Introduction to Airbreathing Propulsion
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Lecture - 60
Radial Flow Turbine and Module Matching

So let us come towards the last part of this discussion we have done complete discussion theoretical discussion on the design on axial flow turbine. Now there are a couple of small components which are left one we will do a quick discussion on radial flow turbine. And then finally we will look at the compressor and turbine matching.

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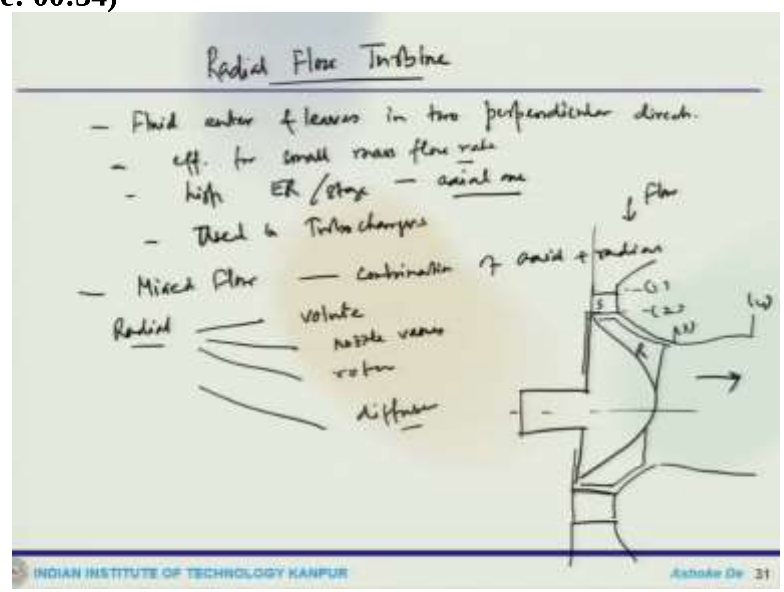

So this is what we are going to talk about on radial flow turbine. So, this is typically like your similar to your centrifugal compressor, where you have the fluid. Here fluid, enters and leaves in 2 perpendicular directions. So, the here also for radial turbine the flow enters radially and leaves accelerate close to axis of rotation. So, the turning of the flow takes place in rotor passage which is relatively long and narrow and for small mass flow rate the radial turbine is efficient for small mass flow rate and which is relatively also, this is better or other preferred compared to axial one.

So, this produces also high develops high pressure ratio per state. So, high expansion ratio par stage, this is compared to axial one but obviously when you talk about the radial one, the multi staging would be a problem. But with the axial one we can go for multi staging and can produce high expansion ratio. So, these are used in typically in used in turbo chargers for small gas turbines turbocharger for cars buses, trucks railway locomotives and diesel powered generator cryogenic and process expander.

Now also apart from the radial there could be another type which is called the mixed flow machine or mixed flow turbine. So, that is characterized is a combination so, this is a combination of axial plus radial type. So, radial flow turbine is composed so the now when you talk about radial one this has 4 elements. So like volute this we have already seen for centrifugal then the nozzle vanes rotor diffuser or sometimes this can be identified as diffuser so, flow enters the diffuser at a large radius and leaves in smaller radius.

So now we can look at the quick aero thermodynamics of the turbine, but let us see this a plot quickly. This is already we have so, this goes with them and then it goes like this. So, we have a some like that. Similarly, this side also we have and this is connected. So this is also connected. So this is the direction of the flow, and there are different station numbers. So, let us say this is 1, this is 2 this is rotor blade. This is the stator blade flow goes out like this. So, this is station 3. This is station 4 so, the these are the station number that we can use. And at the same time, we can see the TS diagram.

