Introduction of Airbreathing Propulsion
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Lecture - 53
Axial Compressor (Contd.,)

So let us continue the discussion on axial flow compressor, so what we are middle of the discussion is the design concept. So we have been talking about different parameters like design parameters and the impact of these parameters so and how you change different parameters from the design consideration. So just in conceptual design this is where we are talking about that.

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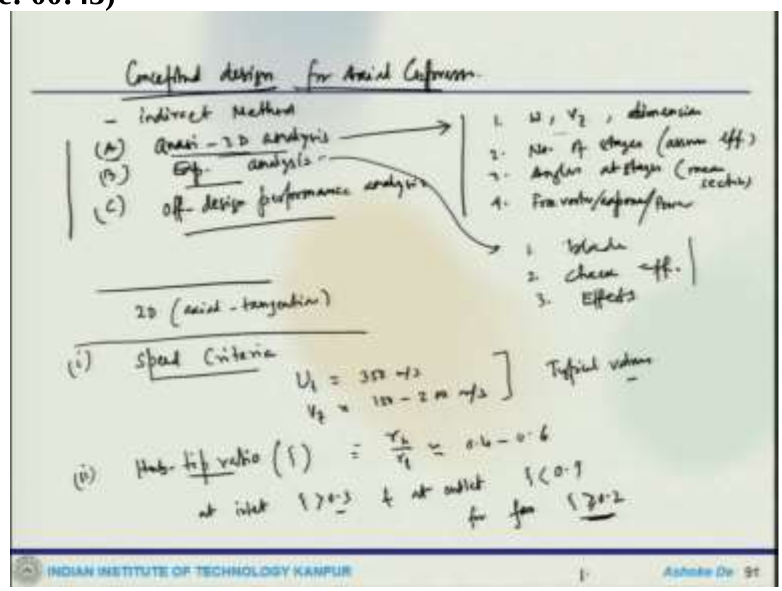

Different indirect method, so these are starting with your quasi-3D analysis and then you do experimental analysis and finally the off-design performance analysis. Now the first step which is the quasi-3D analysis that starts with 2D analysis and which is essentially the 2D analysis means the axial tangential and then extended to define the flow properties at different station blade height and then with those information move to the 3D analysis.

So these are the things that we have already discussed. Now some of the parameters that we can talk about or rather the choice of data so let us say, one of them would be the speeds or speed criteria. So would be the criteria for those speed and all these things. So this is from different experiments and previous existing experience one can determine the maximum speed for example tip speed could be around somewhere 350 meter per second and then the axial velocity could be of 150 to 200 meter per second.

So these are again these parameters are not rigid number or fixed number because the different engine to engine this number can vary but these are some of the sort of typical values that are used for design and all these purposes. Now the second one is the Hub-tip ratio that means ξ so the typical value of

$$\xi = \frac{r_h}{r_t}$$

would be 0.4 to 0.6. So at inlet these ξ is greater than 0.3 and at outlet this ξ is less than 0.9 and if you think about fan; for fan this guy would be somewhere 0.2, so these are the some of these values which could be taken.

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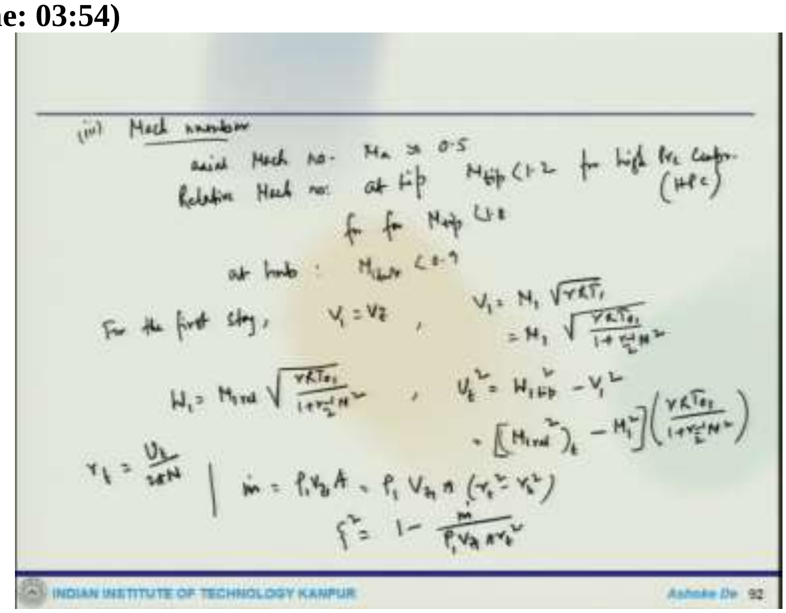

