1 + \frac{(\gamma - 1)}{2}\frac{u^2}{a^2}$$

So, this will have

$$\frac{T_0}{T_1} = 1 + \frac{(\gamma - 1)}{2} M^2$$

so that is equation number (8). Now if we say the process is isentropic then what we can write that

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma - 1}} = \left(\frac{\rho_0}{\rho}\right)^{\gamma}$$

so this is P, T ρ relationship for isentropic flow.

And once we put it back in equation 8 this gives another relationship for

$$\frac{p_0}{p} = \left(1 + \frac{(\gamma - 1)}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$

which is equation (9) and another one on density

$$\frac{\rho_0}{\rho} = \left(1 + \frac{(\gamma - 1)}{2} M^2\right)^{\frac{1}{\gamma - 1}}$$

which is equation (10). So, all these relationships we get.

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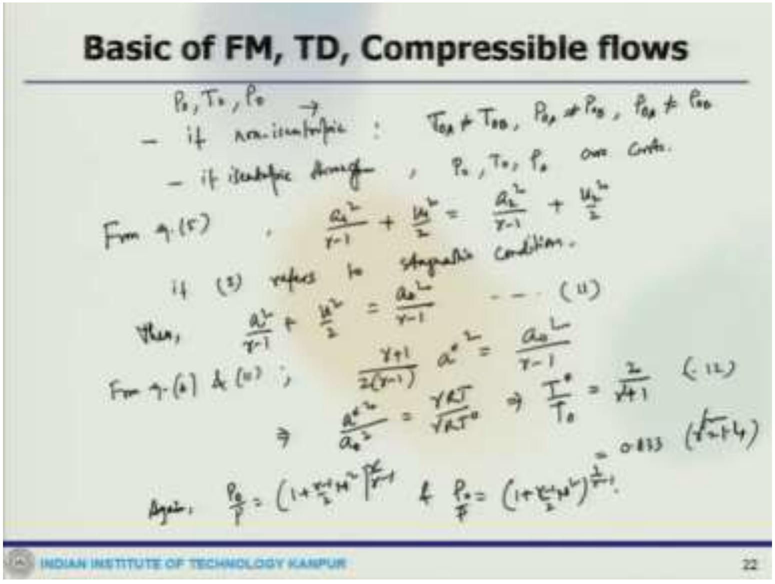

Now the actual flow field does not need to be isentropic to calculate p_0, T_0, ρ_0 etc because these are imagined and defined quantities if the flow field is non isentropic that is it if non isentropic that means not adiabatic and or irreversible then what will have happened $T_{0A} \neq T_{0B}$, $p_{0A} \neq p_{0B}$ and $\rho_{0A} \neq \rho_{0B}$. So, these properties are not going to be same.

Now if the flow field is isentropic throughout if let us say isentropic throughout then p_0, T_0, ρ_0 these are constants okay. So then from equation (5) what we can write

$$\frac{a_1^2}{(\gamma - 1)} + \frac{u_1^2}{2} = \frac{a_2^2}{(\gamma - 1)} + \frac{u_2^2}{2}$$

If let us say if 2 refers to stagnation or conditions then what we get

$$\frac{a^2}{(\gamma - 1)} + \frac{u^2}{2} = \frac{a_0^2}{(\gamma - 1)}$$

that is equation (11).

Now from equation (6) and (11) we get

$$\frac{\gamma+1}{2(\gamma-1)}+a^{*2}=\frac{a_0^2}{(\gamma-1)}$$

In the other way one can write

$$\frac{a^{*2}}{a_0^2}=\frac{\gamma RT^*}{\gamma RT_0}$$

which is

$$\frac{T^*}{T_0}=\frac{2}{\gamma+1}$$

so if we put γ=1.44 this would be roughly 0.833 equals to if we use for γ=1.44

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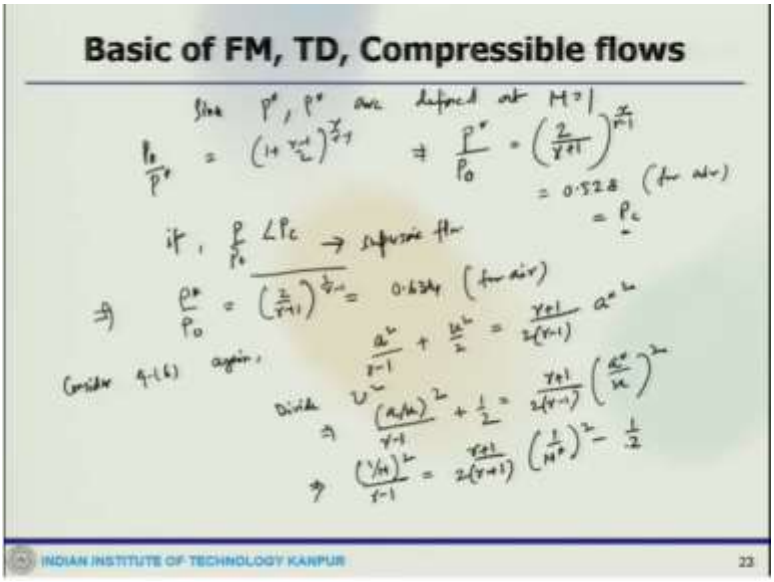

