Introduction to Airbreathing Propulsion

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Lecture – 56
Axial Turbine (Contd.,)

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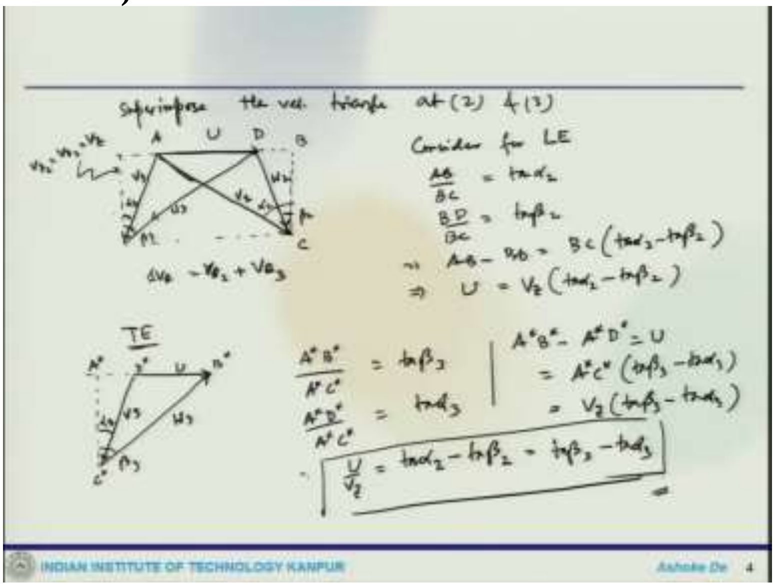

So, we are looking at the velocity triangle of the axial flow turbine and what we have already looked at the leading and trailing edge and the velocity component and this is where we stopped where we get the relationship between the angles alpha, beta at different stages. So, this is what we obtained.

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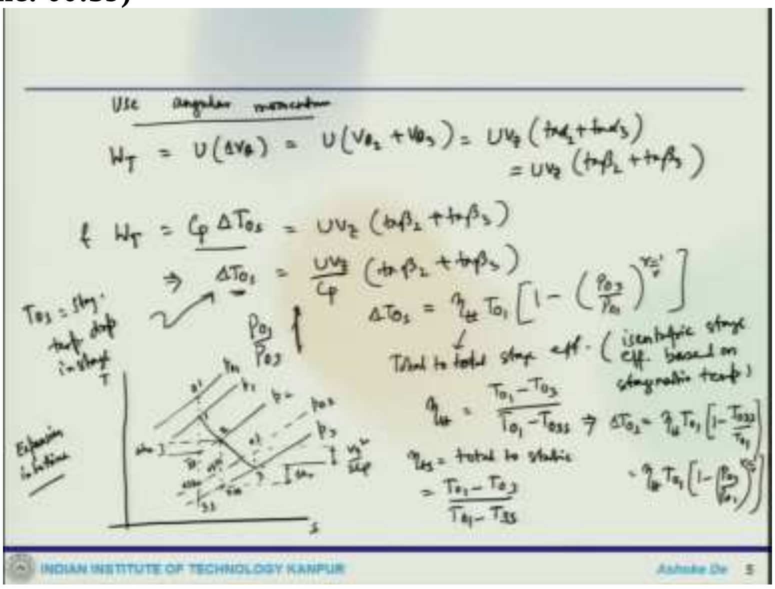

Now, we use the angular momentum. So, using that what we can write is

$$W_T = U(\Delta V_\theta) = U(V_{\theta 2} + V_{\theta 3}) = UV_z(\tan\alpha_2 + \tan\alpha_3) = UV_z(\tan\beta_2 + \tan\beta_3)$$

all this one can write from this velocity triangles that we have already drawn or discussed. So, this

$$W_T = C_p \Delta T_{02s} = UV_z(\tan\beta_2 + \tan\beta_3)$$

then we get

$$\Delta T_{02s} = \frac{UV_z}{C_p}(\tan\beta_2 + \tan\beta_3)$$

So, this is nothing but the T_{0s} is stagnation temperature drop in stage. So, this is the difference in stagnation temperature drop and also we have like P_{01}/ P_{03} also can be correlated with this delta T_{0s} like if you use that, then we can write this

$$\Delta T_{0s} = \eta_{tt} T_{01}\left[1 - \left(\frac{p_{03}}{p_{01}}\right)\right]^{\frac{\gamma-1}{\gamma}}$$

this. So, η_{tt} is total to total stage efficiency.