Now third Mach number so that is another one, so the axial Mach number is 1. So axial Mach number which Ma would be 0.5 typical high subsonic. Now there are relative Mach number so in that case at tip it would be less than 1.2 for this is also for high pressure ratio P_{rc} compressor or rather HPC and for fan M_{tip} would be less than 1.8 and at hub the M_{1hub} this should be less than 0.9 so this also depends on the engine.

Because the aero engine which is whether the supersonic or hypersonic flight depends on that and also the inlet temperature depends on all these, so that the Mach number value suggested here which develops unaffordable stresses in the blade. So consequently these values are drastically reduced even to hub it suggested value or alternatively specify values from the mean blade and axial speeds which can be deployed.

Let us say for the first stage the flow is axial so V_1 would be V_z then we can calculate the Mach number so

$$V_1 = M_1\sqrt{\gamma R T_1}$$

which is

$$V_1 = M_1\sqrt{\frac{\gamma R T_{01}}{1+\frac{\gamma-1}{2}M^2}}$$

So similarly the relative speed at the blade tip which can be calculated

$$W_1 = M_{1,rel}\sqrt{\frac{\gamma R T_{01}}{1+\frac{\gamma-1}{2}M^2}}$$

Now the rotational speed can be calculated then

$${U_t}^2 = {W_{1,tip}}^2 - {V_1}^2$$

which is

$${U_t}^2 = \left({M_{1,rel}}^2 - {M_1}^2\right)\left(\frac{\gamma R T_{01}}{1+\frac{\gamma-1}{2}M^2}\right)$$

So the outer diameter and the rotational speed which are connected like

$$r_t = \frac{U_t}{2\pi N}$$

So either the tip speed or the rotational speed is specified so then the hub tip ratio is calculated from the equation of the mass flow rate equation or other continuity equation

$$\dot{m} = \rho_1 V_{z1} A = \rho_1 V_{z1}\pi({r_t}^2 - {r_h}^2)$$

so this is

$$\xi^2 = 1 - \frac{\dot{m}}{\rho_1 V_{z1}\pi {r_t}^2}$$

So once we get this you can calculate the density.

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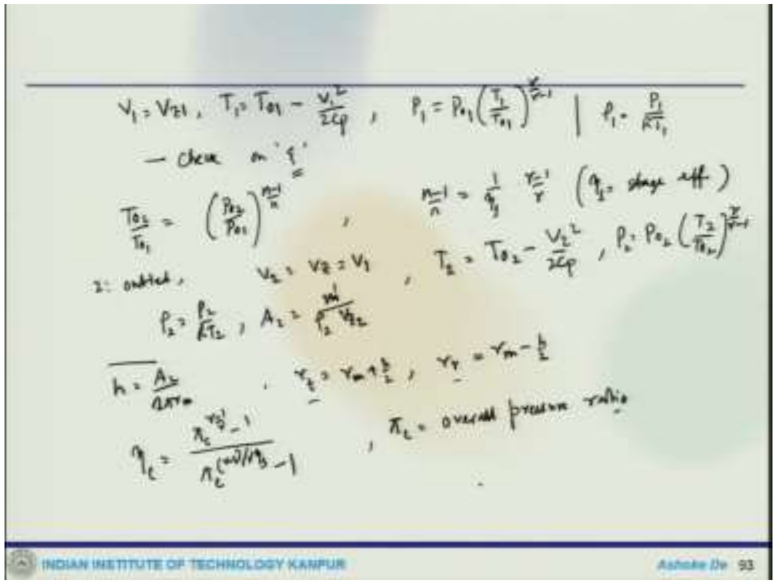

