Introduction to Airbreathing Propulsion
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Lecture – 50
Axial Compressor (Contd.,)

Okay so let us continue the discussion of the degree of reaction in axial flow compressor.

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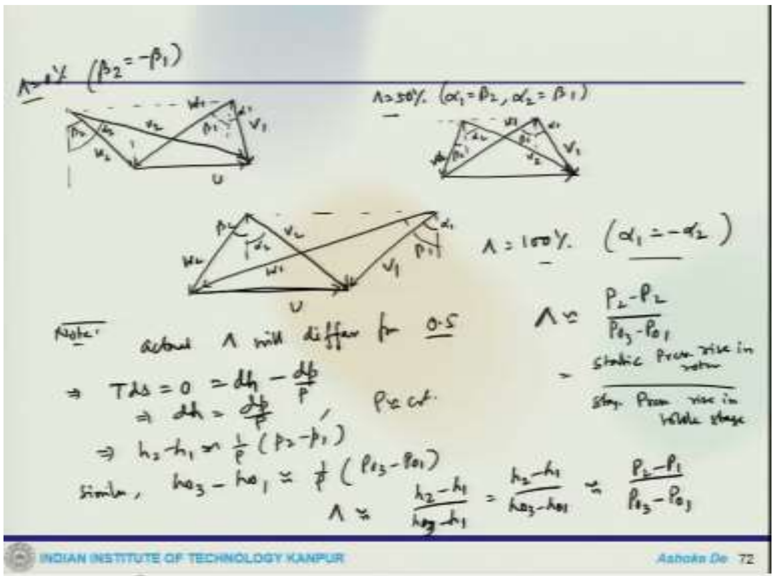

And this is what we stopped at that the different velocity triangle for different degrees of reaction and so these are one can see how the blade angles that varies for 0% degrees of reaction, 50% and 100% and what would be the balding pattern or the angles that one get for the; so this is very important aspect of; when you look at the blade design and talking about the degrees of reaction because these are very important in the sense this dictate the rotor blade design and loading on rotor blade and things like that.

So another thing just to note here is that we have assumed reversible work done in the stage, so work done factor is unity but due to the presence of irreversibilities actual degrees of reaction will differ from 0.5, okay. So for an axial compressor stage in which the change in density is small so this Λ would be approximated as

$$\Lambda = \frac{p_2 - p_1}{p_{03} - p_{01}}$$

so which is essentially the static pressure rise in the rotor by stagnation pressure rise in whole stage, okay.

So this could be very easily shown, I mean it one can just do by assuming just like a flow is approximately isentropic in the rotor, so if you write the

$$Tds = 0 = dh - \frac{dp}{\rho}$$

Now since ρ is pretty much constant one can write

$$dh = \frac{dp}{\rho}$$

$$h_2 - h_1 = \frac{1}{\rho}(p_2 - p_1)$$

and similarly

$$h_{03} - h_1 = \frac{1}{\rho}(p_{03} - p_1)$$

so we can get a

$$\Lambda = \frac{h_2 - h_1}{h_{03} - h_{01}} = \frac{p_2 - p_1}{p_{03} - p_{01}}$$

okay, so which is that means the degrees of reaction can be also estimated.

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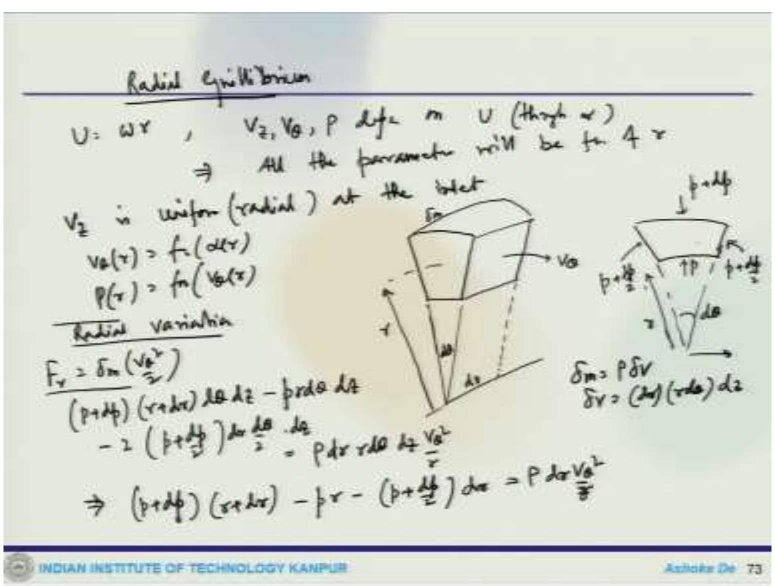