So, which is essentially one can think about isentropic stage efficiency based on stagnation temperature. So, one can define like

$$\eta_{tt} = \frac{T_{01} - T_{03}}{T_{01} - T_{03s}}$$

we will look at the curve and then probably it would be bit easier. So if we draw the let us say Ts diagram. So that is the starting point of somewhere one. So now that is go there, this is P_{01}, this is P_1 and then from here it comes to here.

So that is P_2 from there, it comes to here which is 3 this is P_3. And in between that would be P_{03}. So this is 01. This is 1 this is 2 this is 3 and this is 03. So, this is $\frac{V_3^2}{2C_p}$. Now, this is 2s and this guy is 3s this is 03s then this is 03s this is 3s. Now this 3ss on these are Δh_r. So, in between these this is Δh_r. So, this is an expansion process, this is essentially the expansion in turbine.

So, that is how it looks. Now, this is how one can define

$$\Delta T_{0s} = \eta_{tt} T_{01}\left[1 - \left(\frac{T_{03s}}{T_{01}}\right)\right] = \eta_{tt} T_{01}\left[1 - \left(\frac{p_{03}}{p_{01}}\right)\right]^{\frac{\gamma-1}{\gamma}}$$

So, once you draw the T-s diagram then it is bit easy. So, everything now you can correlate from this T-s diagram what you get.

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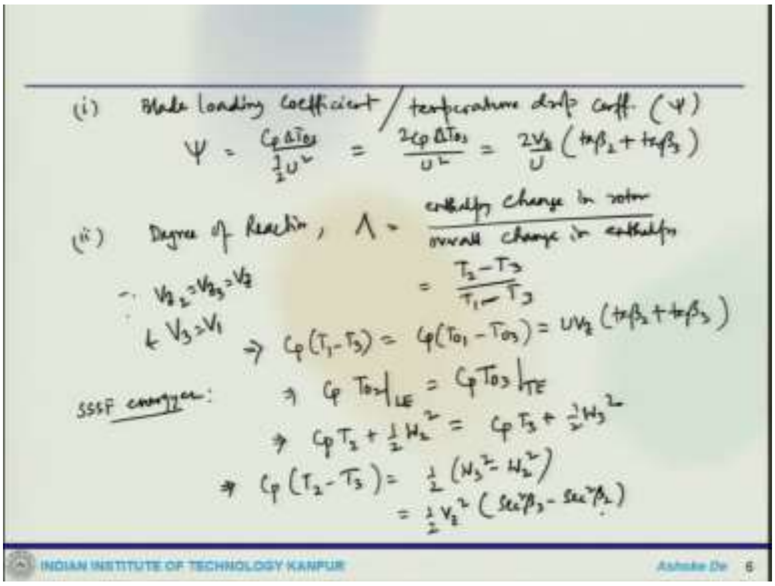

Now, we talk about some of the important dimensionless parameter number 1 which we have also looked at so, you can see the clear similarity while talking about the axial flow compression like blade loading coefficient. So, this already we have seen it or one can say it same representation of temperature drop coefficient. So, which is just ψ, which is

$$\psi = \frac{C_p \Delta T_{0s}}{\frac{1}{2}U^2} = \frac{2C_p \Delta T_{0s}}{U^2} = \frac{2V_z}{U}(\tan\beta_2 + \tan\beta_3)$$

Now, second again degree of reaction which is defined like enthalpy change in rotor overall change in enthalpy. So, this will be

$$\Lambda = \frac{T_2 - T_3}{T_1 - T_3}$$

Since, we have

$$V_{z2} = V_{z3} = V_z$$

$$V_3 = V_1$$

we can write

$$C_p(T_1 - T_3) = C_p(T_{01} - T_{03}) = UV_z(\tan\beta_2 + \tan\beta_3)$$

So, related to rotor blades the flow does no work. Now, if I write steady state steady flow energy equation that yields C_pT_{02} at leading edge is C_pT_{03} at trailing edge.