Also like $V_1 = V_{z1}$,

$$T_1 = T_{01} - \frac{V_1^2}{2C_p}$$

$$p_1 = p_{01}\left(\frac{T_1}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$

So from here you get the density

$$\rho_1 = \frac{p_1}{RT_1}$$

so once you do that but at the same time one has to put an check on hub tip ratio because this has to be within the allowable limit. So here one has to make a call like the type of variation of the compressor passage and its successive stage or one can decide whether the compressor will have constant mean or constant tip or constant hub radius all the stages.

So having said that the next to determine the outlet dimensions of the compressor assuming the stage efficiency or the polytrophic efficiency of the compressor and then we can look at those things. Like

$$\frac{T_{02}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{n-1}{n}}$$

Where,

$$\frac{n-1}{n} = \frac{1}{\eta_s}\frac{\gamma-1}{\gamma}$$

where η_s is stage efficiency or here now assuming the air leaves the stator of the last stage of the compressor axially so then the outlet station of the compressor.

Let us say 2 is outlet so we can write $V_2 = V_z = V_1$. So the static pressure can be estimated as

$$T_2 = T_{02} - \frac{V_2^2}{2C_p}$$

and

$$p_2 = p_{02}\left(\frac{T_2}{T_{02}}\right)^{\frac{\gamma}{\gamma-1}}$$

So the density also can be calculated which is

$$\rho_2 = \frac{p_2}{RT_2}$$

and it would be

$$A_2 = \frac{\dot{m}}{\rho_2 V_{z2}}$$

Now if the constant mean radius option is selected then the mean radius is already known and the blade height and outlet is also can be calculated like h would be

$$h = \frac{A_2}{2\pi r_m}$$

and that tip radius would be

$$r_t = r_m + \frac{h}{2}$$

And

$$r_r = r_m - \frac{h}{2}$$

so these are tip radius and root radius or the hub.

Now then the compressor efficiency is related to the polytropic stage efficiency which would be

$$\eta_c = \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\pi_c^{\frac{\gamma-1}{\gamma\cdot\eta_s}} - 1}$$

where if you recall from the second analysis this is the overall pressure ratio so you can find out that thing. So that is once you have that you can now go and find in.

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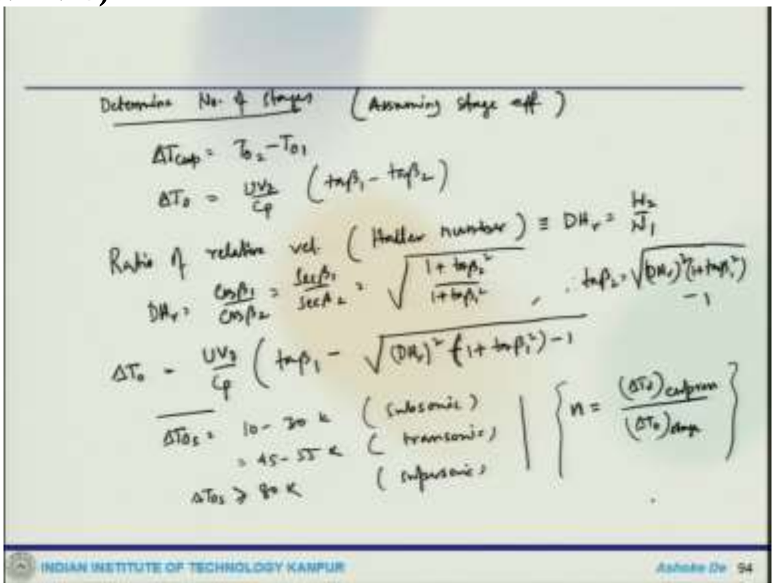

Or either determine number of stages so here one has to assume the assuming stage efficiency. So by assuming stage efficiency one can, so to determine or other to obtain the number of stages the overall temperature rise within the compressor is first determined which is

$$\Delta T_{compressor} = T_{02} - T_{01}$$

and secondly the stage temperature is also determined and then divide both the values to obtain the number of stages.

