Introduction to Airbreathing Propulsion
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Lecture - 58
Axial Turbine (contd.,)

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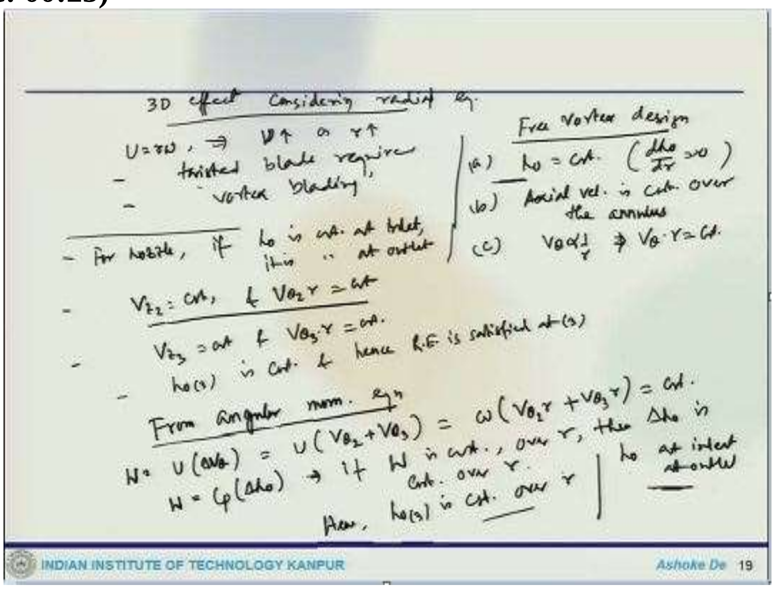

So, let us continue to discuss about this radial free vortex design of the axial flow turbine blades and this is what we have been talking so far that. These are the conditions and we have reached to this when looking at the angular momentum equation.

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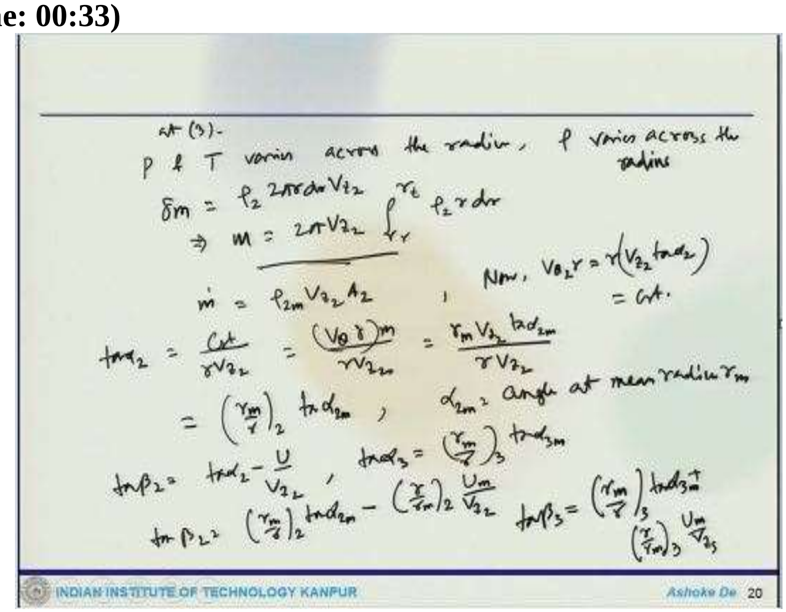

Now, if we move on since the radial equation is satisfied at 3. So, P and T varies across the radius and ρ also varies across the radius. So for a elementary mass what we can write

$$\delta m = 2\pi r dr V_{z2}$$

and we get

$$m = 2\pi V_{z2} \int_{r_r}^{r_t} r dr$$

So, one can see the variation of density is not straight forward hence, one has to go for some sort of a numerical methods or computational methods to find out that thing.