So, which is nothing but

$$C_pT_2 + \frac{1}{2}W_2^2 = C_pT_3 + \frac{1}{2}W_3^2$$

So, now, if you rearrange that, so, what do we get

$$C_p(T_2 - T_3) = \frac{1}{2}(W_3^2 - W_2^2) = \frac{1}{2}{V_z}^2(sec^2\beta_3 - sec^2\beta_2)$$

So, this is nothing but the trigonometry.

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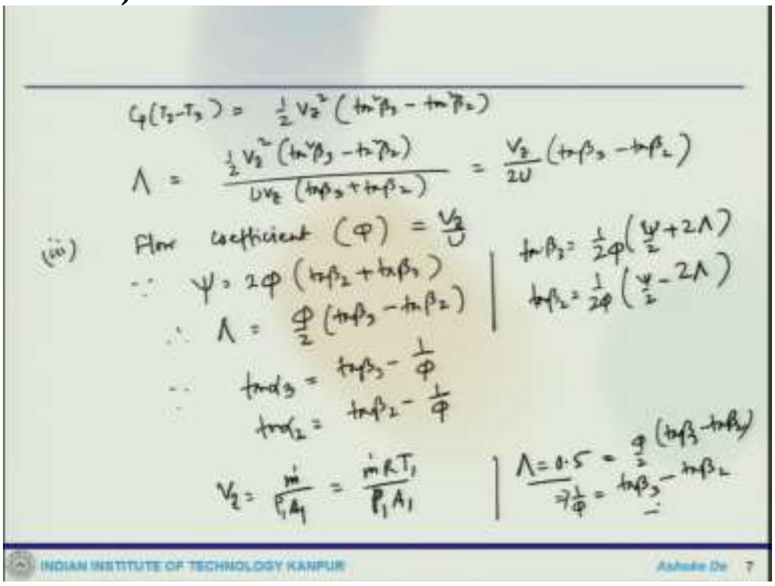

So, I can write like that

$$C_p(T_2 - T_3) = \frac{1}{2}{V_z}^2(tan^2\beta_3 - tan^2\beta_2)$$

Now, we have these definition of degrees of reaction, so

$$\Lambda = \frac{\frac{1}{2}{V_z}^2(tan^2\beta_3 - tan^2\beta_2)}{UV_z(\tan\beta_2 + \tan\beta_3)} = \frac{V_z}{2U}(\tan\beta_3 - \tan\beta_2)$$

And the third non dimensional number is the flow coefficient which is called ϕ nothing but

$$\phi = \frac{V_z}{U}$$

Since we had

$$\psi = 2\phi(\tan\beta_2 + \tan\beta_3)$$

So, we can have degrees of reaction is

$$\Lambda = \frac{\phi}{2}(\tan\beta_3 - \tan\beta_2)$$

Since we can write

$$\tan\beta_3 = \frac{1}{2\phi}\left(\frac{\psi}{2} + 2\Lambda\right)$$

And

$$\tan\beta_2 = \frac{1}{2\phi}\left(\frac{\psi}{2} - 2\Lambda\right)$$

So this gives me

$$\tan \alpha_3 = \tan \beta_3 - \frac{1}{\phi}$$

and

$$\tan \alpha_2 = \tan \beta_2 - \frac{1}{\phi}$$

So,

$$V_z = \frac{\dot{m}}{\rho_1 A_1} = \frac{\dot{m} R T_1}{p_1 A_1}$$

Now, typical experience so those 50% reactions machines are most efficient that means, degree of reaction of 0.5 is more efficient. So, in this case, the expansion is equally divided between the stator and the rotor. Now, if we consider psi equal to or 0.5 and the mean radius, which is

$$\Lambda = 0.5 = \frac{\phi}{2}(\tan \beta_3 - \tan \beta_2)$$

we get

$$\frac{1}{\phi} = \tan \beta_3 - \tan \beta_2$$

so that we have already seen that. That this is going to be the situation.