So, now what will happen to the stage temperature? The rise of the stage temperature can be find out

$$\Delta T_0 = \frac{UV_z}{C_p}(\tan\beta_1 - \tan\beta_2)$$

Now we can have the relative velocity ratio or which is ratio of relative velocity or called De Haller number. So this is represent at DH number which is W_2 / W_1. Then what we can write

$$DH_r = \frac{\cos\beta_1}{\cos\beta_2} = \frac{\sec\beta_1}{\sec\beta_2} = \sqrt{\frac{1+\tan{\beta_2}^2}{1+\tan{\beta_1}^2}}$$

$$\tan\beta_2 = \sqrt{{DH_r}^2\left(1+\tan{\beta_2}^2\right)} - 1$$

So my stage temperature rise would be

$$\Delta T_0 = \frac{UV_z}{C_p}\left(\tan\beta_1 - \sqrt{DH_r{}^2\left(1+\tan\beta_2{}^2\right)} - 1\right)$$

so this is what you get. Now as a guide vane the temperature rise per stage for the available compressor can have the typical fall range like for example delta 2 is would be 10 to 30 Kelvin for subsonic stages this could be 45 to 55 Kelvin for transonic stage and delta T s would be always 80 Kelvin for supersonic range and then we can find out the number of stages like which is

$$n = \frac{\Delta T_{0,compressor}}{\Delta T_{0,stage}}$$

this will give us the number of stages. So this will give you an idea how many stages you require now once you calculate that

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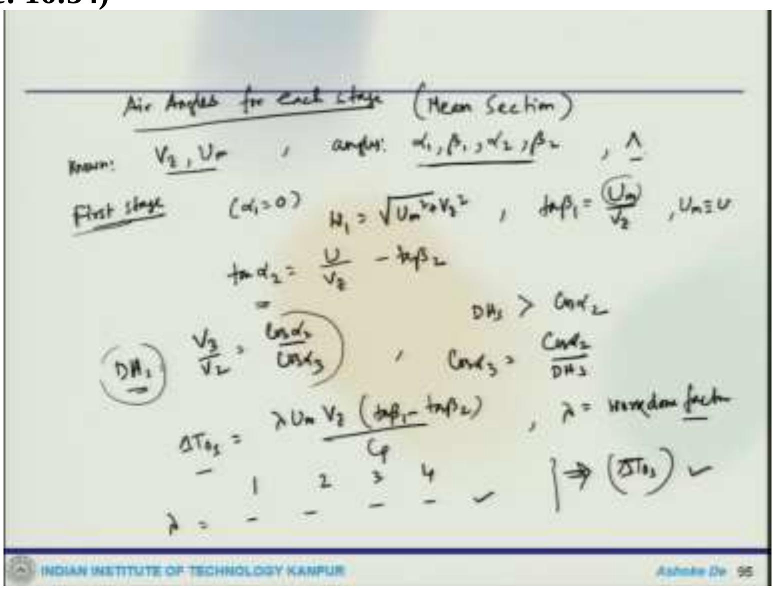

Then one can calculate the air angles for each stage. So these are at the mean section this calculation at the mean section and find out the air angle. So once you have let us say for a constant mean diameter and once you and known axial and mean blade speed that means this is known V_z and U_m. So one can find out the angles of α_1, β_1, α_2 and β_2 so, this can be for each stages these are values can be calculated and also the degree of reaction would be calculated and then check whether that is inacceptable range or not.