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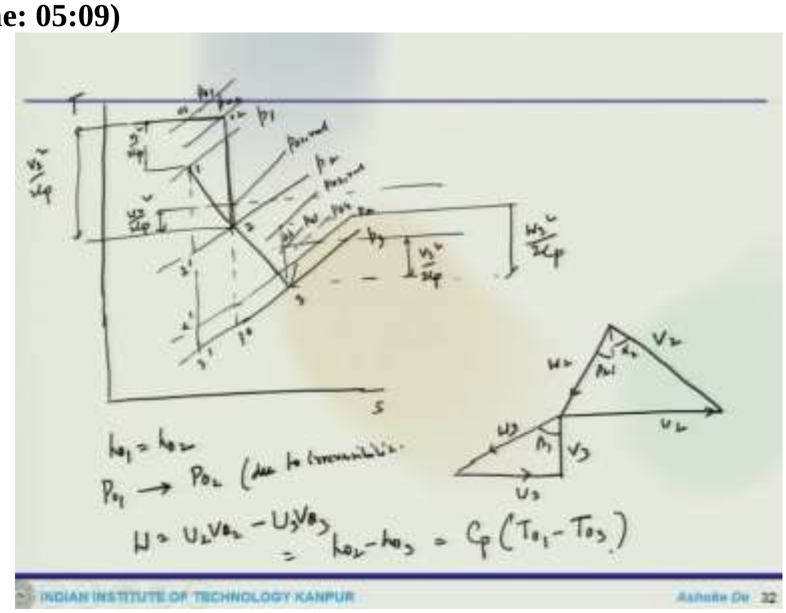

So the TS diagram so that our analysis would be much easier once we look at the TS diagram. So that is one from one it comes to 2. So, then 2 to 3. So this is P_1. Then you can have P_{01} P_{02} we can connect them from here. So let us this is P_{02} and this is P_{01}. So, these are the connecting lines, this is 1, this is 2, this is 3 and 3 to 4. So, this is P_3. So, this could be P_a, this is P_2 and then this is P_{02} relative. So, from one if I come down so, these are extended.

So, these are $2', 3', 4', 3''$ then you we could have from 3 there is a line which is called 3 relative then this is P_{03} then we can have P_{04}. So this is 02 01. So, this point is $\frac{V_1^2}{2C_p}$ this is $\frac{V_2^2}{2C_p}$. Now, this is 03. So, $\frac{V_3^2}{2C_p}$. Now, this is $\frac{W_3^2}{2C_p}$, this is $\frac{W_2^2}{2C_p}$. So, these are the things are velocity triangle if we draw. So, this is how we can draw it. So, that is W_2 α_2, β_2 V_2 U_2. So, this is U_3, β_3 V_3, W_3. So, we have the both the velocity triangle and the TS diagram.

Now, as no work done in the nozzle, so, we get $h_{01} = h_{02}$ now, the stagnation pressure drops from P_{01} to P_{02}, this is due to irreversibility. So, the specific work given by the Euler turbine equation

$$W = U_2 V_{\theta 2} - U_3 V_{\theta 3}$$

So, this is what we get and we can write this as same

$$h_{02} - h_{03} = C_p (T_{01} - T_{03})$$

So, now if the flow at the rotor inlet is radial.

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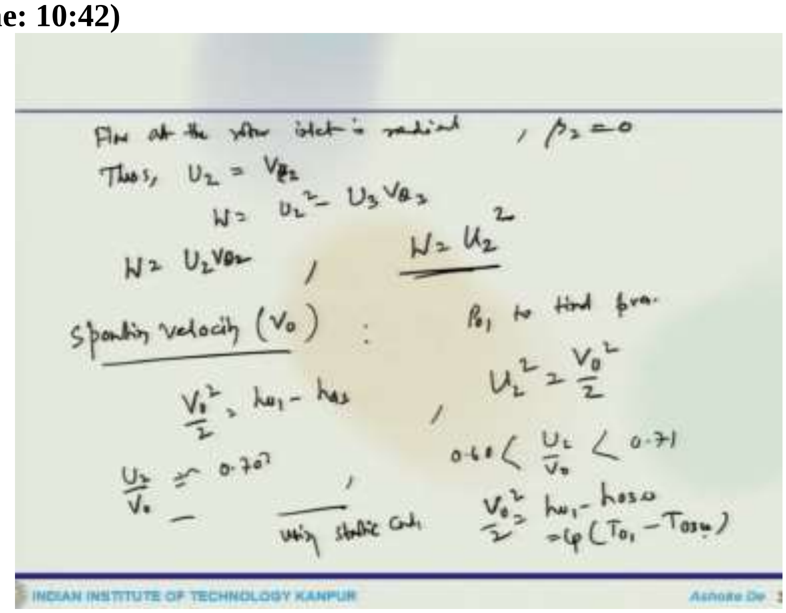

So, the flow at the rotor inlet is radial then the relative angle $\beta_2 = 0$ so, we get thus, we have

$$U_2 = V_{z2}$$

So, we get

$$W = {U_2}^2 - U_3 V_{\theta 3}$$

If the swirl velocity is 0 at the exit, then we get we write

$$W = U_2 V_{\theta 2}$$

and if the flow is radial at the inlet and accelerate outlet, so, also we get

$$W = {U_2}^2$$

So, now, we can look at the spouting velocity which is V_0.