So, what we get since $p^*, \rho^*$are defined at M = 1 condition we get \

$$\frac{p_0}{p^*}=\left(1+\frac{(\gamma-1)}{2}\right)^{\frac{\gamma}{\gamma-1}}$$

So which means

$$\frac{p^*}{p_0}=\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

which would be 0.528 for air this also that critical pressure ratio now if P/P0 is less than critical pressure ratio then the flow is supersonic flow okay now that in the similar way we get for

$$\frac{\rho^*}{\rho_0}=\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}$$

which would be 0.634 for air.

So, these are the now finally we consider equation (6) again and what we can write is

$$\frac{a^2}{(\gamma-1)}+\frac{u^2}{2}=\frac{\gamma+1}{2(\gamma-1)}a^{*2}$$

Now it divides by u square so you can write

$$\frac{(\frac{a}{u})^2}{(\gamma-1)}+\frac{1}{2}=\frac{\gamma+1}{2(\gamma-1)}(\frac{a^*}{u})^2$$

So, which

$$\frac{(\frac{1}{M})^2}{(\gamma-1)}=\frac{\gamma+1}{2(\gamma-1)}(\frac{1}{M^*})^2-\frac{1}{2}$$

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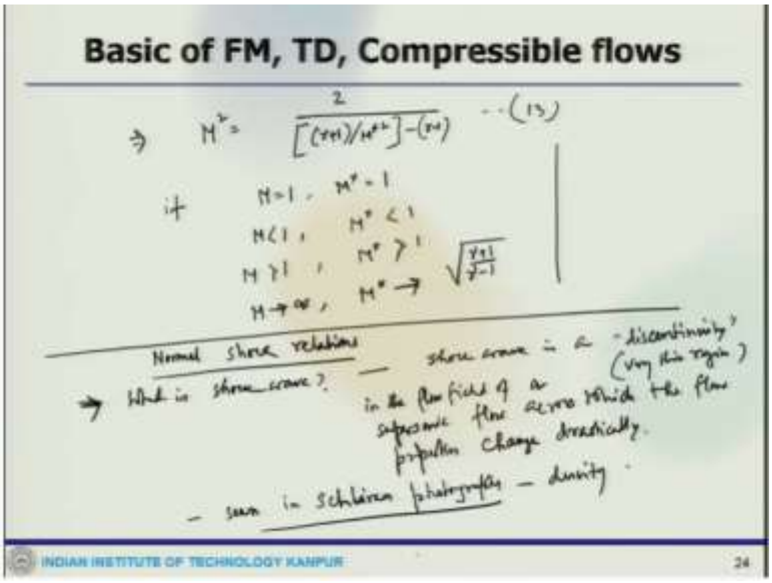

So this gives an fantastic relationship between M and M^* so this

$$M^2=\frac{2}{\left[\frac{\gamma+1}{M^{*2}}\right]-(\gamma-1)}$$

So this is an relationship that you get between M and M^*. Now if M = 1 then M^* also becomes 1 if M < 1 which is subsonic so M^* also less than 1 if even greater than 1 M^* also becomes greater than 1 if M tends to infinity M^* tends to $(\gamma+1)/(\gamma-1)$

So, these are the correlation that you can obtain from the M and M^* relationship and this is for 1D dimensional steady compressible flow you can derive. Now moving ahead so will other thing that we will talk about is the normal shock relations. Okay so the first question which comes to somebody mind what is shockwave? So that is an very pertinent question now a single way one can answer that shockwave is a discontinuity which is a of very thin region in the flow field of a supersonic flow across which the flow properties change drastically.

So means this is an discontinuity in the flow field which is of a very thin region order of 10^{-5} or 10^{-6} centimetres depends on the geometry and the condition in a supersonic flow across which the flow properties changes drastically. Now this can be seen or measured in schlieren photographs or images because of so it can be measured or seen in schlieren photograph because of density variation.