So let us start with let us say first stage, so the first stage is characterized as the flow is also axial so α would be 0. So the relative velocity would be

$$W_1 = \sqrt{{U_m}^2 + {V_z}^2}$$

where

$$\tan\beta_1 = \frac{U_m}{V_z}$$

Now we can for simplicity we can write that U_m now, since everything is in mean radius we can replace that with the U so now, the absolute angle at the rotor outlet would be

$$\tan\alpha_2 = \frac{U}{V_z} - \tan\beta_2$$

Now this is also the inlet angle to the stator and the outlet angle of the stator would be α_3 which can be calculated from the this ratio

$$\frac{V_3}{V_2} = \frac{\cos\alpha_2}{\cos\alpha_3}$$

which is known as again the de haller number DH. So now the value of this de haller number has to be selected or something assumed or if this is not reasonable you can see this is a ratio of $\frac{\cos\alpha_2}{\cos\alpha_3}$ then the values could be completely imaginary.

So one has to satisfy that

$$\cos\alpha_3 = \frac{\cos\alpha_2}{DH_s}$$

Now we can calculate the stage temperature rise so that is

$$\Delta T_{0s} = \frac{\lambda U_m V_z(\tan\beta_1 - \tan\beta_2)}{C_p}$$

here λ is work done factor. So now at different stages let us say first second third fourth that would be different values of lambda using these values now one can find out the temperature rise across the stages and once that is then from here you get ΔT_{0s}.

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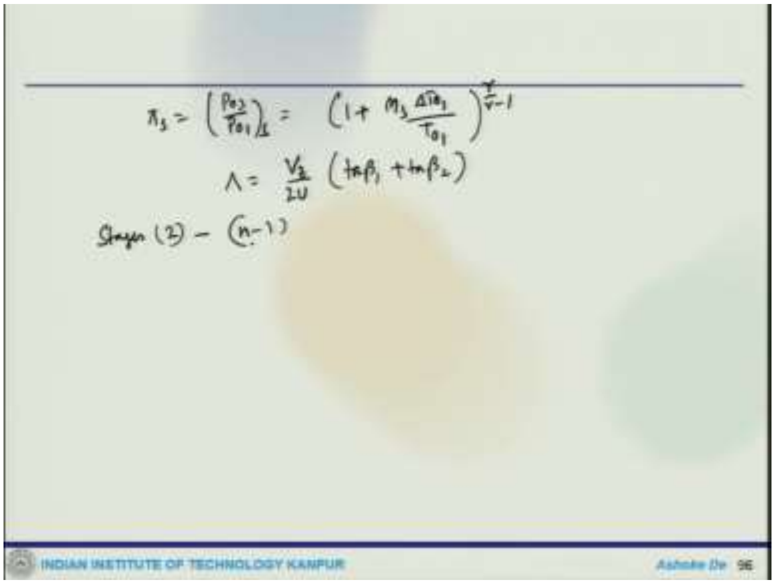

And once you get that then you move to calculate the pressure issue for the stage

$$\pi_s = \left(\frac{p_{03}}{p_{01}}\right)_s = \left(1 + \eta_s \frac{\Delta T_{01}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$

and degrees of reaction is calculated like

$$\Lambda = \frac{V_z}{2U}(\tan\beta_1 + \tan\beta_2)$$

So, whatever the equation system that we have derived for the stage dynamics that all of them are in use for calculation of all this now, there is first stage now from stages 2 to $n-1$.