So, now, as a good approximation, we can calculate this density at the mean radius and then we can estimate the other things so let us say

$$\dot{m} = \rho_{2m} V_{z2} A_2$$

So that is at the mean radius that is what we can write. Now we have

$$V_{\theta 2} r = r(V_{z2} \tan \alpha_2)$$

which is constant. So, we get

$$\tan \alpha_2 = \frac{constant}{r V_{z2}} = \frac{(V_{\theta 2} r)_m}{r V_{z2}} = \frac{r_m V_{z2} \tan \alpha_{2m}}{r V_{z2}}$$

so that gives me

$$\tan \alpha_2 = \left(\frac{r_m}{r}\right)_2 \tan \alpha_{2m}$$

where α_{2m} is the angle at mean radius r_m.

So that is what you get. And then you get

$$\tan \beta_2 = \tan \alpha_2 - \frac{U}{V_{z2}}$$

$$\tan \alpha_3 = \left(\frac{r_m}{r}\right)_3 \tan \alpha_{3m}$$

And we can write

$$\tan \beta_2 = \left(\frac{r_m}{r}\right)_2 \tan \alpha_{2m} - \left(\frac{r_m}{r}\right)_2 \frac{U_m}{V_{z2}}$$

$$\tan \beta_3 = \left(\frac{r_m}{r}\right)_3 \tan \alpha_{3m} - \left(\frac{r_m}{r}\right)_3 \frac{U_m}{V_{z3}}$$

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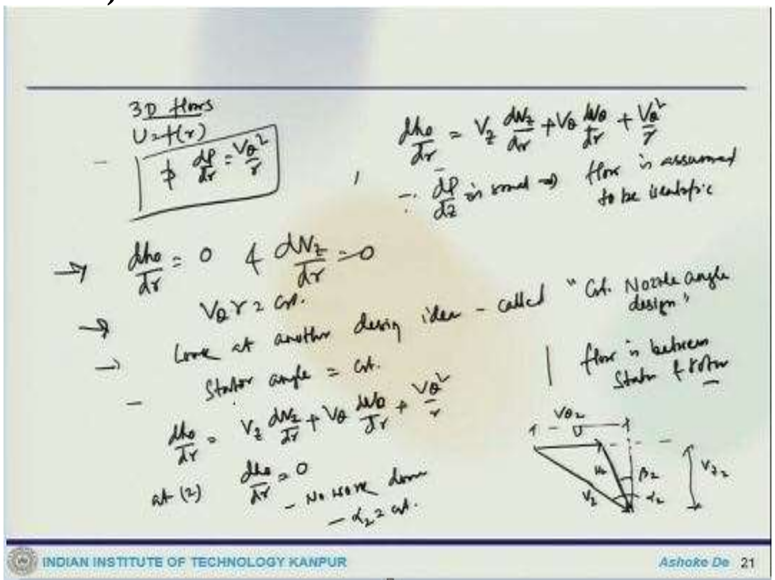

Now, once we look at the 3D flows since the blade velocity U is function of r the velocity triangle changes from root to tip of the rotor obvious the reason, the flow properties also vary with r and the radial equilibrium condition has to be fulfilled which is

$$\frac{1}{\rho}\frac{dp}{dr} = \frac{{V_\theta}^2}{r}$$

So, this has to be satisfied. Now, we have already seen for exhale compressor that vortex flow equation that

$$\frac{dh_0}{dr} = V_z\frac{dV_z}{dr} + V_\theta\frac{dV_\theta}{dr} + \frac{{V_\theta}^2}{r}$$

So, this is the equation vortex flow equation that we have seen since, $\frac{dp}{dz}$ is it is small. So, the flow is assumed to be an isentropic. Now, in free vortex design we had assumed that

$$\frac{dh_0}{dr} = 0$$

And

$$\frac{dV_z}{dr} = 0$$

So, then for radial equation which will get $V_\theta r$ = constant. Now same is true for free vortex design of turbine. So, following the procedure, which we have already adopted in the axial compressor this design method can be involved for free vortex case of turbine stage.