So, this term spouting velocity originally came from the hydraulic turbine design practice is defined as the velocity that has associated kinetic energy equal to the isentropic enthalpy. So, this is associated kinetic energy associated kinetic energy equal to the isentropic enthalpy drop from turbine inlet stagnation pressure P_{01} to final pressure. The exhaust pressure here can have several interpretations depending upon whether the total or static conditions are used in the related efficiency definition and upon whether or not a diffuser included in the turbine.

Thus, if the flow is assume ideal, isentropic through the turbine with specific diffuser, then the specific work can be defined as

$$h_{01s} - h_{4s} = \frac{{V_0}^2}{2}$$

So, the velocity ratio what we get $\frac{U_2}{V_0}$ would be around 707 in practice this lies between 0.68 to 0.71. So, that is how it lies between when no diffuser is used and if the total condition are used instead of static, then we can using static condition you get

$$\frac{{V_0}^2}{2} = h_{01} - h_{03ss} = C_p(T_{01} - T_{03ss})$$

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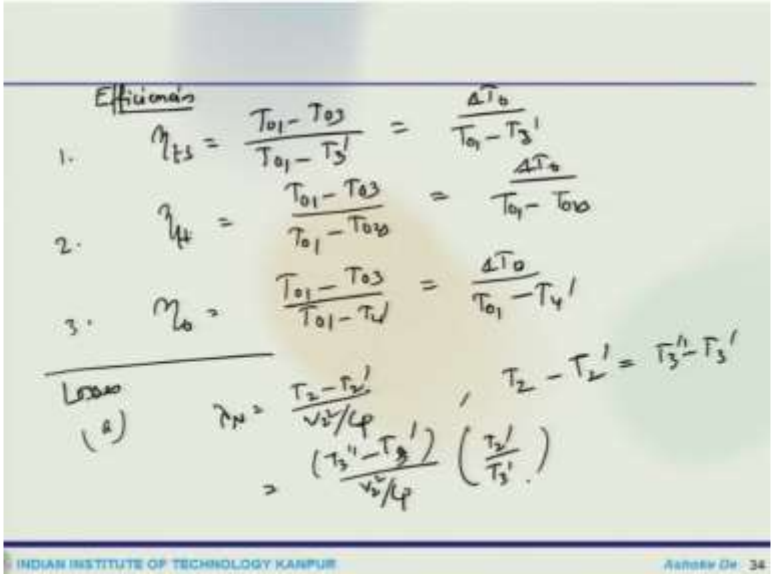

Similarly, there are efficiencies for radial flow turbine which can be defined one is that turbine total to static efficiency which will be

$$\eta_{ts} = \frac{T_{01} - T_{03}}{T_{01} - T_{03}{}'} = \frac{\Delta T_0}{T_{01} - T_{03}{}'}$$

these are all corresponding to that this diagram total to total efficiency this is

$$\eta_{tt} = \frac{T_{01} - T_{03}}{T_{01} - T_{03s}} = \frac{\Delta T_0}{T_{01} - T_{03s}}$$

So, the combined efficiency or the overall efficiency

$$\eta_0 = \frac{T_{01} - T_{03}}{T_{01} - T_4{}'} = \frac{\Delta T_0}{T_{01} - T_4{}'}$$

Now, same there are also losses like we can have nozzle coefficient losses. So, which is nozzle coefficient loss is defined as

$$\lambda_N = \frac{T_2 - T_2{}'}{\frac{V_2^2}{2C_p}}$$

assuming

$$T_2 - T_2{}' = T_3{}'' - T_3{}'$$

So, we can replace that

$$\lambda_N = \frac{T_3{}'' - T_3{}'}{\frac{V_2^2}{2C_p}}\left(\frac{T_2{}'}{T_3{}'}\right)$$