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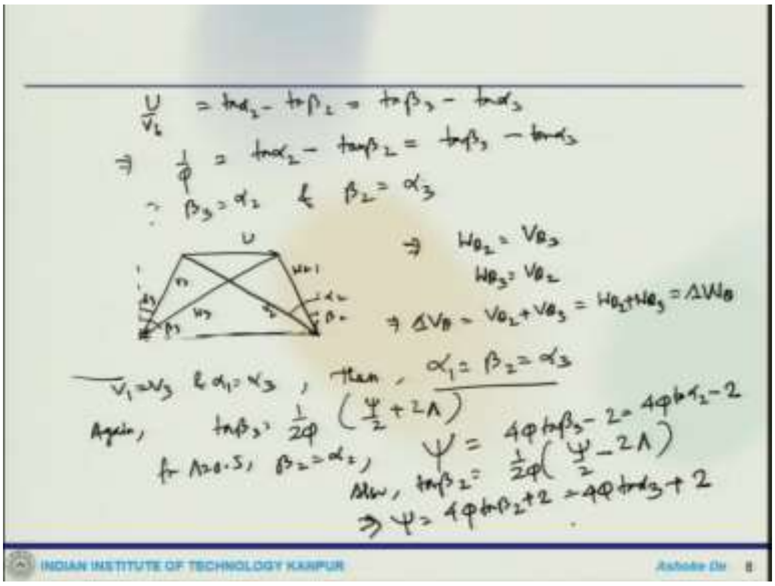

Now, if you write

$$\frac{U}{V_z} = \tan \alpha_2 - \tan \beta_2 = \tan \beta_3 - \tan \alpha_3$$

So,

$$\frac{1}{\phi} = \tan \alpha_2 - \tan \beta_2 = \tan \beta_3 - \tan \alpha_3$$

since

$$\beta_3 = \alpha_2$$

$$\beta_2 = \alpha_3$$

So, the velocity diagram would be symmetrical. So, if we draw that. So, this is U. So, W_2 V_2 and α_2 β_2 W_3 V_3 α_3 β_3. So, which gives $W_{\theta 2}$ is $V_{\theta 3}$ and $W_{\theta 3}$ is $V_{\theta 2}$.

So my change in this $V_{\theta 3}$ Now further, if we consider repeating stages, then we have $V_1 = V_3$ and $\alpha_1 = \alpha_3$, then we get $\alpha_1 = \beta_2 = \alpha_3$. So the stator and rotor blade have same inlet and outlet angles. Now, again looking back since

$$\tan \beta_3 = \frac{1}{2\phi}\left(\frac{\psi}{2} + 2\Lambda\right)$$

this now for this equals to 0.5 and $\beta_2 = \alpha_2$ we get

$$\psi = 4\phi \tan \beta_3 - 2 = 4\phi \tan \alpha_2 - 2$$

Also, we get

$$\tan \beta_2 = \frac{1}{2\phi}\left(\frac{\psi}{2} - 2\Lambda\right)$$

this so that gives us back equals to

$$\psi = 4\phi \tan \beta_2 - 2 = 4\phi \tan \alpha_3 - 2$$

So these are the different relationships that you can get.