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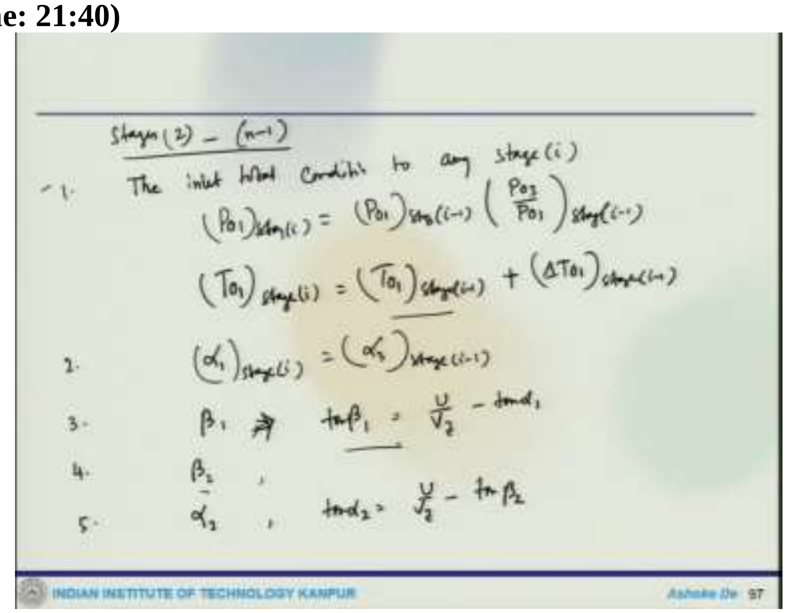

So stages 2 to $n-1$ so that you can follow this procedure. First the inlet total conditions to any stage let us say I can be calculated

$$(p_{01})_{stage\ i} = (p_{01})_{stage\ (i-1)} \left(\frac{p_{03}}{p_{01}}\right)_{stage\ (i-1)}$$

and similarly

$$(T_{01})_{stage\ i} = (T_{01})_{stage\ (i-1)} (\Delta T_{01})_{stage\ (i-1)}$$

Second the flow is no longer axial but the outlet angle α_3 of any stage will be equal to the inlet absolute angle α_1.

So what one can write is that, α_1 for stage i is α_3 for stage i - 1 so that is what you get. Then third the inlet relative angle that β_1 can be calculated. So to calculate β_1 we need

$$\tan \beta_1 = \frac{U}{V_z} - \tan \alpha_1$$

so that what you get when you get to the β_1. Now once you get β_1 the outlet relative angle that is β_2 can be obtained so, by using the de haller number this can be obtained. So then you have β_1 and β_2 then the absolute outlet angle α_2 can be calculated like

$$\tan \alpha_2 = \frac{U}{V_z} - \tan \beta_2$$

and by that time we already have the β_2 known so that can be done.

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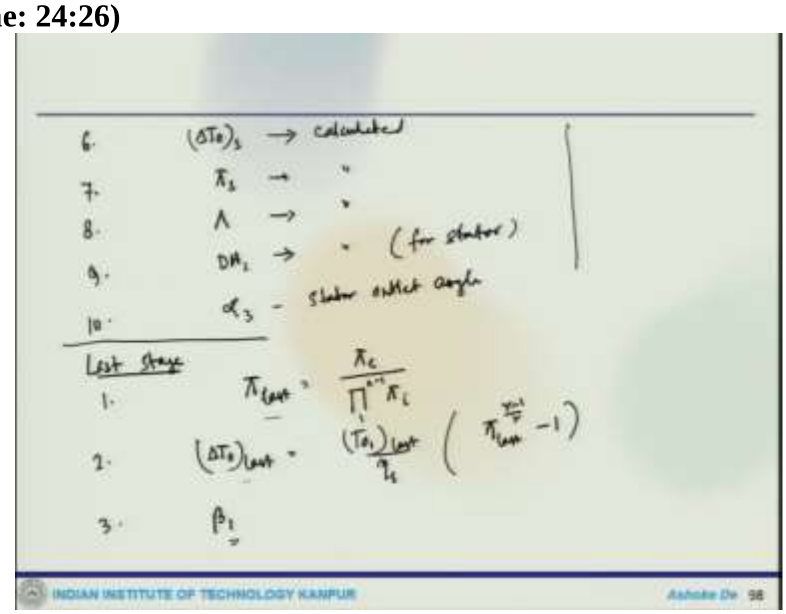

Then the state temperature rise which is T_{0s} is this can be calculated. So, once we by using some approximate work done factor and then the stage pressure ratio that is π_s which can be calculated, then at 8 degree of reaction is calculated, then 9 for approximate De haller number for stator that is calculated this is for stator and then the stator outlet angle α_3 which can be calculated so α_3 which is a stator outlet angle so all the angles then calculated.