Now we will move to another theory called the radial equilibrium theory. So, so far the dimensions were in mean radius, we have neglected the variation of the proportion but our $U = \omega r$ and our $V_z, V_\theta\ and\ p$ depends on U through α. So all the parameters would be essentially all the parameters will be function of r. So V_z is uniform that means radial at the inlet, so large variation can developed of sub-stages so V_θ would be function of $\alpha(r)$ and P(r) would be function of $V_\theta(r)$ or rather.

So we can check this element for the radial equilibrium theory, okay. So this is dz, this is dθ, now this distance is r, this is V_θ, that is δm. So in a another schematic, if you look at in that viewpoint, so one can write this side is $p + \frac{dp}{2}, p + dp$ this is P, this is dθ, so this distance would be r, okay. So we can find out this radial variation. Now consider a small fluid element, this is small fluid element of mass δm with tangential velocity component V_θ.

Now the force would be

$$F_r = \delta m \left(\frac{{V_\theta}^2}{r}\right)$$

Now from the force balance of this element which is here, we can write that

$$(p + dp)(r + dr)d\theta\ dz - prd\theta\ dz - 2\left(p + \frac{dp}{2}\right)dr\frac{d\theta}{2}dz = \rho dr\ rd\theta\ dz\ \frac{{V_\theta}^2}{r}$$

okay. So essentially here the centripetal force is here we can write

$$\delta m = \rho \delta V$$

$$\delta V = (dr)(rd\theta)dz$$

So forces on the side faces in radial minus axial plane considering the average pressure. So once we simplify this we can write

$$(p + dp)(r + dr) - pr - \left(p + \frac{dp}{2}\right)dr = \rho dr\ \frac{{V_\theta}^2}{r}$$

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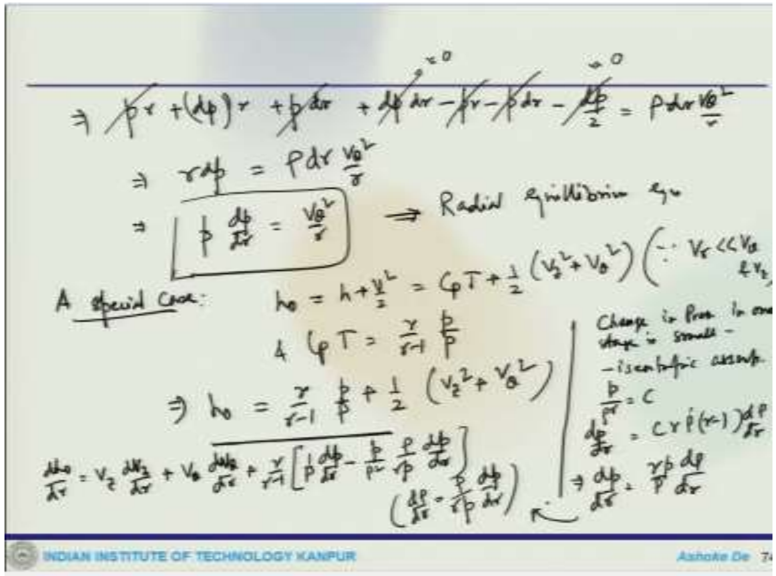

So now further simplification would give

$$rdp = \rho dr \frac{{V_\theta}^2}{r}$$

$$\frac{1}{\rho}\frac{dp}{dr} = \frac{{V_\theta}^2}{r}$$

so that is what your radial equilibrium equation, okay, so actual velocity distribution must satisfy this behavior, so this is important.