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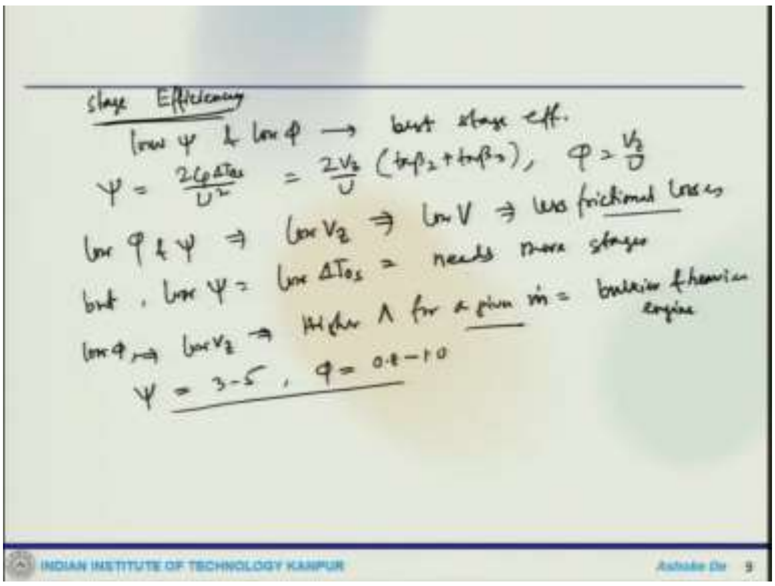

Now will look at the how do estimate the stage efficiency. So that we will try to estimate. So typically this is obtained by wind tunnel test of cascade of blades where we measure the temperature and pressure across the blades and then with the losses and everything else. We get the theta S, So this test show the design having low ψ and low ϕ yield the best stage efficiency. Where

$$\psi = \frac{2C_p \Delta T_{0s}}{U^2} = \frac{2V_z}{U}(\tan\beta_2 + \tan\beta_3)$$

Now what happens low ψ and low ϕ means low V_z which means low V less frictional losses. But low ψ is low ΔT_{0s} is needs more stages so one hand we have low frictional losses on the other hand because of this low psi we need more stages. Now also low ψ and low ϕ, low ϕ means low V_z which means higher degrees of reaction for a given $\dot{m}$ which means bulkier and heavier engine.

So, optimum values often aircraft engine. So, this means larger turbine annulus area. So, typical value of these goes between 3 to 5 while phi goes between 0.8 to 1. So, just to keep the frontal area of the aircraft engine and the width on the lower side. Now for industrial gas turbine is feel is vital, so, low phi, low psi desired and last stage a low axial velocity a small swirl angle alpha 3 are desirable to keep down the losses in the exhaust diffusion.

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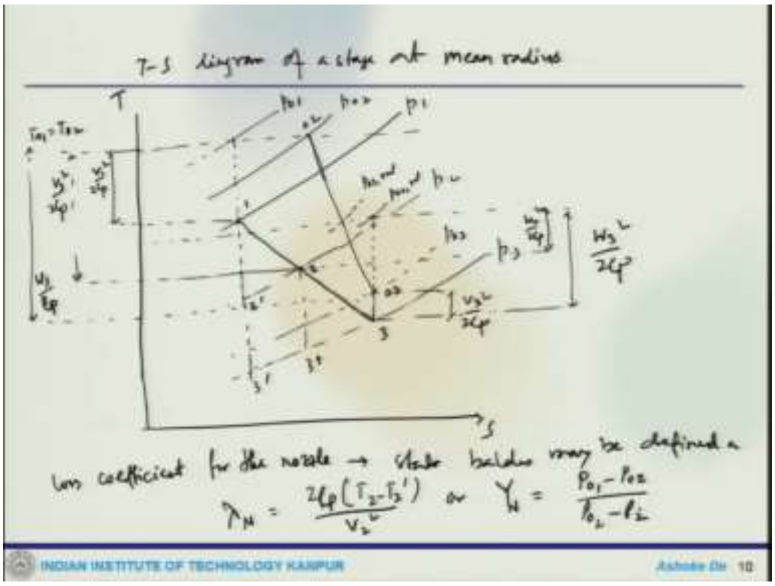

Now, we can actually estimate the frictional losses from each state. Now let us look at this previous diagram again in details T-S diagram of a stage at mean radius. So, we have this. So, that is so this is p_{01}, p_{02}. So this is p_1. So this is 02. This is 1 then this p_2 then it is goes to 3. So p_3. So this is 2 this is 3 then we have p_{03} we get here from 3 we get 2 and then. So, this is 03.