Now, let us look at another design idea which is called constant nozzle angle design. So, this is another one that we can think about so here the nozzle angle nozzle or stator angle. So, the stator angle is constant or assumed to be constant. So, avoid having to manufacture nozzles of varying outlet angle. So, this is what we can do for the constant nozzle. Now, we consider this equation where

$$\frac{dh_0}{dr} = V_z \frac{dV_z}{dr} + V_\theta \frac{dV_\theta}{dr} + \frac{{V_\theta}^2}{r}$$

we are considering and then we will put back the now, flow is between stator and rotor.

So, some flow is also uniform across the annulus at inlet of the nozzles data. So that can give us a velocity triangle like this which is like this. This is U W_2 V_2. So we can go a little bit further. So we do so this is β_2, this is α_2 this is V_{z2} this component is $V_{\theta 2}$. So, that is what this is U now, what happens at 2

$$\frac{dh_0}{dr} = 0$$

So, the flow is just done there is no work done. So, the constant nozzle angle design, which means α_2 is constant.

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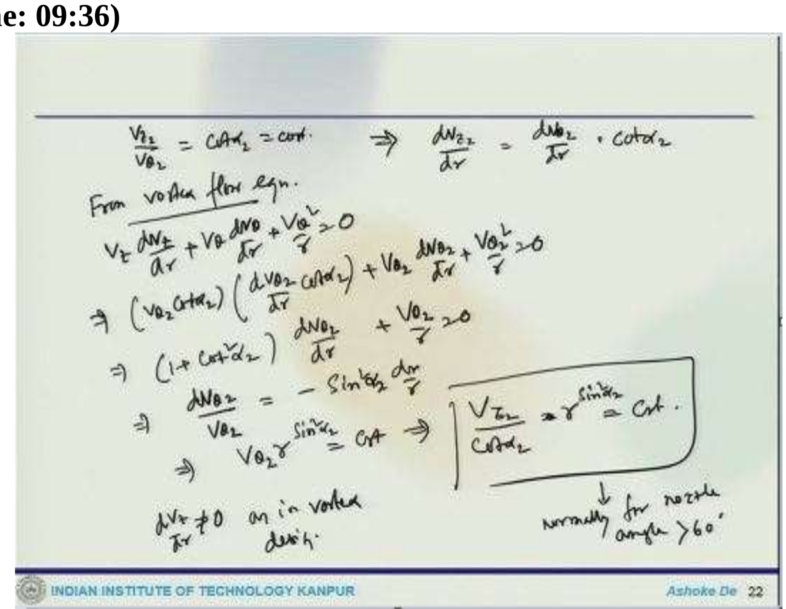

So, what we can write is that

$$\frac{V_{z2}}{V_{\theta 2}} = \cot \alpha_2$$

which is constant. So, that is what we can write from the triangle that we have in there. Now, once we write that that lead us to

$$\frac{dV_{z2}}{dr} = \frac{dV_{\theta 2}}{dr} \cdot \cot \alpha_2$$

now, from vortex flow equation, what we get

$$V_z \frac{dV_z}{dr} + V_\theta \frac{dV_\theta}{dr} + \frac{{V_\theta}^2}{r} = 0$$

Now that we will replace now

$$(V_{\theta 2} \cot \alpha_2) \left(\frac{dV_{\theta 2}}{dr} \cot \alpha_2 \right) + V_{\theta 2} \frac{dV_{\theta 2}}{dr} + \frac{{V_{\theta 2}}^2}{r} = 0$$