Now one can again consider a special case. What it could be? Is that the any radial direction r the stagnation enthalpy is given by

$$h_0 = h + \frac{V^2}{2} = C_pT + \frac{1}{2}\left({V_z}^2 + {V_\theta}^2\right)$$

Now since

$$V_r < V_\theta \; \& \; V_z$$

what we can write

$$C_pT = \frac{\gamma}{\gamma - 1}\frac{p}{\rho}$$

so which turns out to be

$$h_0 = \frac{\gamma}{\gamma - 1}\frac{p}{\rho} + \frac{1}{2}\left({V_z}^2 + {V_\theta}^2\right)$$

So once we differentiate this one with respect to r, what we will get? So before we do this another thing which one can write that the change in pressure is one stage is small. So change in pressure in one stage is small.

$$\frac{dh_0}{dr} = V_z\frac{dV_z}{dr} + V_\theta\frac{dV_\theta}{dr} + \frac{\gamma}{\gamma-1}\left[\frac{1}{\rho}\frac{dp}{dr} - \frac{p}{\rho^2}\frac{\rho}{\gamma p}\frac{dp}{dr}\right]$$

Now this we write

$$\frac{d\rho}{dr} = \frac{\rho}{\gamma p}\frac{dp}{dr}$$

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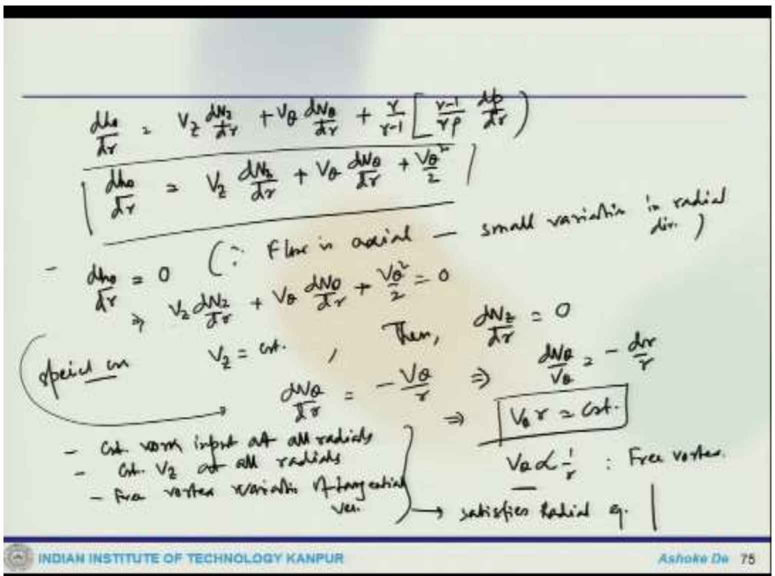

So if you use that what we will get

$$\frac{dh_0}{dr} = V_z\frac{dV_z}{dr} + V_\theta\frac{dV_\theta}{dr} + \frac{\gamma}{\gamma-1}\left[\frac{\gamma-1}{\gamma p}\frac{dp}{dr}\right]$$

$$\frac{dh_0}{dr} = V_z\frac{dV_z}{dr} + V_\theta\frac{dV_\theta}{dr} + \frac{{V_\theta}^2}{r}$$

So this is another way one can find out. Now let us assume apart from the regions near the walls of the annulus the stagnation enthalpy or other temperature will be uniform across the annulus 2 into 2 compressor.

So what that happens

$$\frac{dh_0}{dr} = 0$$

since flow is axial or rather predominantly axial, so small variation in radial direction, so that means constant work input and all radial hence h naught will progressively increase in axial direction. So this is

$$V_z \frac{dV_z}{dr} + V_\theta \frac{dV_\theta}{dr} + \frac{{V_\theta}^2}{r} = 0$$

So again special case if V_z is maintained constant across the annulus there then we can write

$$\frac{dV_z}{dr} = 0$$

so which get us back from here is that

$$\frac{dV_\theta}{dr} = -\frac{V_\theta}{r}$$

So which means

$$\frac{dV_\theta}{V_\theta} = -\frac{dr}{r}$$

so that gives here $V_\theta r = constant$ after integration, so which means tangential velocity is inversely proportional so V_θ would be inversely proportional to the r so condition known as free vortex, constant work input all radials, constant V_z all radials, free vortex variation of tangents and velocity. So these are the situation so that means constant work input at all radials, constant V_z at all radials, free vortex variation of tangential velocity. So these are all satisfying, so this satisfies radial equilibrium. Also this V_θ into r constant also satisfies the radial equilibrium.