So, this is p_{03} relative this is p_{02} relative. And then this is 2 prime, 3 prime, 3 double prime. So, this one is the $\frac{V_1^2}{2C_p}$. This is $\frac{V_2^2}{2C_p}$. So that is W_s/C_P So, here $T_{01} = T_{02}$. Then so this is $\frac{V_3^2}{2C_p}$. So, this is $\frac{W_3^2}{2C_p}$ this one $\frac{W_2^2}{2C_p}$. So, these are the complete diagram of that. So, the loss coefficient for the nozzle is the stator blades may be defined as

$$\lambda_N = \frac{2C_p(T_2 - T_2{}')}{V_2^2}$$

Or

$$Y_N = \frac{(p_{01} - p_{02})}{(p_{02} - p_2)}$$

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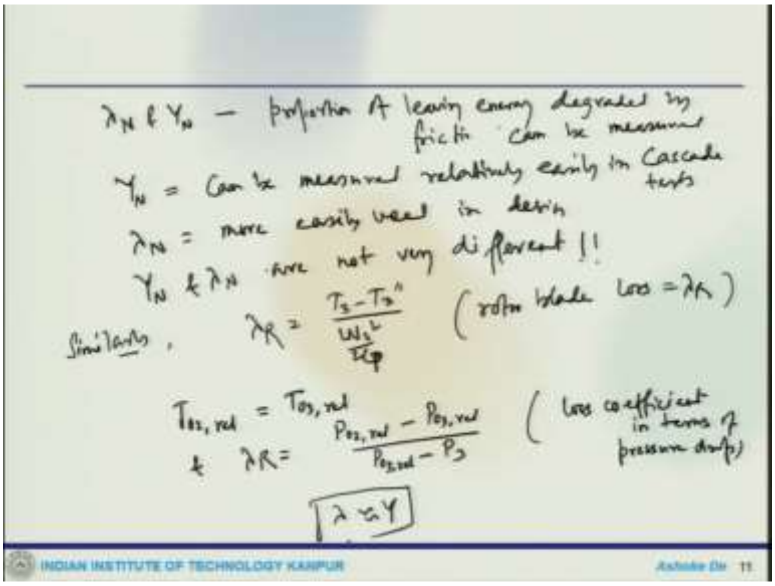

So, here we have this λ_N and Y_N these are proportion of leaving energy degraded by friction, which can be measured Y_N is can be measured relatively easily in cascade tests. This is more easily used in design and Y_N and both λ_N are not very different. So, one can show that doing some now similarly

$$\lambda_R = \frac{(T_3 - T_3{}'')}{\frac{W_3^2}{2C_p}}$$

So, which is a rotor blade loss which is λ_R.

So, is defined as a proportion of the leaving kinetic energy relative to or leaving kinetic energy relative to the ρ. So, it can be related to the cascade test measure. So, one can note here that no work is done by that gas relative to the blades. So, T_{02} relative should be T_{03} relative and

$$\lambda_R = \frac{(p_{02,rel} - p_{03,rel})}{p_{03,rel} - p_3}$$

So, essentially the last coefficient in terms of pressure drop.

So, as I said 1 can show that λ sort of would be equivalent to Y and this one can easily show with some argument also by doing some analytical analysis through this Ts diagram that we have drawn here. So, this is globe a picture for a particular stage you get on how the absolute velocity, relative velocity and all these corresponding stages, they are connected. So now, this how we can show this lambda and Y.

This could be another important argument, but we can look at that thing in the next lecture. So, we stop it here and continue that discussion of the correlation in the next lecture.