So we replace all these with there at station 2 that is $V_{\theta 2}$. So, this we can write

$$(1 + \cot^2 \alpha_2) \frac{dV_{\theta 2}}{dr} + \frac{V_{\theta 2}}{r} = 0$$

so $V_{\theta 2} \neq 0$. So that gives us

$$\frac{dV_{\theta 2}}{V_{\theta 2}} = -\sin^2 \alpha_2 \frac{dr}{r}$$

So what do we get

$$V_{\theta 2} r^{\sin^2 \alpha_2} = constant$$

That means

$$\frac{V_{z2}}{\cot \alpha_2} r^{\sin^2 \alpha_2} = constant$$

So which; will get you this important. So, this is valid for normally for nozzle angle greater than 60 degree. Now, the $\frac{dV_z}{dr} \neq 0$ as in vortex design. So, this is what you get when you get these things.

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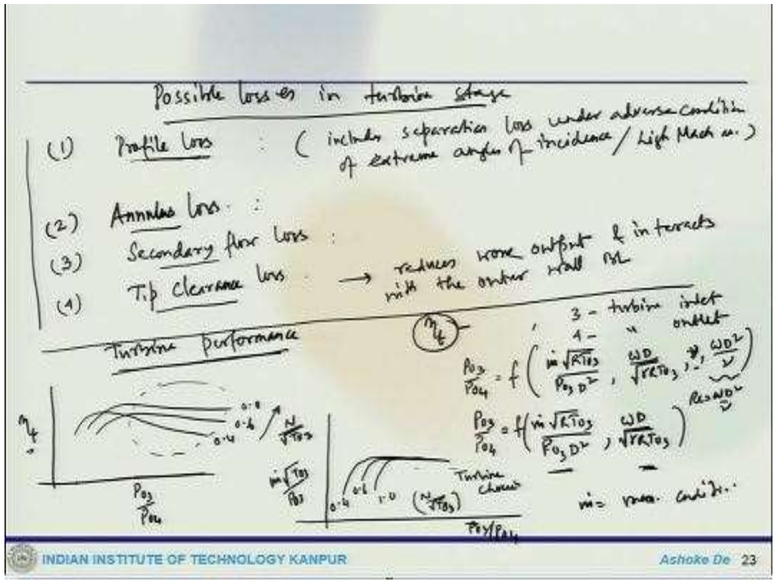

Now having said that we will move to the other thing is that what are the possible losses in turbine stage. So, like compressor, we can also have some losses in turbine stage. So, number 1 which one can think about is the profile loss what is that this profile loss is nothing but which is associated with the boundary layer growth over the blade profile. So, that is what we call it a sort of a profile loss. So, this is associated with the boundary layer and that is growing over the blade profile.

So, this includes separation loss under adverse pressure gradient or other under adverse condition of extreme angles of incidence or high Mach number .So that is one of these things that cannot possibly happen second is the annulus loss. What is that? This is again associated with the boundary layer growth on inner and outer loss of the annulus, so, this is what is termed as annulus loss.

Third, there is secondary flow loss. So, this is from the secondary flow when a wall boundary layer is turned through an angle by an adjusting curved surface. So, that time will have some loss which is due to the secondary flow are known as secondary flow loss fourth one tip clearance loss. So, this is near the rotor blade tip the gas does not follow the intended path. So, this is happening at the rotor blade tip fluid gas or the fluid does not follow the desired path.

So, what it does it reduces work output and increases or rather interact with the outer wall boundary layer. So, this losses are estimated I mean how do you estimate these things, I mean there are certain obvious things, but typically if you conduct the test in a test rig, so, you can estimate these losses or you can carry out the detail computational study and also can estimate these losses. Now, then it comes down another important thing, which is the turbine performance.

So, this we have extensively discussed while talking about the compression. So, similarly, for turbine there could be in performance can and one can again estimate these things like using those simple non dimensional analyses of Buckingham pi theorem. So, now the eta t this one can obtained from the cyclic calculations or measured let us say 3 is the turbine inlet and 4 is turbine outlet. So, this is the turbine efficiency.