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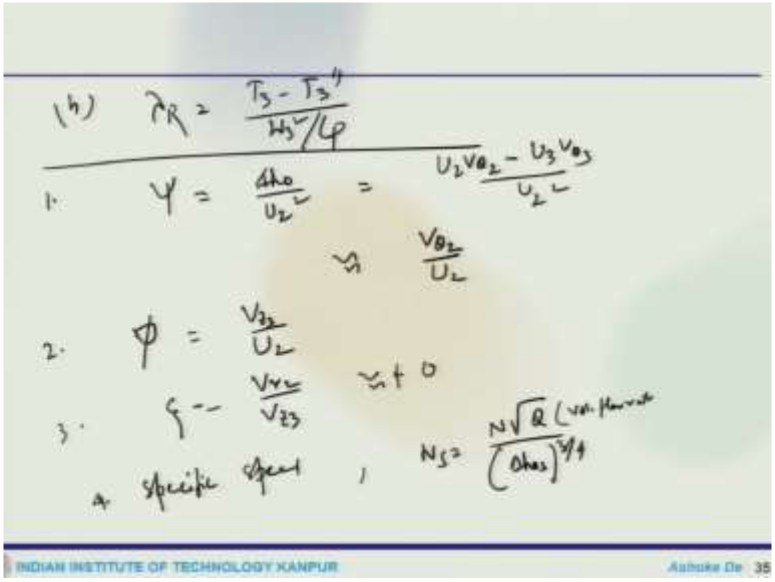

Now, rotor loss coefficients this could be also written as

$$\lambda_N = \frac{T_3 - T_3{}''}{\frac{W_3^2}{2C_p}}$$

So, this can be used. Now there are some dimensionless parameter one is stage loading, which is

$$\psi = \frac{\Delta h_0}{U_2^2}$$

So, these are similarly that we have defined for axial case and this one can show that this would be

$$\psi = \frac{U_2 V_{\theta 2} - U_3 V_{\theta 3}}{U_2^2} \approx \frac{V_{\theta 2}}{U_2}$$

there could be flow coefficient, which is the $\frac{V_{z3}}{U_2}$ rotor non dimensional velocity.

So, this is $\frac{V_{r2}}{V_{z3}}$, which would be roughly 1 and a specific feat was applied almost exclusively to incompressible flow machine and which is defined as

$$N_s = \frac{N\sqrt{Q}}{\Delta h_{0s}{}^{3/4}}$$

So, this is the volume flow rate this Q is the volume flow rate at the outlet.

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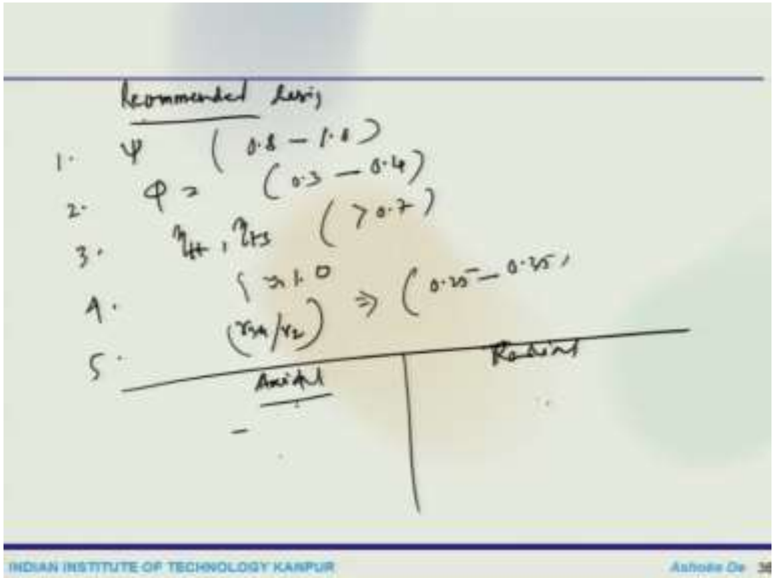

So, one can have some recommended design practice. So, based on the design first you need to calculate the stage loading which is would be range of 0.8 to 1, then you can design some flow coefficient which would be range of 0.2 to 4, then you have total to total and all the efficiency total to total, total to static efficiencies would be greater than 0.7, you have velocity ratio, which would be close to 1 then the ratio between hub to inlet radius would to be also range of 0.25 to 0.35.

