Introduction to Airbreathing Propolusion
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Lecture – 51
Axial Compressor (Contd.,)

So, let us continue the discussion on axial flow compression. So, now we have come across different efficiencies and difference between different efficiencies.

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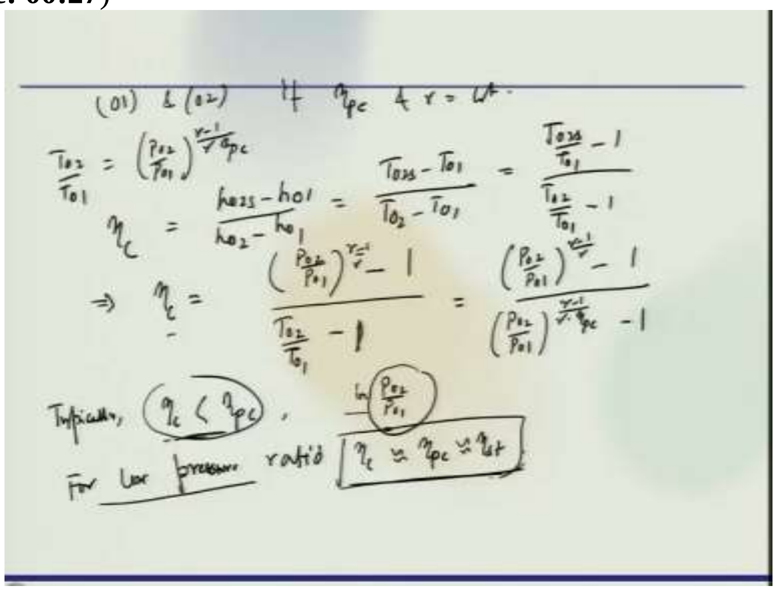

So, this is where we looked at that the relationship between adiabatic efficiency and polytropic efficiency. So, and we said that for low pressure rise that means that when the pressure rise is low this adiabatic polytropic and stage efficiency they are quite approximately same.

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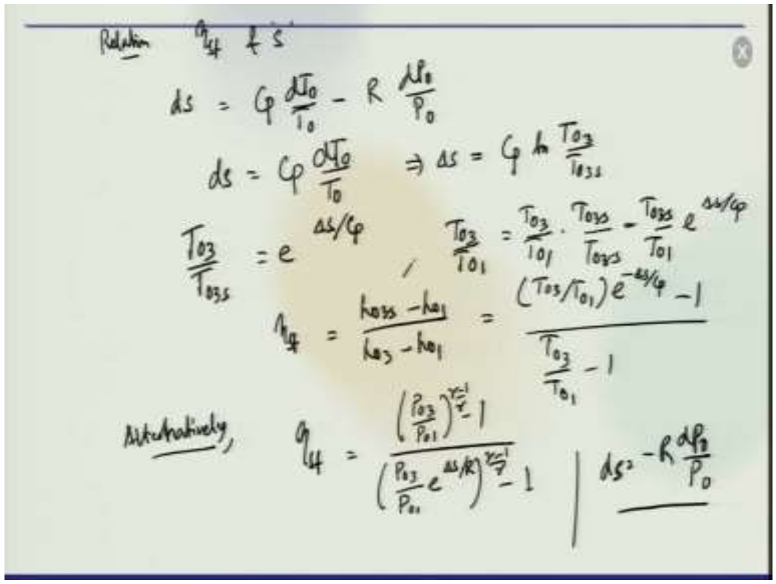

Now we can now look at the relation between stage efficiency and entropy. So, we can, so, the entropy diagram, the ds diagram that we have already drawn we can refer to that and what we can write that the

$$ds = C_p \frac{dT_0}{T_0} - R\frac{dp_0}{p_0}$$

So, it corresponds to same pressure. So,

$$ds = C_p \frac{dT_0}{T_0}$$

So, which means that

$$\Delta s = C_p \ln \frac{T_{03}}{T_{03s}}$$

So, one can write

$$\frac{T_{03}}{T_{03s}} = e^{\frac{\Delta s}{C_p}}$$

Similarly,

$$\frac{T_{03}}{T_{01}} = \frac{T_{03}}{T_{01}}\frac{T_{03s}}{T_{03s}} = \frac{T_{03s}}{T_{01}} e^{\frac{\Delta s}{C_p}}$$