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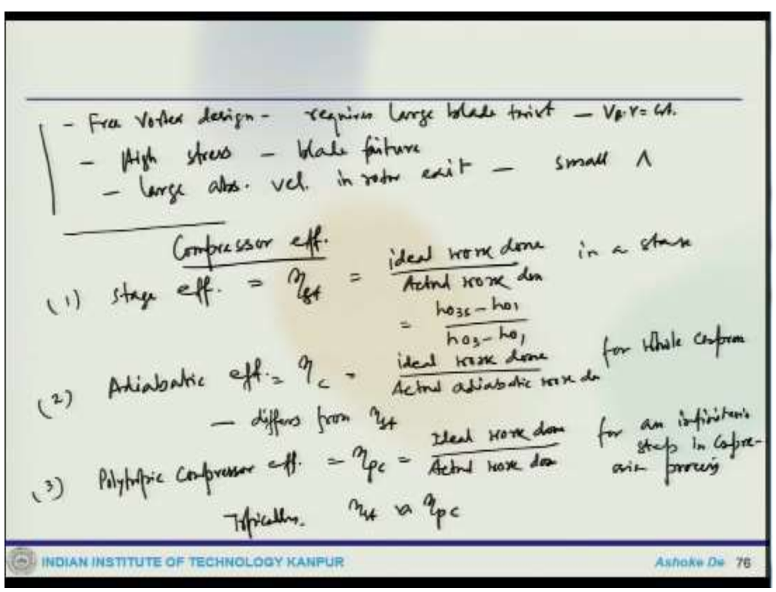

Now what important is that when we have free vortex design this requires large blade twist. So this requires large blade twist in order to maintain $V_\theta * r = constant$, so that means high structural stress which may lead to blade failure, large absolute velocity in the rotor exit so that could be another issue so that means small pressure rise in rotor which means small degrees of reaction but widely used though in axial turbines, so free vortex design widely used in axial turbine compared to compressor.

Now with that note we will move it move to the different kind of efficiencies that one can define for compressor. So there are three different kinds which actual and ideal work one can do one is the stage efficiency which is η_{st} which is ideal work done divided by actual work done in a stage, so that one can write

$$\eta_{st} = \frac{h_{3s} - h_{01}}{h_{03} - h_{01}}$$

Second it could be adiabatic efficiency which is η_c again idle work done divided by actual adiabatic work done for whole compressor, okay. So this was discussed in already in cycle analysis.

So this differs significantly, so differs from stage efficiency, third could be polytropic compression efficiency which is η_{pc} that means idle work done divided by actual work done for an infinitesimal step in compression air and process, so this is typically your stage efficiency maybe with the polytropic efficiency. So we can always find out a relationship between η_{pc} and η_c.

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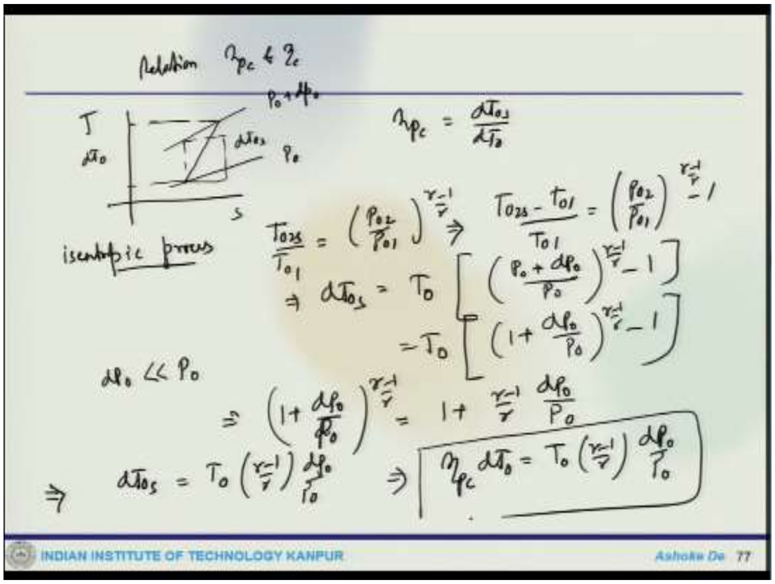