And now, finally, if you compare both the axial and radial so, the axial quick comparison and radial the axial one is used for large engine, this is used for small engine this is for large this is for axial for a large mass flow rate this is for small mass flow rates, better efficiency, this has a lower efficiency, low pressure ratio per state the says high pressure ratio positive higher overall efficiency lower overall efficiency. Multistage is possible monitor staging is difficult very expensive it is cheap difficult to manufacture this is easy to manufacture. So, these are the some of the comparisons that one can think about between these.

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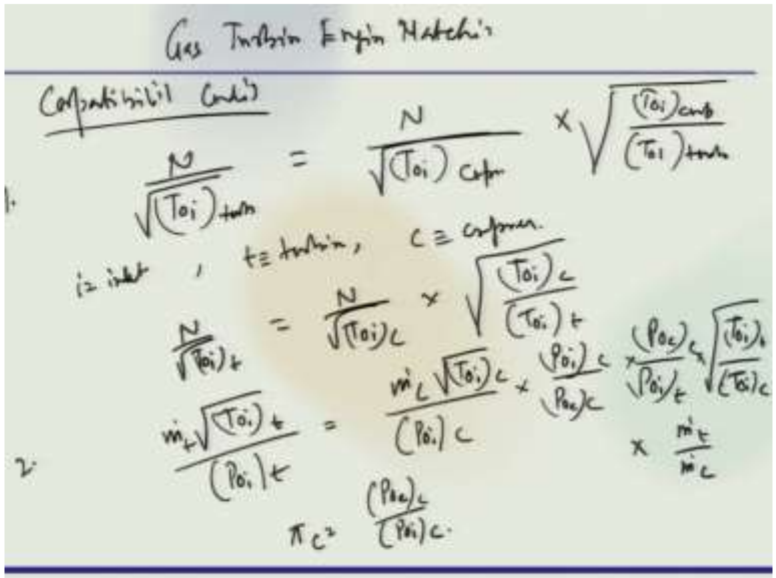

Now finally, we go to the gas turbine engine matching. So, this is like we have to take the first the compatibility condition compatibility conditions so, we have operating speed

$$\frac{N}{\sqrt{(T_{0i})_{turbine}}} = \frac{N}{\sqrt{(T_{0i})_{compressor}}} * \sqrt{\frac{(T_{0i})_{compressor}}{\sqrt{(T_{0i})_{turbine}}}}$$

where i stands for inlet and now we will use T for turbine c for compressor and then this could be return

$$\frac{N}{\sqrt{(T_{0i})_t}} = \frac{N}{\sqrt{(T_{0i})_c}} * \sqrt{\frac{(T_{0i})_c}{\sqrt{(T_{0i})_t}}}$$

So, this could be

$$\frac{\dot{m}_t\sqrt{(T_{0i})_t}}{(p_{0i})_t} = \frac{\dot{m}_c\sqrt{(T_{0i})_c}}{(p_{0i})_c} * \frac{(p_{0i})_c}{(p_{0e})_c} * \frac{(p_{0e})_c}{(p_{0i})_t} * \sqrt{\frac{(T_{0i})_t}{(T_{0i})_c}} * \frac{\dot{m}_t}{\dot{m}_c}$$

So, e stands for outlet the exit condition. Now, the compressor pressure ratio is

$$\pi_c = \frac{(p_{0e})_c}{(p_{0i})_c}$$

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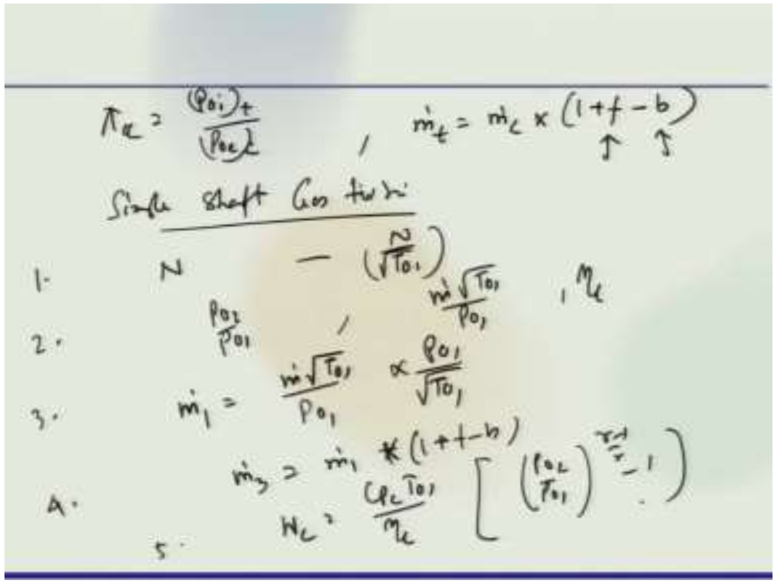