So, stage efficiency would be

$$\eta_{st} = \frac{h_{03s} - h_{01}}{h_{03} - h_{01}} = \frac{\frac{T_{03s}}{T_{01}} e^{\frac{\Delta s}{C_p}} - 1}{\frac{T_{03}}{T_{01}} - 1}$$

So, further simplification or rather alternatively, one can write that stage efficiencies

$$\eta_{st} = \frac{\left(\frac{p_{03}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(\frac{p_{03}}{p_{01}} e^{\frac{\Delta s}{R}}\right)^{\frac{\gamma-1}{\gamma}} - 1}$$

So, what it happens that the losses in compressor blades row may be radially related to entropy generation by the adiabatic form which is

$$ds - R\frac{dp_0}{p_0}$$

So, that is what it does.

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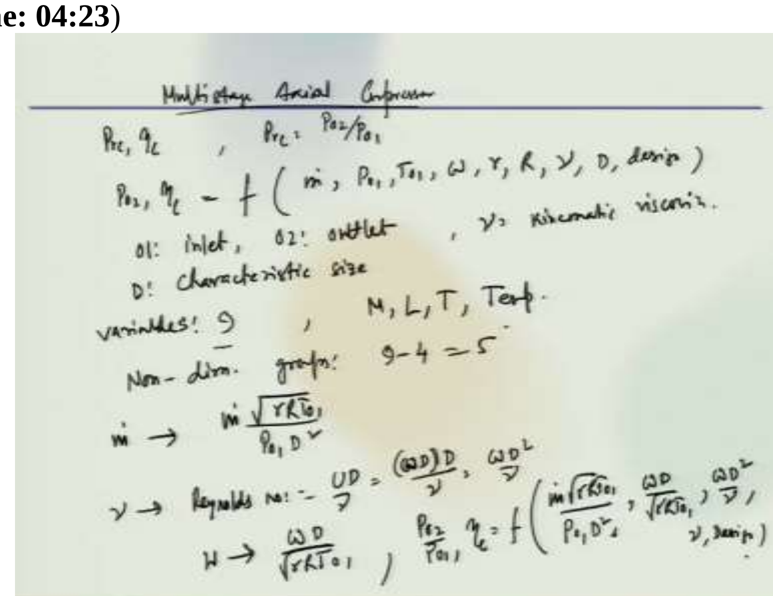

Now, we can similarly like centrifugal compression who can actually look at the characteristics performance of multistage axial compression. So, this is performance. So, what we have to do as like we have to identify the variables so, we are interested to know p_{rc} η_c have to compressor where p_{rc} is the pressure rise and so, my P_{02} and it η_c this would be function of

$$\eta_c = f(\dot{m}, p_{01}, T_{01}, \omega, \gamma, R, v, D, Design)$$

So, these are the complete specification of the geometric shape of a machine.

Our 01 is inlet 02 is outlet and υ is kinematic viscosity, D is the characteristic size or length typically this is the tip diameter of the first rotor. So, we got the total number of variables here are 9 and we have mass, length, time and temperature, these are the primary, these are the independent variables. So, we have non dimensional groups of 9 - 4 which is 5. Now, gamma and design are dimensionless. So, we need 3 more dimensional groups.