So, what you have again these are the P_{03}/P_{04} the pressure issue, they are function of some non-dimensional group, which are instead of going into the details that we have already done that for the compressor rather extensive fashion. So, just quickly go through these things. So,

$$\frac{p_{03}}{p_{04}} = F\left(\frac{\dot{m}\sqrt{RT_{03}}}{p_{03}D^2}, \frac{\omega D}{\sqrt{\gamma RT_{03}}}, \upsilon, \frac{\omega D^2}{\upsilon}\right)$$

So, this is υ and this now Reynolds number which is essentially this $\frac{\omega D^2}{\upsilon}$ is not an important variable and also and the same way the υ to it we can drop those term and because we have already discussed about the justification at the high turbulent conditions, they are pretty much independent. So, these walls down to

$$\frac{p_{03}}{p_{04}} = F\left(\frac{\dot{m}\sqrt{RT_{03}}}{p_{03}D^2}, \frac{\omega D}{\sqrt{\gamma RT_{03}}}\right)$$

so, there is only 2 non-dimensional group and so, these like, this is the sort of non-dimensional mass flow rate and this is the non-dimensional speed.

And we can look at the variation, for example, eta t with the change in pressure ratio. So, that some sort of end like it goes like this. So these are 0.4 0.6 0.8 something like that, and that is are $\frac{N}{\sqrt{T_{03}}}$. So that means, changing the rotational speed and then how the pressure ratio changes, it changes the turbine efficiency. So, what one can see here is that eta t is almost constant over a wide range of operating conditions. So, I mean, if you look at this portion of the curve, so, they are pretty much remain constant.

So, since the flow is accelerating in the loss coefficient is also low, because that already we have discussed, because the turbine the things are expanding so, when the flow is expanding, it actually extracting the energy out and the speed also increases. So, this is where the flow is always accelerating. Now, the other component is mass flow rate $\frac{\dot{m}\sqrt{T_{03}}}{p_{03}}$. So, this is again the car goes like some sort of m.

So, they said let us say 0.4 6 1 again these are the non-dimensional speed and this is the zone where you can see there is hardly any change in the mass flow rate, this is called turbine choking. So, that means m. dot is the maximum condition. So that is what it calls them choking. So, this can occur in nozzle and all this.

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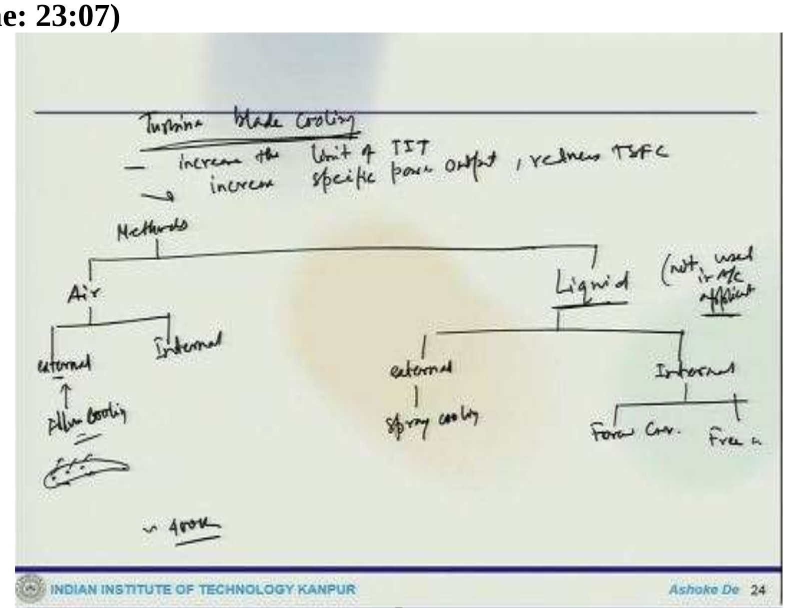

Now, other aspect of it because these blades are exposed to the high thermal gradient or the hot gases are coming out of this blade. So, turbine blade cooling is another important aspect. So, why the cooling is required? Because you want to increase the limit of turbine inlet temperature and also you can increase specific power output or increased the specific or output and reduces TSFC. So, there are broadly 2 ways of cooling. So, the methods are broadly classified into 2 larger one, one is the air cooling system, other case is the liquid cooling system.