So this particular procedure of 1 to 10 these are repeated for all the stages till the last stages except the last stages. Now when you go to last stage then what do we get? Then first thing is that the pressures ratio at the last stage that needs to be calculated though first thing that we calculate is the pressure ratio at the last stage. That means the pressure ratio of the last stage is calculated from the overall pressure ratio of the compressor.

And the product of the pressure rise which is obtained in the till the previous stage then, the corresponding temperature rise needs to be calculated. So

$$(\Delta T_0)_{last} = \frac{(\Delta T_{01})_{last}}{\eta_s}\left(\pi_{last}{}^{\frac{\gamma-1}{\gamma}} - 1\right)$$

so, then once we do this then we calculate the β_1. So and the β_1 we can calculate like this

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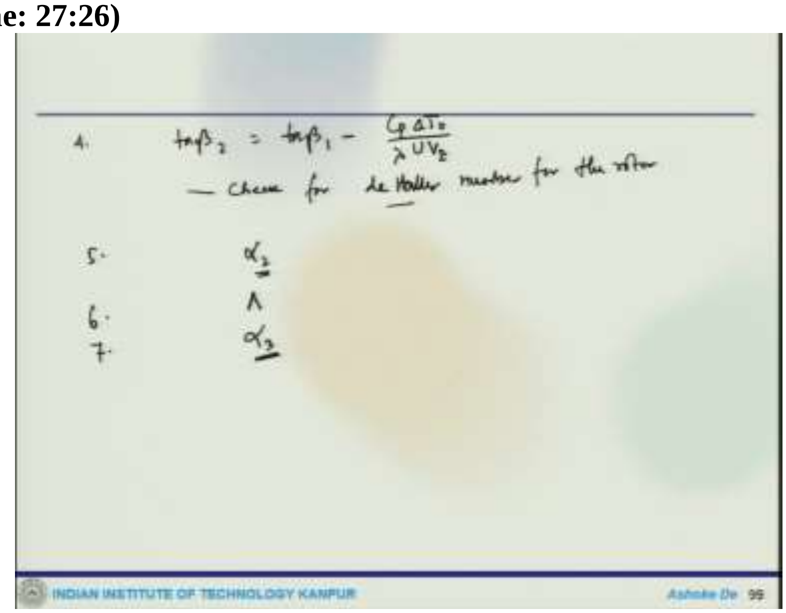

We calculate β_2 like

$$\tan\beta_2 = \tan\beta_1 - \frac{C_p\Delta T_0}{\lambda U V_z}$$

where the outlet blade angle is obtained. Now here also we have to check for de haller number for the rotor. So we will check that and once that is done then the outlet absolute angle can be calculated. So α_2 can be calculated already we have estimated that how to estimate the, these things and then we get degrees of reaction and finally we get the α_3.

So that is how and once we get that we can cross check the de haller number and all these what we have taken into consideration. Now this is how all the flow angles which are there from the so the whole idea is that we have to calculate so given the known quantity like the axial velocity and the mean velocity we need to calculate all of flow angles like alpha 1 beta 1 and the degrees of reaction and all these just to do that we go stage by stages.

So first we do the first stage where we calculate all the parameters and once we calculate all the parameters we obtain the data for the first stage and we calculate all these angles and then we move to the 2 to n-1 that means subsequent stages. First stage calculations we do independently then, using those data we do this calculation for the later stages till n-1 and once we carry out all these so we obtain all the data for the and then with those data we move to the last stage.

Where our last stage pressure ratio, last stage temperature rise and using those data we get β_1 we get β_2 and also we check the de haller number for the rotor and get α_2 degrees of reaction and finally α_3. So this is how we get all these angles for the different stages. Now we can also this is what we do to get first stage, second stage onwards till last but one stage and the last stage so, stop here and continue the discussion in the next lecture.