So,

$$\dot{m} \rightarrow \frac{\dot{m}\sqrt{\gamma R T_{01}}}{p_{01} D^2}$$

Or

$$\upsilon \rightarrow Reynolds\ number = \frac{UD}{\upsilon} = \frac{(\omega D)D}{\upsilon} = \frac{\omega D^2}{\upsilon}$$

So,

$$W \rightarrow \frac{\omega D}{\sqrt{\gamma R T_{01}}}$$

So, that means

$$\frac{p_{02}}{p_{01}}, \eta_c = f\left(\frac{\dot{m}\sqrt{\gamma R T_{01}}}{p_{01} D^2}, \frac{\omega D}{\sqrt{\gamma R T_{01}}}, \frac{\omega D^2}{\upsilon}, \upsilon, Design\right)$$

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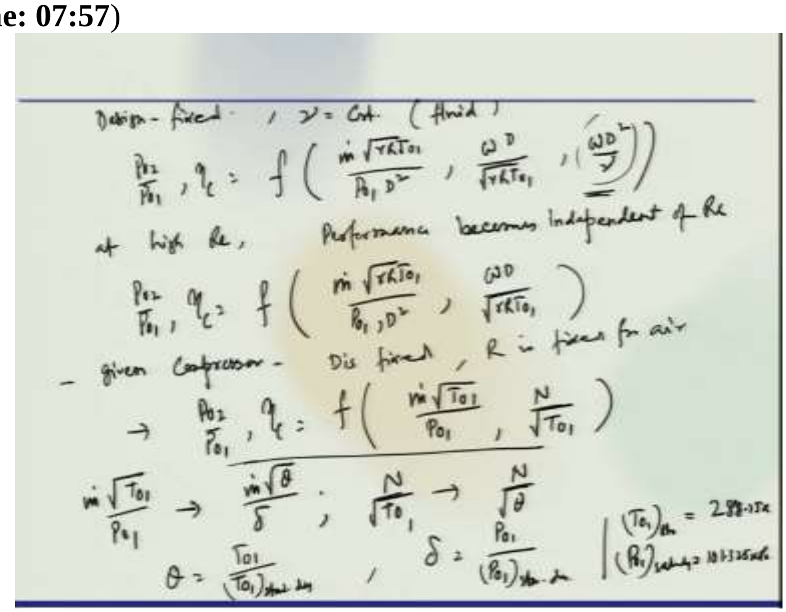

Now for a given machine design is fixed and υ is also almost constant for a particular working fluid over the temperature range. So, that reduces this

$$\frac{p_{02}}{p_{01}}, \eta_c = f\left(\frac{\dot{m}\sqrt{\gamma R T_{01}}}{p_{01} D^2}, \frac{\omega D}{\sqrt{\gamma R T_{01}}}, \frac{\omega D^2}{\upsilon}\right)$$

so typically the compressor operates at very high Reynolds number. So, at that range performance is almost independent of Reynolds number. So, once we say that so at high Re the performance becomes independent of Re.

So, this guy drops out and we end up with the group which is function of

$$\frac{p_{02}}{p_{01}}, \eta_c = f\left(\frac{\dot{m}\sqrt{\gamma R T_{01}}}{p_{01} D^2}, \frac{\omega D}{\sqrt{\gamma R T_{01}}}\right)$$

Since for a given compressor for D is fixed, R is fixed for here. So, we can drop them from this non dimensional group and what we can retain or rather we have is function of

$$\frac{p_{02}}{p_{01}}, \eta_c = f\left(\frac{\dot{m}\sqrt{T_{01}}}{p_{01}}, \frac{N}{\sqrt{T_{01}}}\right)$$

So, we can further modify the choice by

$$\frac{\dot{m}\sqrt{T_{01}}}{p_{01}} \to \frac{\dot{m}\sqrt{\theta}}{\delta}$$

And

$$\frac{N}{\sqrt{T_{01}}} \to \frac{N}{\sqrt{\theta}}$$

So, where we define

$$\theta = \frac{T_{01}}{(T_{01})_{standard\ day}}$$

And

$$\delta = \frac{p_{01}}{(p_{01})_{standard\ day}}$$

where typically that T_{01} standard day would be 288.15 kelvin and P_{01} standard day would be 101.325 kPa. So, this choice is now brings back to these 2 non dimensional group now, can take care of the variation and the main conditions also compared to this standard atmosphere.