Now, again the air cooling system this is where you can have external cooling or you could have internal cooling and one of the example of the external cooling is the film cooling system film cooling approach, which is basically the air is injected disease just like an synthetic jet there are multiple holes which are done through which the air is injected out, it does if you somebody look at the blades like this and these are the holes from which the air comes out and that is actually cools the blood surface.

Now liquid cooling this is not used in aircraft application that is one thing. Because the channeling the liquid into the blade is a bottom leg, there could be corrosion, then there could be formation of deposits. So, that is why this is not a very preferred method for aircraft application. And here also you can have external you could have internal this could have spray cooling; this could be forced convection free convection. So, now with the cooling, one important thing is the operating temperature can be increased even by around 400 Kelvin. So, that is a change one can obtain this cooling method. Now, another way to look at this.

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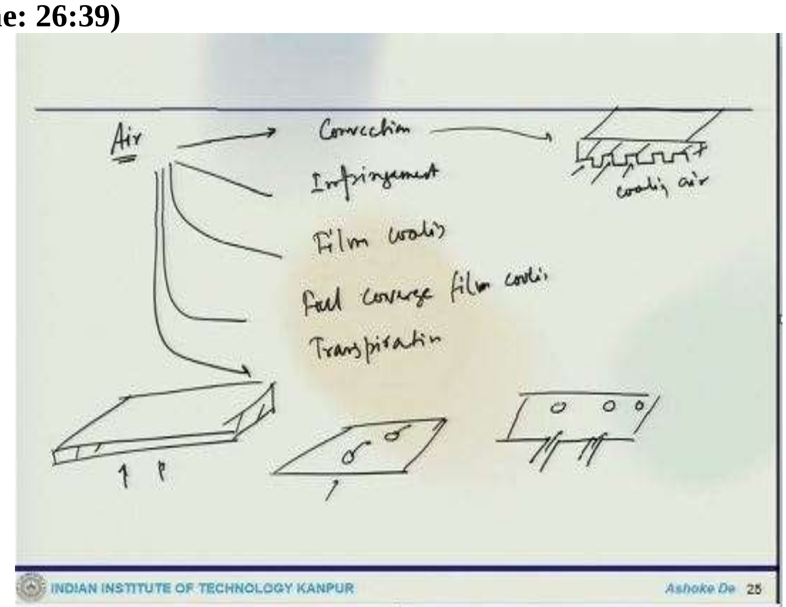

Since the air cooling is a preferred method in aircraft application, so, instead of broadly categorized in external or internal cooling, one can also look at like convection cooling or you can do impingement cooling film cooling, full coverage film cooling or you can have transpiration cooling. So, convection cooling is typically how it looks like is that you have a blade or passage like this and you met such groups like this and this through which the passages, the cooling air goes through it typically done.

Now in the impingement cooling, you have a layer like this and then you do some sort of an the bottom the impingement and film cooling it is already you have a slab like this through where there would be hole through which the cooling air comes through and full coverage film, they are actually you have these passages top of that these holes, so through which it comes. So, this is what you get in the full coverage and transpiration. That is the way you do.

So, these are the different cooling strategies that one can adopt. So, the important thing here is that when you talk about the turbine, I mean apart from all these losses, the important part is that blade cooling parameters because that can improve the efficiency and since the turbine blades are exposed to the high temperature, so, cooling is very, very essential, so, that the life of those blades are increased.

And at the same time, you can improve the efficiency of your whole engine. So, this is what we can talk about axial turbine and we stop the discussion here and continue the rest of the discussion in the next lecture.