So let us consider a TS diagram; this is stage so goes from here to there let us say P_0, this would be P_0+dP_0, okay so that is dr_0, dT_{0s} , so there is an incremental pressure rise from p_0 to $p + dp_0$ so my

$$\eta_{pc} = \frac{dT_{0s}}{dT_0}$$

so we are trying to find out the relation between η_{pc} and η_c. Now for isentropic process what will happen

$$\frac{T_{02s}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_{02s} - T_{01}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1$$

So

$$dT_{0s} = T_0\left[\left(\frac{p_0 + dp_0}{p_0}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$dT_{0s} = T_0\left[\left(1 + \frac{dp_0}{p_0}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

now if we use the binomial expression for

$$\left(1 + \frac{dp_0}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = 1 + \frac{\gamma - 1}{\gamma}\frac{dp_0}{p_0}$$

$$dT_{0s} = T_0\left[1 + \frac{\gamma - 1}{\gamma}\frac{dp_0}{p_0} - 1\right]$$

$$dT_{0s} = T_0\frac{\gamma - 1}{\gamma}\frac{dp_0}{p_0}$$

$$\eta_{pc}dT_0 = T_0\frac{\gamma - 1}{\gamma}\frac{dp_0}{p_0}$$

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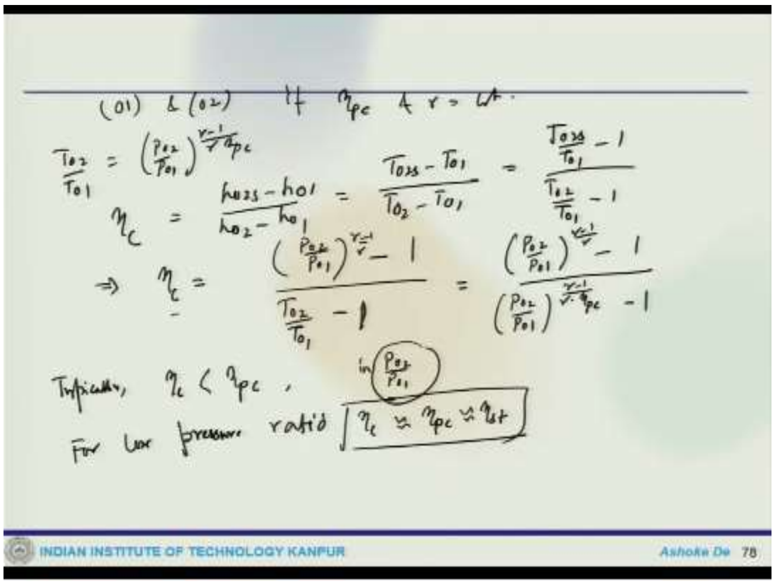

Now when the compressor between two stagnation stages of one and two if η_{pc} and γ constant then this

$$\frac{T_{02}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma\,\eta_{pc}}}$$

so

$$\eta_c = \frac{h_{02s} - h_{01}}{h_{02} - h_{01}} = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}} = \frac{\frac{T_{02s}}{T_{01}} - 1}{\frac{T_{02}}{T_{01}} - 1}$$

so we get

$$\eta_c = \frac{\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{T_{02}}{T_{01}} - 1} = \frac{\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma\,\eta_{pc}}} - 1}$$

now this is for the relationship between adiabatic efficiency and polytropic efficiency. Now typically $\eta_c < \eta_{pc}$, so the difference increases with the increase in $\frac{p_{02}}{p_{01}}$.

So with this; if there is an increase in pressure ratio the differences also increases, for low pressure ratio η_c is pretty much equal to the polytropic efficiency which would be also same for stage efficiency. So this is an important conclusion or rather important information that one should keep in mind is that when you have a low pressure ratio then these three different efficiencies which we have got here stage efficiency, adiabatic efficiency or polytropic efficiency they turn out to be same.

So one can look at the textbook like Hill Peterson or any other textbook for this proof for low pressure. But here adiabatic efficiency would be lower than the polytropic efficiency and this; typically, this is what happens but this difference between the adiabatic efficiency and polytropic efficiency will increase with the increase in pressure ratio. So if the pressure ratio actually increases then this difference also increases. So we will look at the other relationship in the subsequent lecture and we will stop it here.