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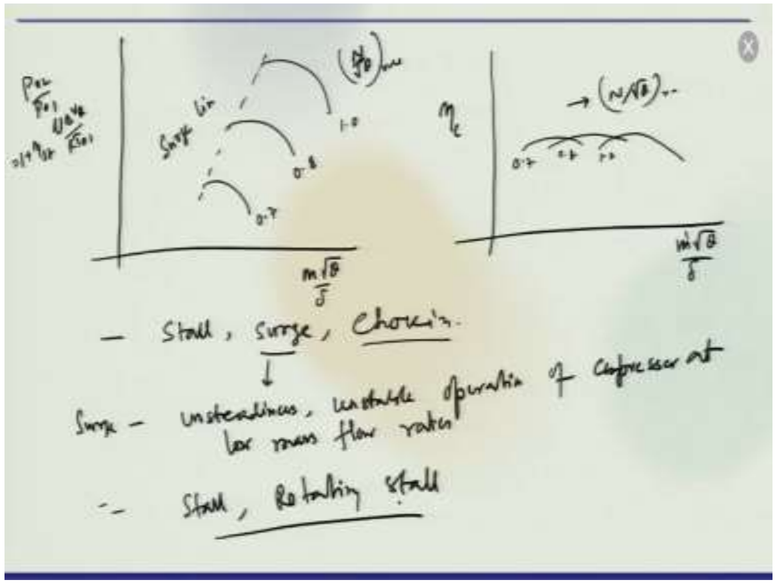

Now, once we plot these curves, let us say $\frac{p_{02}}{p_{01}}$ which one can also write $1 + \eta_{st}\frac{U\Delta V_{\theta}}{RT_{01}}$ and here $\frac{\dot{m}\sqrt{\theta}}{\delta}$. This is what we get and this line is the, so this is the variation of, so these are 0.7, 0.8 roughly 1. So, this is my surge line and when I look at eta c what says m dot delta. So, they are like this, this is 0.7 0.8 1. So, that also variation with this. So, performance characteristics of an axial flow compressor is same as the centrifugal compressor.

So, they also suffers from stall, surge, choking. So, in compressor choking flow does not necessarily reach to the sonic velocity. So, keep that in mind. This is different condition compared to compressible flow, when you talked about the typical convergent divergence nozzle when the flow choking means it reaches that sonic condition but in compressor that does not necessarily mean it reaches sonic velocity as we have talked earlier, the possibility of flow separation is higher at later stages.

Now, flow separation may reduce the effective flow passage says that the later passage work as throttling valve and also in the surge, it is violent, unstable condition or rather operation of the compressor at low mass flow rate. So one can think about this is unsteadiness. It is unstable operation of compressor at low mass flow rates. So, the best operation point is very close to the surge point.

Now as a designer one has to prevent the surge and also extend this surge limit by shifting the surge line to the left that means this line it should be shifted more towards the left so that we have a better operational line, as in case of centrifugal compressor rotating stall or stall is also present in axial flow compressor. So, you have stall you have rotating stall. So, those things what we have already discussed while talking about the centrifugal compressor they also do present in axial compression. So, obviously this possible flow separation occurs at the high or very low angle of attacks.