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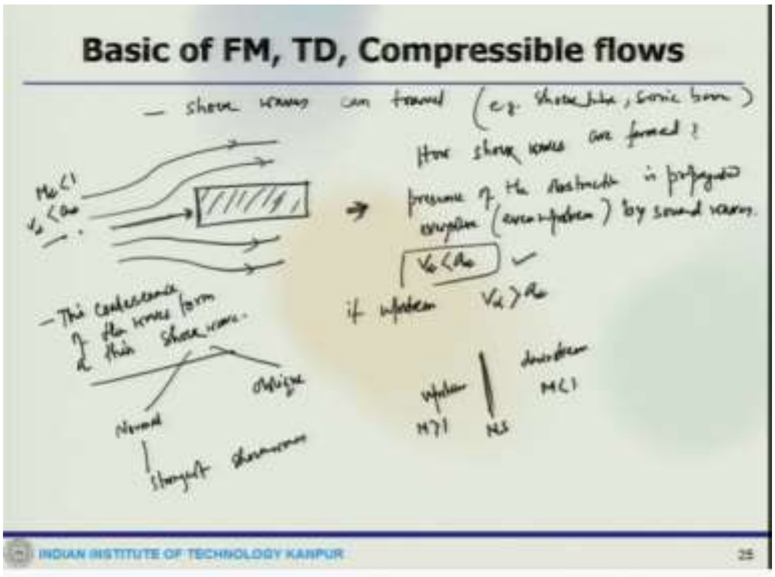

Now shock waves can travel so shock waves can travel for example shock tube, sonic boom these are the example almost explosive compression process. So, there is a dramatic increase in pressure now let us see how so you have a object inside the flow field. So, these are the pattern of the streamline let us say M < 0 less than a infinity so this is how we can define how shock waves are formed okay.

So, we consider a flat plate or rather this object this is mounted in a flow and the flow field consists of individual molecules some of which impact on the face of this let us say cylinder. Now in general there will be change in molecular energy and momentum due to impact. Now this change in momentum is seen as an obstruction by the molecules.

So, then the fluid particle will sense there is an obstruction sitting there and because of change in that momentum. Now the random motions of these molecules communicate this obstruction to the other region of the flow through collision that means with the through the molecular collision these presence of this object is being communicated to the other region. So, the presence of obstruction is propagated everywhere even upstream by sound waves.

So, this is very important that the presence of the obstruction is propagated everywhere even upstream by sound waves. Now this is for the situation if the incoming stream is subsonic like

$$V_\infty < a_\infty$$

then the sound waves can work upstream and convey the presence of this body to the fluid particle sitting here. So, from this to this position this information can be passed and the flow properties and the flow field change accordingly to accommodate these changes and that is why you can see the streamlines goes like that.

Now this is alright if the upstream condition is this but if the upstream condition is supersonic then what will happen this sound wave cannot propagate from this position to the upstream position and the information of this presence of this body cannot be passed to the point which is sitting in the upstream. So, this cannot no longer propagate upstream instead what will happen they tend to coalesce a short distance ahead of the body.

So, if the body is here around a short distance ahead of that they will coalesce and this coalescence of the waves form a thin shock wave okay. So, this is what happens when the flow field is supersonic and there is a body which is placed inside the flow field then the information of this presence of this body cannot be passed to the upstream and then this shock wave is found.

And there could be two types of shock waves one could be normal shock wave other one could be oblique. Now in normal shock wave the it is perpendicular to the flow field and this is very strong or rather strongest shock waves. Now flow behind the normal shock waves is also subsonic let us see if you have a normal shock here then this is upstream which is greater than 1 then this is downstream.

So, this would be subsonic so the upstream flow does not know about the presence of shock waves and the flow properties have to change abruptly across the shock wave to adjust this variation between upstream and downstream. So, this is what a shock wave is all about and how shock wave is formed and all these.

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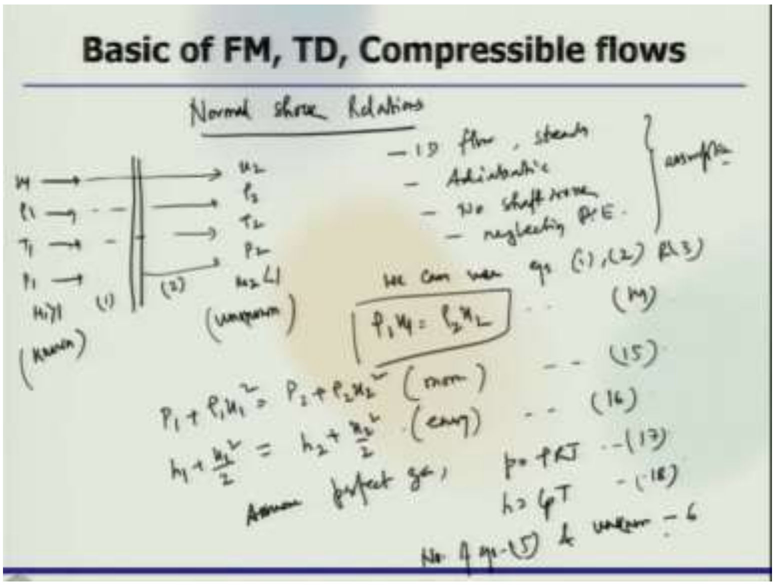