And combustion pressure issue

$$\pi_{cc} = \frac{(p_{0i})_t}{(p_{0e})_c}$$

conservation of mass which will get

$$\dot{m}_t = \dot{m}_c(1 + f - b)$$

f is fuel air ratio, this is bleed ratio bleed air ratio then we can have conservation of energy or power balance. Now, just to look at some single shaft gas turbine engine, so, these are the following steps you select a rotational speed N. So, that the from ambient conditions we can calculate

$$\frac{N}{\sqrt{T_{01}}}$$

who is defined speed lines, then you can compress a pressure ratio assume this P_{02}/P_{01}.

From the compressor map we can find the operating point and the mass flow rate parameter like

$$\frac{\dot{m}\sqrt{T_{01}}}{p_{01}}$$

and compressor efficiency, then the compressor mass flow rate is calculated by this relation

$$\dot{m}_1 = \frac{\dot{m}\sqrt{T_{01}}}{p_{01}} * \frac{p_{01}}{\sqrt{T_{01}}}$$

and then the turbine mass flow rate is calculated

$$\dot{m}_3 = \dot{m}_1 * (1 + f - b)$$

and we calculate the specific work of the compressor which is

$$W_c = \frac{C_{pc} T_{01}}{\eta_c}\left[\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

So, that is how you calculate. So, from the map you get all this.

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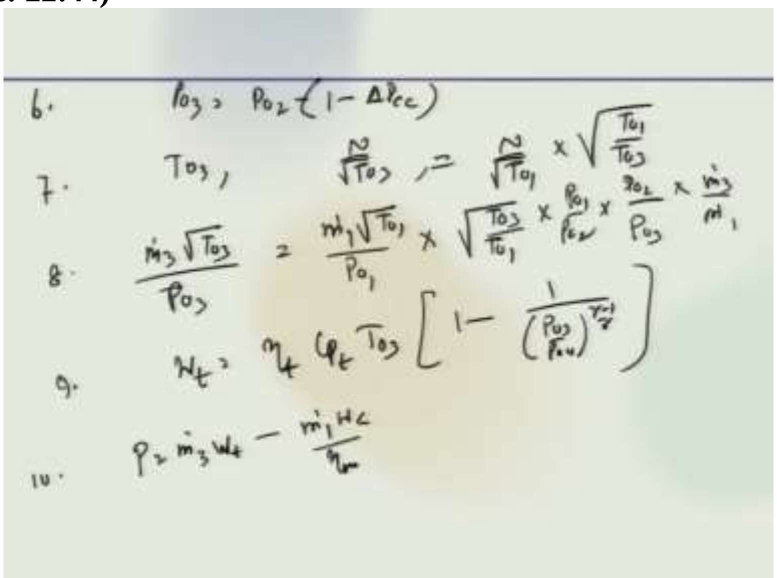

Then, you get the pressure loss in the combustion chamber which is

$$p_{03} = p_{02}(1 - \Delta p_{cc})$$

Then assume the turbine inlet temperature T_{03} we calculate

$$\frac{N}{\sqrt{T_{03}}} = \frac{N}{\sqrt{T_{01}}} * \sqrt{\frac{T_{01}}{T_{03}}}$$

so, this will determine the speed line for the turbine map. And then finally turbine mass flow rate is determined

$$\frac{\dot{m}_3\sqrt{T_{03}}}{p_{03}} = \frac{\dot{m}_1\sqrt{T_{01}}}{p_{01}} * \sqrt{\frac{T_{03}}{T_{01}}} * \frac{p_{01}}{p_{02}} * \frac{p_{02}}{p_{03}} * \frac{\dot{m}_3}{\dot{m}_1}$$

From these mass flow parameter and operating speed line turbine efficiency can be calculated from the under specific work which is output for the turbine would be at

$$W_t = \eta_t C_{pt} T_{03}\left[1 - \frac{1}{\left(\frac{p_{03}}{p_{04}}\right)^{\frac{\gamma-1}{\gamma}}}\right]$$

And then finally, the power would be

$$P = \dot{m}_3 W_t - \frac{\dot{m}_1 W_c}{\eta_m}$$

So, a check has to be performed, so that you can. So, this is for a single stage single shaft engine one can do then no one can have off design of free turbine engine also free turbine engines which is like a gas generator and similarly, you can find out this mapping and then this is used for these things. So, the free turbine or the gas generator also one can do like.