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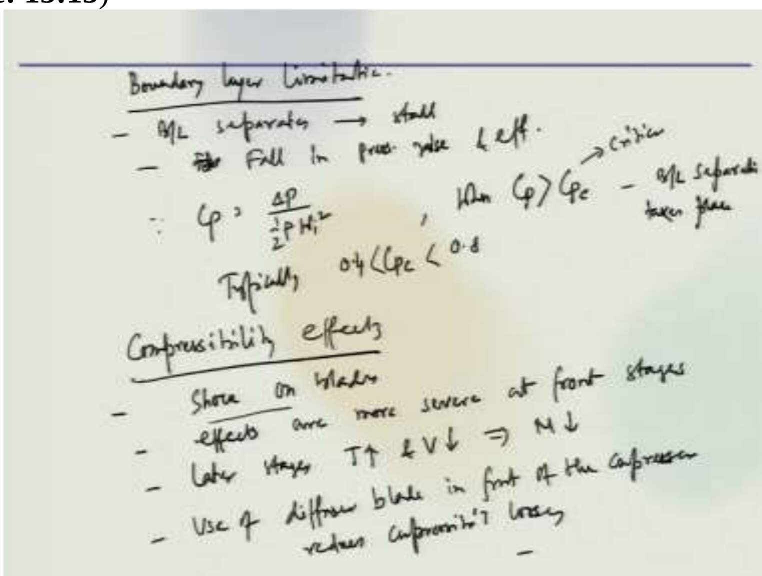

Now, there would be boundary layer limitation. So, when boundary layer separates, it causes the compressor to stall. So, separation can cause fall in pressure rise and efficiency. So, there would be fall in pressure rise and efficiency. So, boundary layer separation can occur at end walls or at the blade surface. The blade surface can be passages can be considered to be diffuser here has to move against a negative pressure gradient. So, boundary layer separation can occur same

$$C_p = \frac{\Delta p}{\frac{1}{2}\rho W_i^2}$$

Now, when $C_p > C_{pc}$ which is the critical one. So, boundary layer separation takes place typical value of C_{pc} 0.4 and this. So, now one can think about how these can be prevented by proper blade design and also or some section in blades. Now, there could be another factor which is the compressibility effect, this is also possibly compressibility effects. So, this could be there. So, once we have high speed flows which could be detrimental to the blade transonic and supersonic flows induce shock.

So, shock on blades this will have excessive lead to excessive performance loss and shock induced flow separation. So, when there would be high speed flows like transonic and high speed or supersonic flows, there could be oblique that could have shock induced on the blades which may lead to the shock induced separation or something like that. So, these effects are more severe at front stages. So, this kind of issues which would be the shock induced separation or blade or stresses are loading on blade would be more severe at the front stages.

In later stages what happens, the temperature goes up and velocity goes down which means the Mach number also goes down and compressibility effects become insignificant. So, obviously there is always a limit to the speed of an aircraft due to compressibility effect. So, use of diffuser blade in front of the compressor reduces some of the compressibility. So, use of diffuser blade in front of the compressor reduces compressibility process. So, these are the different factors or effects that one can think about considering them in the system.

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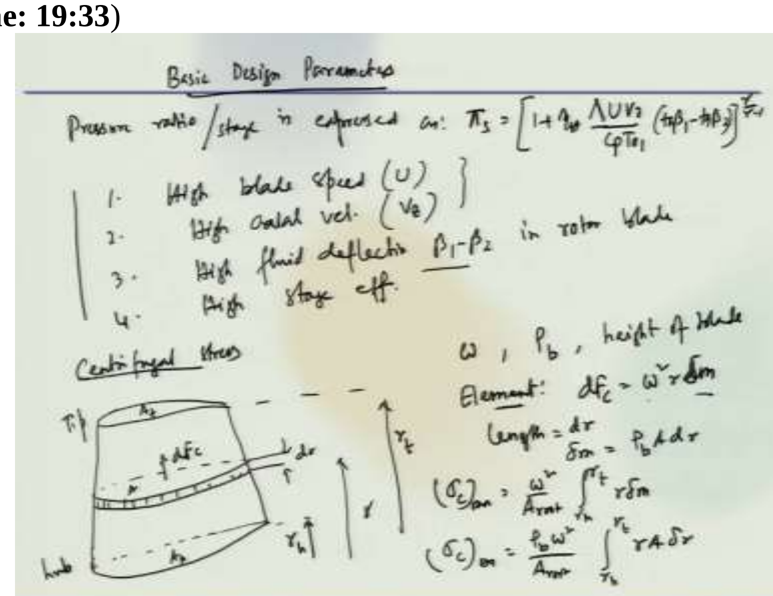