Now we will look at the normal shock relations. So normal shock relations will have this small shock let us say so $u_1, \rho_1, T_1, p_1, M_1$. So, this is stagnation 1 and this is u_2, ρ_2, T_2, p_2, and $M_2 < 1$ this is at point (2). So, these informations all are known and downstream information are sort of unknown now again the situation here is this is 1D flow steady adiabatic flow no shaft work and neglecting potential energy.

So, these are the some of the assumptions which are associated so that we can derive the simple equations now whatever we have done so that we can use derived equation 1, 2, and 3 directly. So, what we write

$$\rho_1 u_1 = \rho_2 u_2$$

that is form continuity. So, let us say equation (14) or mass conservation equation and we can write

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

which is momentum that is (15) and

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

which is energy that is (16). Now we assume perfect gas so that we can write

$$p = \rho RT$$

And

$$h = C_p T$$

So, what we have we have 1, 2, 3, 4, 5. So number of equations are 5 and unknowns are 6. So, what we will do?

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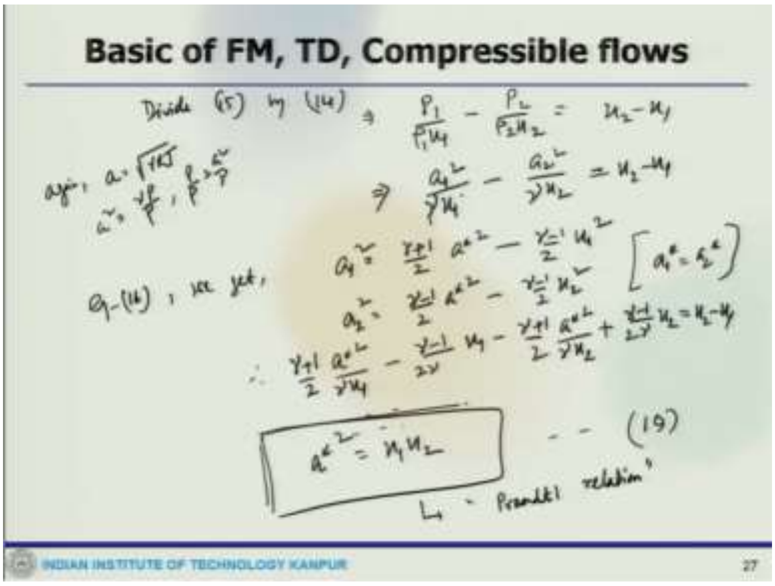

We divide (15)/(14) and what we get

$$\frac{p_1}{\rho_1 u_1} - \frac{p_2}{\rho_2 u_2} = u_2 - u_1$$

$$a = \sqrt{\gamma RT}$$

$$a^2 = \frac{\gamma p}{\rho} \text{ and } \frac{p}{\rho} = \frac{a^2}{\gamma}$$

So, if you use that this would becomes

$$\frac{a_1^2}{\gamma_1 u_1} - \frac{a_2^2}{\gamma_2 u_2} = u_2 - u_1$$

Now from equation (16) get

$$a_1^2 = \frac{\gamma+1}{2} a^{*2} - \frac{\gamma-1}{2} u_1^2$$

So, what you can write

$$a_2^2 = \frac{\gamma+1}{2} a^{*2} - \frac{\gamma-1}{2} u_2^2$$

this is happening because since the flow is adiabatic $a_1^* = a_2^*$

So, we can get

$$\frac{\gamma+1}{2}\frac{a^{*2}}{\gamma u_1} - \frac{\gamma-1}{2\gamma} u_1 - \frac{\gamma+1}{2}\frac{a^{*2}}{\gamma u_2} - \frac{\gamma-1}{2\gamma} u_1 = u_2 - u_1$$

So further if we do the little bit of mathematics what we get finally is

$$a^{*2} = u_1 u_2$$

So, this is equation number (19) and this relationship is known as or called as Prandtl relation. So, this is Prandtl relation okay so this is a very important relation for normal shock where you get this a star and the velocity upstream of the shock and the relationship between the both the upstream and downstream velocities. So, we can see how one can obtain this but we will continue that discussion in the next lecture so we will stop it here.