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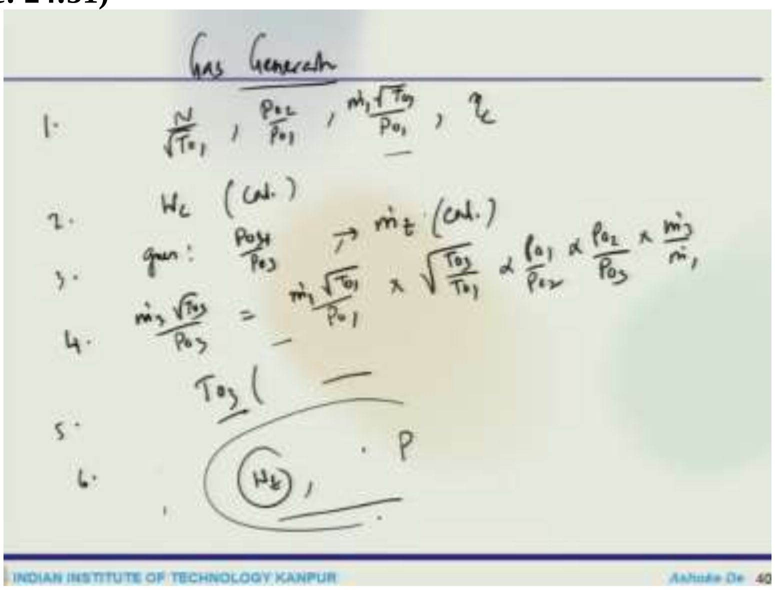

So, these are the steps first you can select some speed line on the compressor map and choose any point on line. So on the compressor parameter like $\frac{N}{\sqrt{T_{01}}}$, these are one has to choose $\frac{p_{02}}{p_{01}}$, $\frac{\dot{m}_1\sqrt{T_{01}}}{p_{01}}$ and it η_c. So, once you choose that, then you finally calculate the compressor specific output this is calculated then you get some value of pressure ratio for turbine and the value of the turbine mass flow rate through the turbine m dot t.

This would be calculated with this case value and the turbine mass flow rate is once it is determined already we have shown the equation then the turbine mass flow rate which is kind of mapped with the compressor that can be done which is exposed like

$$\frac{\dot{m}_3\sqrt{T_{03}}}{p_{03}} = \frac{\dot{m}_1\sqrt{T_{01}}}{p_{01}} * \sqrt{\frac{T_{03}}{T_{01}}} * \frac{p_{01}}{p_{02}} * \frac{p_{02}}{p_{03}} * \frac{\dot{m}_3}{\dot{m}_1}$$

So, then next step, so turbine inlet temperature could be defined.

So, this would be can be defined and which the speed line which the speed line or turbine efficiency that can be determined then finally, we can match the temperature ratio and then finally, find out the other possible steps to follow which we have already shown, find the output of the turbine power and like we have done and the total power. So, essentially this is how the mapping is done.

You first assume certain parameters, then go to compressor map, pick up some values then go to turbine calculate everything then match the mass flow rate, because the mass flow rate has to be properly mapped. And then finally, look at the inlet temperature of what has been assumed that is correct or not and once this converges, you find out the power and the turbine. So, that is how one do the mapping for the compressor and turbine and this is also very, important.

The compressor and turbine mapping because these are important for the design of the gas turbine unit so, if you look at that, we have first initially looked at the cycle analysis. Then when you move to the rotating component like turbo machineries, we have looked at axial compressor axial centrifugal compressor axial turbine to some discussion on regular turbine and finally, this mapping the whole point is that turbine produces the power and some of the power.

Which is produced by the turbine, it goes to operate the fan and the compressor in the gas turbine engine. And that is why this mapping is very, important because they have particular characteristics of operational line. And these should restrict their operation. So that pretty much concludes the theoretical discussion of all these airbreathing propulsion. Thank you very much.