Now, little bit more on to the design aspect. So, if we look at some of the basic design parameters we have now the pressure ratio per stage is expressed as

$$\pi_s = \left[1 + \eta_{st}\frac{\Lambda U V_z}{C_p T_{01}}(\tan\beta_1 - \tan\beta_2)\right]^{\frac{\gamma}{\gamma-1}}$$

So, to obtain high pressure ratio per stage, it is needed to have. So, if you want high pressure ratio number 1 high blade speed that is U, second high axial velocity that is V_z, third high fluid deflection that is $\beta_1 - \beta_2$ in that rotor blade.

Fourth high stage efficiency. So, these are some of the requirement, but one can see when you increase the rotational speed which is obviously these are there is a limit to that what one can do because there is a structural load of the stresses would increase in the rotor hub. So, further increase in the rotational of the axial velocities that means U and V_z is also limited by the tip Mach number. So, compressor in the old days had a maximum tip Mach number out typically less than point eight, which has nearly been doubled for the present transonic compressor.

So, in the old days, this Mach number tip Mach number used to be 0.8 now it is 1.4 to 1.7 also, this fluid deflection angle has upper limit which is determined by the diffusion factor and stage efficiency has several loss limitations. Now, just to look at these kinds of things when we talk about high blade speed or axial velocity, we have different loads some of them like one is the centrifugal stress. So, what happens?

So let us say if we have a rotor blade like this and so let us say surface. So this is platform area. dF_c, this is hub, this is tip, hub. Now, this is dr, this is r, this is r t. So, centrifugal stage depend on the rotational speed omega blade materials rho b and height of blade those are the maximum centrifugal stage occurs in the root. So, now as you can see, so, the centrifugal force which arises from this particular element.

So, that element if we look at it

$$dF_c = \omega^2 r \delta m$$

where omega is the rotational speed, r is the radius of the blade element and having a mass of delta m. Now, if it is the length is dr then my

$$\delta m = \rho_b A dr$$

Then the maximum centrifugal stress at the rotor is summation of the centrifugal forces of all the elements of the blade from the hub to tip. So, that can be divided by the cross sectional area. So, one can write

$$(\sigma_c)_{max} = \frac{\omega^2}{A_{root}} \int_{r_h}^{r_t} r\delta m$$

similarly,

$$(\sigma_c)_{max} = \frac{\rho_b \omega^2}{A_{root}} \int_{r_h}^{r_t} rA\delta r$$

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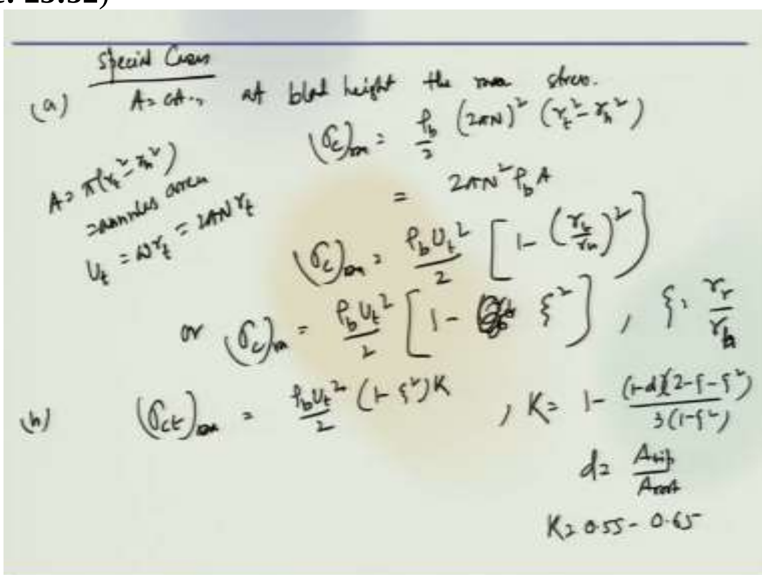

Now, you can have some special case you can consider some special cases for example, let us say the blade has constant area A is constant. And then at blade height the maximum stress would be

$$(\sigma_c)_{max} = \frac{\rho_b}{2}(2\pi N)^2(r_t^2 - r_h^2)$$

$$= 2\pi N^2 \rho_b A$$

$$(\sigma_c)_{max} = \frac{\rho_b U_t^2}{2}\left[1 - \left(\frac{r_t}{r_h}\right)^2\right]$$

$$(\sigma_c)_{max} = \frac{\rho_b U_t^2}{2}[1 - \xi^2]$$

where ξ is define the root to hub to radius ratio.

Now, the other case could be for linear variation of the cross section area with radius or $(\sigma_c)_{max}$ could be

$$(\sigma_c)_{max} = \frac{\rho_b U_t^2}{2}[1 - \xi^2]K$$

$$K = 1 - \frac{(1-d)(2-\xi-\xi^2)}{3(1-\xi^2)}$$

So, these are the tip to root area ratio and typical values of K goes between 0.55 to 0.65 for tapered blades.

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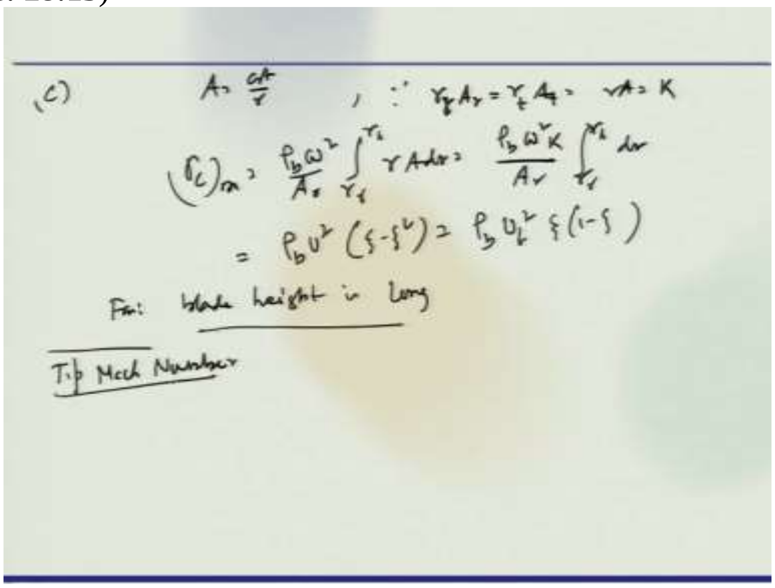

And other case could be area or the area is inversely proportional to the radius. So, this would be constant A / r or inversely proportional to the thing since

$$r_r A_r = r_t A_t = rA = K$$

then my $(\sigma_c)_{max}$ would be

$$(\sigma_c)_{max} = \frac{\rho_b \omega^2}{A_r} \int_{r_h}^{r_t} rA\delta r = \frac{\rho_b \omega^2 K}{A_r} \int_{r_h}^{r_t} \delta r$$

$$= \rho_b U^2(\xi - \xi^2) = \rho_b {U_t}^2 \xi(1-\xi)$$

So, the same equation can be applied for both fan and compressor for fan the blade height is long.

So, which develops large tip rotational speed and the hub to tip ratio is small. Thus the tensile stresses are expected at blade root. So, the same thing one can apply to both compressor and the fan. So, now, the other parameter that one can look at is the tip Mach number. This is also quite important because the most of the axial flow compressors are operated at transonic range where there could be a tendency of shock. So, the maximum Mach number in the axial flow compressor occurs are the blade tip.

So, we should look at this parameter also in details, what could be the blade tip Mach number and all this. So, we will just stop here and continue this discussion of the blade tip Mach number and other factors in the next section.